Int. Agrophysics, 1994, 8, 113-130

A RESPONSE SURFACE METHODOLOGY FOR DETERMINATION OF ENGINEERING PROPERTIES OF SOIL *IN SITU*

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A b s t r a c t. Since in situ soils seldom behave like remolded laboratory soils or disturbed field samples, it is important to 'identify' or 'calibrate' the engineering properties of field soil by means of in situ tests. A response surface methodology based on an orthogonal regression in the parameter space has been developed to 'identify' engineering properties of any material based on in-situ tests. The proposed methodology was verified for the case of a two parameter hypo-elastic model for soil as well as a complex five parameter model for soil which includes nonlinear material behavior in elastic range, yield based on Drucker-Prager yield criteria and associated plastic flow upon yield.

K e y w o r d s: soil, engineering properties, surface methodology

INTRODUCTION AND REVIEW OF LITERATURE

One of the challenges in the design of an off-road vehicle is to equip it with a traction device (tire or track) which can develop high traction efficiently (i.e. optimum tractive efficiency) while deterring soil compaction. Even an increase of one percentage point in the tractive efficiency leads to an annual savings of over 100 million liters (about 25 million gallons) of fuel in U.S. alone [7]. On the other hand, soil compaction has been recognized as a worldwide problem with serious implications on agricultural sustainability [10]. Although, certain amount of soil compaction may even be desirable for some crops under certain environmental conditions (optimum soil compaction), excessive soil compaction can lead to diminished soil porosity, reduced water infiltration, increased resistance to root penetration, increased tillage energy requirements, decreased biological activity, and a reduction in crop yield [11]. A necessary pre-requisite for the successful design of a traction device is a sound mathematical model for the soil-traction interaction process. This interaction is an extremely complex, dynamic process. A key ingredient of such a model is a constitutive relationship which describes the stress-strain behavior for soil. Schafer *et al.* [13] stated that an accurate description of soil constitutive relationship is necessary for the integrity and robustness of the model. Soil is perhaps one of the most complex material from an engineering point of view [4].

Numerous constitutive models are currently available for soils. Among these are the elasticity models, higher order nonlinear elasticity models, hypoelasticity models, plasticity models and visco-plasticity models. Desai [4], Desai and Siriwardane [5] and Chen and Baladi [3] have discussed these models and their applicability to a specific loading situation in detail. These constitutive models require material parameters which describe the elastic behavior of soil, onset of yield and subsequent plastic flow, material hardening or softening rules etc.

Typically these parameters are determined using laboratory tests. Often remolded soils are employed in the laboratory tests which may not behave like field soils. Even if field samples are obtained, these samples undergo disturbances during excavation and testing, and may not behave like *in situ* soil under actual loading conditions in field. It is preferable to determine the soil material parameters based on undisturbed *in situ* tests. The technique of obtaining material parameters based on actual system response is called 'back analysis', 'inverse solution', 'identification', or 'calibration procedure'. The process of 'calibrating' actual field response to model behavior is expected to 'identify' the material parameters which can accurately predict system response in subsequent analysis which utilize the same constitutive model.

The back analysis technique has been successfully used in geomechanics in studying tunnelling problems in rocks and in investigating settlement problems [15]. If a closed form solution exists for the underlying differential equation describing the physical problem, then back analysis to obtain the material parameters involves optimizing the difference between the analytical and experimental responses. However, most real life problems in geomechanics are either geometrically or materially nonlinear, and an analytical solution may not exist. In such cases a numerical procedure such as a finite element method (FEM) may be used to obtain solutions to the governing differential equation. When finite element analysis is used, back analysis may take one of the two forms: 1) inverse method, 2) direct method.

In the inverse method nodal values of displacements and stresses obtained by a FEM technique are used as known boundary conditions and the unknown displacements and stresses are eliminated from the global matrix equation by reduction. This inverse technique is quite sensitive to experimental error and may not converge at all in some cases [2,8,12]. The direct approach results in more accurate parameter values. In the direct method, nodal values of the response are computed using a finite element method for a set of assumed parameter values. The actual values of response at the same nodes can be obtained by field or *in situ* tests. The difference between the finite element predictions and experimentally measured values at these nodes are optimized to obtain soil parameters. The direct method can be computationally very expensive since at each iteration a new FEM analysis with updated parameters needs to be carried out [2].

Our objective is to develop a methodology to determine material properties of soil based on a given constitutive law using *in situ* field tests. A response surface will be built based on the outcome of a FEM analysis using an orthogonal regression technique. The response surface will be a function of unknown material parameters. This response surface will be used to predict the response corresponding to the experimental values (i.e. at the same load and nodal point). The main advantage of our technique is that once the response surface is created using FEM analysis, there is no need to go back to the FEM analysis. During the optimization technique only the response surface is used. This approach is expected to make this technique computationally very efficient.

MATHEMATICAL MODELING

Response surface development

Let us consider a general material constitutive model for soil (or any other material) consisting of m parameters: $p_1, p_2, p_3, ..., p_m$. For example, if we select a nonlinear constitutive model with extended Drucker-Prager yield criteria and associated flow rule, then six parameters will be involved [1]. These parameters are p_1=logarithmic bulk modulus, κ; p_2= Poisson's ratio, ν; p_3= yield surface shape factor (i.e. related to the third invariant of stress), K; p_4=cohesion, c; p_5= internal angle of friction, Φ; P_6=initial void ratio, e. The last parameter, e is really related to initial stress condition. The response of a system to applied load depends on its geometry, material properties and the load itself. If the applied load and the geometry are fixed (i.e. for a given geometry and loading), the system response is a function of material constants used in the constitutive equation. There is a function $\Phi = \Phi\ (p_1, p_2, p_3, \dots p_m)$ which represents the system response as the material properties used in the constitutive equation are changed. Unless the underlying differential equation describing the response of the real system is linear, this function is seldom known explicitly. One of the goals of this study is to find an approximate representation for this real response, Φ. This approximation to the real response is termed the response surface, F in this study. One convenient way of determining the response surface F is to determine the variation of F as one of the material parameter, p_i is changed while all other parameters are held constant. Let this response function for the single variable p_i be $f_i(p_i)$. If we repeat this process for each of the m material parameters (i.e. for $i=1,2$ ......, m), then one easy way of obtaining the response surface is simply to multiply these component equations, $f(p_i)$, i.e.:

$$\Phi \cong F = Cf_1(p_1)f_2(p_2)f_3(p_3) \dots f_m(p_m) \tag{1}$$

where F - response surface, f_i - a component equation which is a function of parameter p_i only, C - constant.

In general such a representation is accurate only in a small region due to geometric and/or material nonlinearities in the system. The error is expected to be small if the range of p_i is small for each of the m parameters.

Thus the process of building the response surface requires holding all relevant factors except parameter p_i constant (i.e., geometry, loading, all other material properties p_j, $j=1,2$ .... m but $j \neq i$) and determining the component equation $f(p_i)$. Once all the component equations are determined, Eq. (1) can be used to build the response surface. It should be recognized that for each given geometry and loading there will be one response surface. In the case of plate sinkage tests, for a given plate size and load level there will be a response surface. Since there are m unknown parameters, at least m field measurements are needed to solve for these m parameters. In practice, it is preferable to have more than m points (i.e. n>m) so that the m parameters can be determined by an optimization scheme. Since each unreplicated *in situ* measurement corresponds to a given geometry and loading, each of these experimental values correspond to a point (or contour) on one response surface. Thus each of the n unreplicated measurements will correspond to a point (or contour) on one of the n distinct response surfaces. Note that more than one observations at a given geometry and loading refer to the same point (or contour) on a response surface that corresponds to that geometry and loading. Thus replicates do not provide additional equations to solve for the parameters, but help in controlling experimental error. Suppose we have n distinct combination of geometry and load level there will be n response surfaces, F_i, $i=1,2$ .....n. From Eq. (1) we get:

$$F_1 = C_1 f_{11} f_{12} \cdots f_{1m}$$

$$F_2 = C_2 f_{21} f_{22} \cdots f_{2m}$$

$$\vdots$$

$$F_n = C_n f_{n1} f_{n2} \cdots f_{nm} \tag{2}$$

where f_{ij} is the component equation corresponding to response i and parameter p_{ij} and C_i is the constant corresponding to the same response surface i. Since each of the same response surface i. Since each of the material parameter has its own range, some properties such as Poisson's ratio, ν vary in a very narrow range (0.0 to 0.5) whereas others such as Young's modulus, E may vary over a very large range (thousands of kPa). From the point of optimization as well as orthogonal regression, it is preferable to map each of the parameter to the same range through scaling. Each of the unknown parameter was nondimensionalized and mapped to vary from -1 to +1 by the following transformation:

$$p'_i = \frac{2(p_i - \bar{p}_i)}{p_{i\,max} - p_{i\,min}} \tag{3}$$

where p'_i - nondimensional value of parameter i, $\bar{p}_i$ - mid point value of parameter i, $p_{i\,max}$ - upper bound value of parameter i, $p_{i\,min}$ - lower bound value of parameter i.

The value of the mid point is zero, upper bound is 1 and the lower bound is -1 for each of the nondimensionalized parameter.

Let f'_{ij} be the nondimensionalized component equation corresponding to the nondimensionalized parameter p'_j and test condition i. The relationship between f'_{ij} and f_{ij} is given by:

$$f'_{ij} = \frac{f_{ij}}{\bar{\bar{F}}_i} \tag{4}$$

where $\bar{\bar{F}}_i$ - computed value of the response surface F_i for test i when all the parameters are set equal to the mid point value of zero.

Moreover, it is convenient if we nondimensionalize the system response to avoid numerical problems in the analysis. The nondimensionalized response surface is given by:

$$F'_i = C'_i f'_{i1} f'_{i2} \dots f'_{im} \quad i = 1, 2, 3, \dots, n \tag{5}$$

where F'_i - nondimensionalized response surface values corresponding to the i^{th} test condition, C'_i - correction constant, approximately equal to 1.

The data for the creation of response surfaces can be obtained from any analytical or numerical models. We propose to use an orthogonal regression technique to determine the component equation f'_{ij}. The use of an orthogonal regression technique not only provides an equation to accurately predict the overall system response, but also provides an accurate estimate of regression parameters [9,14]. An accurate estimation of regression parameters is essential in order to identify the unknown material parameters by optimization. The function f'_{ij} is an orthogonal polynomial of parameter p'_j and is given by:

$$f'_{ij} = \sum_{r=0}^{k} a_{ir} p'^{r}_j \tag{6}$$

The values of a_{ir}, $r=1,2 \dots k$ are determined by using model response (analytical or numerical such as FEM) and orthogonal regression techniques. Only requirement for the use of orthogonal regression in curve fitting is that p'_j be equally spaced during model evaluation while all other material parameters be held at the mid point values. The theoretical value of the correction constant, C'_i in Eq. (5) is one. However, when curve fitting is employed to determine the regression coefficients, a_{ir}, the value of this correction constant may be slightly different than one. The actual value of C'_i can be found by employing a linear regression technique between F'_i and $(f'_{i1}, f'_{i2} \dots f'_{im})$. To accomplish this linear regression, model response at orthogonal points used in building the response surface and some additional random points may be used.

Higher order correction

As stated previously, in general the orthogonal response surface is expected to be close to the true model response only near the mid point and the parameter axes (p'_i axis). As we start moving away from the origin and the parameter axes, the two surfaces will depart from each other. At large distances from the origin and the parameter axes this error can be significant. The relation between the nondimensionalized true response, Φ'_i and the response surface, F'_i is given by:

$$\Phi'_i = F'_i + \varepsilon_i \tag{7}$$

where ε_i is the error in our approximation, $\Phi'_i = F'_i$.

By assuming that the function Φ'_i is 'well behaved' (i.e. analytic everywhere in the parameter space), this function can be represented in a Taylor series as follows:

$$\Phi'_i = 1+b_1p'_1+b_2p'_2+...+b_mp'_m+b_{11}p'^2_1+ b_{12}p'_1p'_2+...+b_{1m}p'_1p'_m+b_{111}p'^3_1+ b_{112}p'^2_1p'_2+...+b'_{11m}p'^2_1p'_m+b_{123}p'_1p'_2p'_3+...+ b_{12m}p'_1p'_2p'_m+... \tag{8}$$

where coefficients, b_i, b_{ij}, b_{ijk}, etc. for $i=1,2, ...m$; $j=1,2 ....m$; $k=1,2,.....m$ are respectively related to the partial derivatives of the function, F'_i with respect to p'_i, p'_{ij}, p'_{ijk} etc. at the origin (mid point). Eq. (8) reduces to f'_{ij} along p'_j axis, i.e.:

$$f'_{ij} = 1 + bjp'j + b_{jj}p'^2_j+b_{jjj}p'^3_j+... \tag{9}$$

Using Eqs (5) and (7-9) we get:

$$\varepsilon_i=d_{12}p'_1p'_2+...+d_{1m}p'_1p'_m+d_{23}p'_2p'_3+...+ d_{2m}p'_2p'_m+...+d_{112}p'^2_1p'_2+...+d_{11m}p'^2_1p'_m+ d_{123}p'_1p'_2p'_3+... \tag{10}$$

where 'd's are constant coefficients related to the cross derivatives of Φ'_i at the origin. It should be noted that strictly from a theoretical point of view, an orthogonal response surface can be created based on Eq. (8) rather than Eq. (5) which relies on the product of component equations. In such a case, very little difference is expected between the real surface and orthogonal response surface. If nine equidistant values of each of the parameter p'_i, $i=1,2,$ ...., m are used in evaluating real surface, 9^m model evaluations will be needed. If $m=2$ then 81 model evaluations are needed. On the other hand, if $m=6$, then an astronomical 531 441 model evaluations are necessary. In most real problems, where FEM evaluation of a complex model is necessary, using Eq. (8) as a basis for the response surface is infeasible except for the case of a two parameter model. The response surface represented by Eq. (5) requires only $[8*m+1]$ model evaluations (i.e., for $m=2$, 17 model evaluations are necessary whereas for $m=6$, 49 model evaluations are necessary).

Second order correction

In practice, Eq. (10) will be truncated at some convenient point. The truncated function is an approximation to e_i and is called the correction function, E_i. If we limit ourselves to only the product of the type $p'_ip'_j$ for $i=1,2,$ ...., m and $j=1,2,$ ...., m, but $i \neq j$, then E_i will be a second order function. This second order function, E_i contains $n_{c\,min}$ unknowns given by:

$$n_{c\,min} = \frac{m(m-1)}{2} \tag{11}$$

In order to determine the second order correction function, E_i model responses are obtained at n_c additional check points, where n_c is greater or equal to $n_{c\,min}$. The additional check points can be selected randomly or in a deterministic way. It can be shown that the form of the second order correction is:

$$E_i = \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} e_{i,\frac{(j-1)(2m-j)}{2}+k-j} p'_j p'_k \quad (12)$$

The 'e' coefficients can be derived from a set of n_c linear equations with $n_{c\,min}$ unknowns. A multiple linear regression technique based on Eq. (12) can be used to estimate the 'e' coefficients. Modification to the response surfaces can be accomplished by adding Eq. (12) to Eq. (5). The resulting improved response surface is given by:

$$F_i = C f_{i1} f_{i2} \ldots f_{im} + E_i \qquad i=1,2,2,\ldots,n \quad (13)$$

It is important to emphasize that the second order correction neglects all higher orders of ε_i. There may be some situations where these higher order corrections are necessary. In such cases, it is possible that the second order correction may even give poor results. In situations like these, use of Eq. (5) may give more accurate results than Eq. (13). More discussion on this important issue will follow when we consider examples.

Third order correction

In order to get more accurate results to the function E_i, we may consider the third order correction. The third order correction consists of all cross product terms of the parameters upto and including the third order terms. The third order function E_i is the summation of $n_{c\,min}$ combination of cross products, therefore we have $n_{c\,min}$ unknown coefficients. It can be shown that the number of combinations, $n_{c\,min}$ is given by:

$$n_{cmin} = \frac{3m(m-1)}{2} + \frac{1}{2}\left[\frac{(m-1)(m-2)}{2} + \sum_{j=1}^{m-2} j^2\right] \quad (14)$$

The function E_i for the third order correction is:

$$E_i = \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} e_{i,\alpha(m,j,k)} p'_j p'_k + \sum_{j=1}^{m-2} \sum_{k=j+1}^{m-1} \sum_{l=k+1}^{m} e_{i,\beta(m,j,k,l)} p'_j p'_k p'_l + \sum_{j=1}^{m} \sum_{k=1}^{m} e_{i,\gamma(m,j,k)} p'^2_j p'_k (1-\sigma_{jk}) \quad (15)$$

where

$$\sigma_{jk} = \begin{cases} 1 & j=k \\ 2 & j\neq k \end{cases} \quad (16)$$

$$\alpha(m,j,k) = \frac{(j-1)(2m-j)}{2} + k - j \quad (17)$$

$$\beta(m,j,k,l) = \frac{m(m-1)}{2} + \frac{1}{2}\left\{\frac{j-1}{2}[2m(m-j-1)+j] + \sum_{r=1}^{j-1} r^2\right\} + \frac{k-j+1}{2}(2m-k+j-2-)+1-k \quad (18)$$

$$\gamma(m,j,k) = \frac{m(m-1)}{2} + \frac{1}{2}\left[\frac{(m-1)(m-2)}{2} + \sum_{r=1}^{m-2} r^2\right] + (m-1)(j-1) + \sigma \quad (19)$$

$$\sigma_{jk} = \begin{cases} k & k<j \\ k-1 & k>j \end{cases} \quad (20)$$

The 'e' coefficients can be derived from a set of n_c ($n_c \geq n_{c\,min}$) linear equations with $n_{c\,min}$ unknowns. A multiple linear regression technique based on Eq. (15) can be used to estimate the 'e' coefficients. The modified response surface is given by Eq. (13).

Estimation of material parameters

Let U_i be one of the n independent experimental observations. In order to make U_i consistent with F'_i, we transform it into a nondimensional value, U'_i. The relation between U_i and U'_i is given by:

$$U'_i = \frac{U_i}{F_i} \tag{21}$$

The subtraction of Eq. (21) from Eq. (13) yields a set of n nonlinear equations in m unknowns:

$$F'_1 - U'_1 = 0$$
$$F'_2 - U'_2 = 0$$
$$\vdots$$
$$F'_n - U'_n = 0 \tag{22}$$

In general Eq. (22) is seldom an equality due to the presence of approximation as well as experimental errors. One method of determining engineering properties of the material involves optimizing sum of squares of residuals, SSR defined by:

$$SSR = \min \left\{ \sum_{i=1}^{n} (F'_i - U'_i)^2 \right\} \tag{23}$$

The expression for the SSR in Eq. (23) is used as an objective function and a nonlinear optimization technique is used to solve for material parameters.

Transformation of experimental results

As explained previously, the accuracy of function F'_i may be high in some region on the response surface and low in another region. This implies that the estimated parameters will be high in accuracy sometimes and poor in accuracy some other times. Generally speaking, the inaccuracy increases as we move away from the origin and parameter axes. At the origin, the nondimensionalized F'_i has a value of unity (cf. Eq. (5)). In some sense, as the values of F'_i change from unity the difference between the real and the response surface values (both corrected and uncorrected) tend to increase. Thus the values of F'_i can be used as a measure of this departure. This argument suggests that there exists a function $\Phi'_i = \Phi'_i(F'_i)$. Inverse of this transformation, $F'_i = F'_i(\Phi'_i)$ is of particular interest in our case. This relationship can be used as a transformation rule for experimental values by replacing the real response, Φ'_i by experimental value, U'_i. If we denote the transformed experimental value which corresponds to F'_i by U^*_i, then we have $U^*_i = U^*_i(U'_i)$. The transformation function can be obtained by conducting a polynomial regression between F'_i and Φ'_i in some acceptable range, $\Phi_{i\ min}$ and $\Phi_{i\ max}$, thus:

$$F'_i = \sum_j g_j \Phi'^j_i \tag{24}$$

where 'g_j's are regression coefficients. The corresponding transformation rule for the experimental values is given by:

$$U^*_i = \sum_{j=0}^{k} g_j U'^j_i \tag{25}$$

The value of U^*_i calculated from Eq. (25) can be used for replacing U_i in Eq. (22) and (23). This modification may significantly improve the accuracy of the parameters obtained through optimization. Following examples study illustrates the methodology.

VERIFICATION OF THE PROPOSED METHODOLOGY

Two example problems from soil mechanics were selected to illustrate the methodology as well as provide some verification to the methodology. The first example is a simple two parameter hypo-elastic model for soil. This example was selected to illustrate the methodology since the real as well as the orthogonal response surface can be plotted in a 3-D space. A more complex five parameter soil model was chosen as the second example to illustrate the application of the methodology to very complicated problems.

Two parameter model

A simple two parameter material model is selected to illustrate the main features of this technique. A cylindrical bar or soil column under uniaxial compression is considered. The bar is made of a hypoelastic material with an incremental constitutive law given by [5]):

$$d\sigma = (E_0 + E_1\sigma)\, d\varepsilon \tag{26}$$

where E_0 and E_1 are the material parameters of interest in this model, σ is axial stress (positive in compression), and ε is axial strain. Integration of Eq. (26) yields:

$$u = \frac{L}{E_1} \ln\left(\frac{E_0 + E_1\sigma}{E_0}\right) \tag{27}$$

where u - deformation of the bar (the contraction), L - length of the undeformed bar.

Eq. (27) will be used as a basis to build the response surface, F'_i. The response surface will be developed in the following range of parameters E_0 and E_1:

$E_{0\,min}$=689.5 kPa (100 psi)
$E_{1\,min}$=10.0
$E_{0\,mid\,point}$=3792.1 kPa (550 psi)
$E_{0\,max}$=6894.8 kPa (1000 psi)
$E_{1\,max}$=100.0
$E_{1\,mid\,point}$=55.0

The nondimensional parameters (Eq. (3)) for this case are:

$$E'_0 = \frac{2(E_0 - E_{0\,mid\,point})}{E_{0\,max} - E_{0\,min}} \text{ and}$$

$$E'_1 = \frac{2(E_1 - E_{1\,mid\,point})}{E_{1\,max} - E_{1\,min}} \tag{28}$$

The response at the mid point of the parameters is (i.e. origin):

$$\bar{F}_i = \frac{L}{E_{1\,mid\,point}} \ln\left(\frac{E_{0\,mid\,point} + E_{1\,mid\,pont}\sigma_i}{E_{0\,mid\,point}}\right) \tag{29}$$

The nondimensional representation of the real surface is obtained by dividing Eq. (27) by Eq. (29). The plot of the nondimensionalized real surface obtained by using an applied stress of σ=689.5 kPa (100 psi), is given in Fig. 1a. The response surface for this case which includes second order correction is:

$$F'_i = f'_1(E'_0) f'_2(E'_1) + e_i E'_0 E'_1 \tag{30}$$

where f'_1 and f'_2 are orthogonal polynomial functions of E'_0 and E'_1 respectively and e_i is second order correction coefficient. The function f'_1, f'_2, and the coefficient e_i were found as described previously. The approximation of the surface without the second order correction is shown in Fig. 1b, and the error of this approximation is shown in Fig. 1c. The response surface describes the real function with reasonable accuracy except at the corners. The error is particularly high as both parameters (E_0 and E_1) approach their minimum values.

The second order correction for this case was obtained by using the edge points, the mid points of the lower and upper range for each parameter - a total of 16 combinations. The second order correction decreases the error in the zones of high error (Fig. 1d). However, this correction to the response surface increased the error in some other regions where the error was negligible previously. In rest of the area the second order correction is not enough and a transformation of the data using Eq. (25) is necessary.

A study of the error distribution shows three different possibilities. In some areas the orthogonal surface represents the real surface with negligible error. In some other

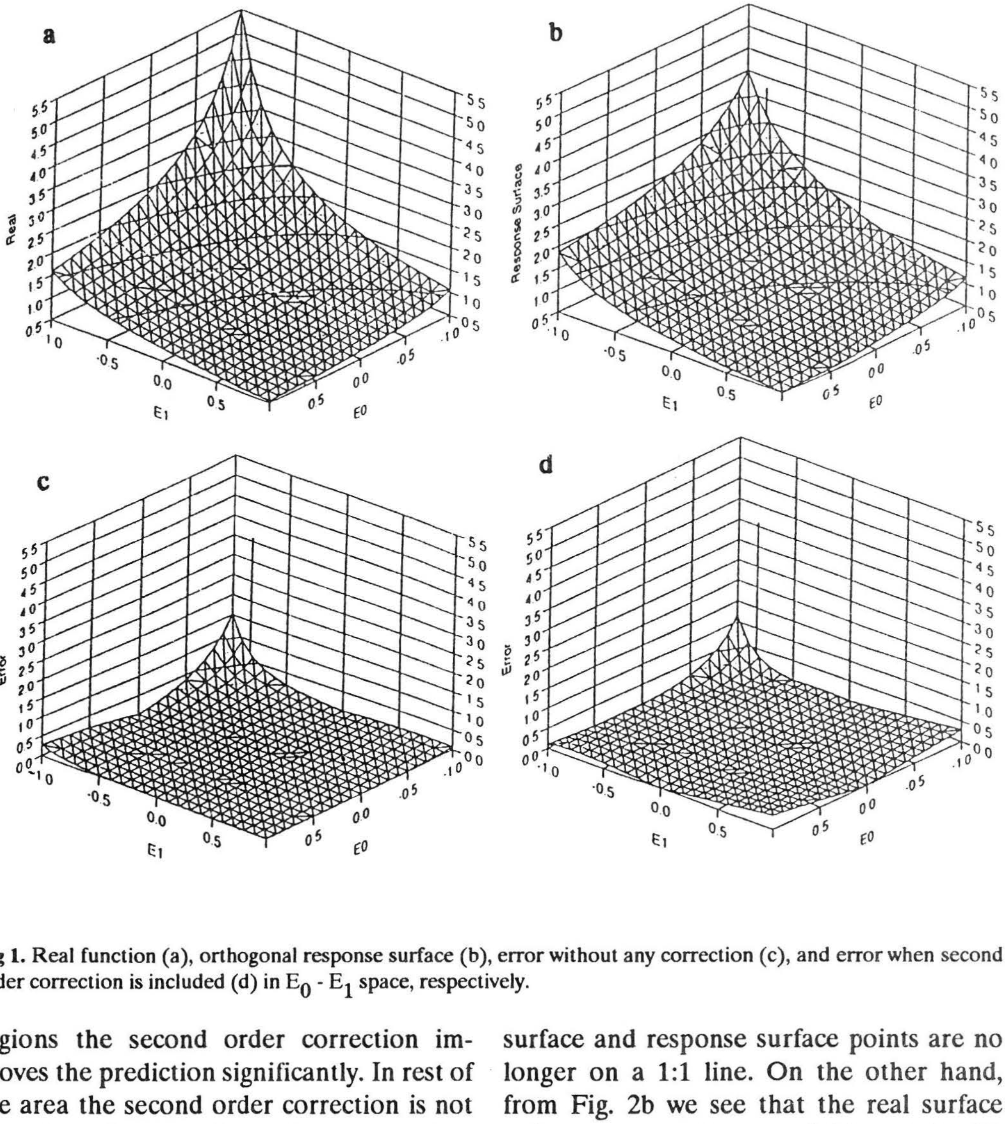


Fig 1. Real function (a), orthogonal response surface (b), error without any correction (c), and error when second order correction is included (d) in E_0 - E_1 space, respectively.

regions the second order correction improves the prediction significantly. In rest of the area the second order correction is not enough and a transformation of the data using Eq. (25) is necessary. The graph of real surface versus orthogonal response surface without any correction is shown in Fig. 2a, and the real surface versus the response surface with second order correction is given in Fig. 2b. From Fig. 2a it is clear that the orthogonal surface without any correction corresponds to the real surface very well (slope 1:1) until a response value of 1.5 is attained. From about a value of 1.5, the real surface and response surface points are no longer on a 1:1 line. On the other hand, from Fig. 2b we see that the real surface points correspond very well (slope 1:1) with response surface points with second order correction from a response surface value of 1.5 until about 2.7. Beyond a value of 2.7 real surface points do not correspond (no more a 1:1 line) to the response surface points with or without any correction. This portion of the graph corresponds to only 3 % of the parameter range. Even in this range there is a curvilinear relationship between the real surface and response surface points

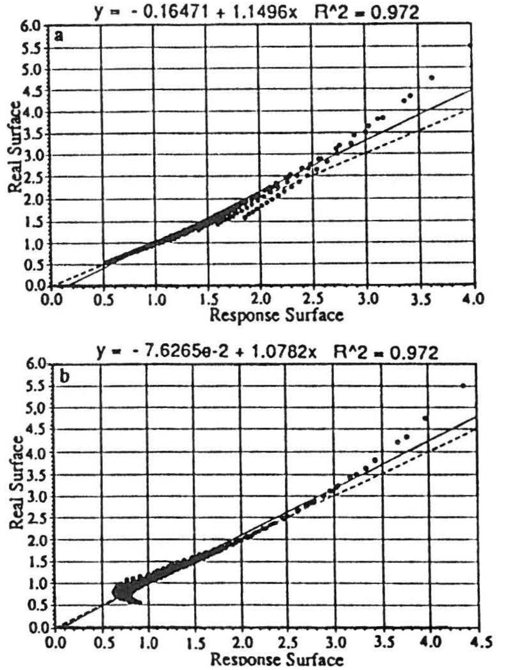


Fig. 2. Real surface versus response surface without any correction (a) and using second order correction (b).

(i.e. transformation Eq. (25) is valid).

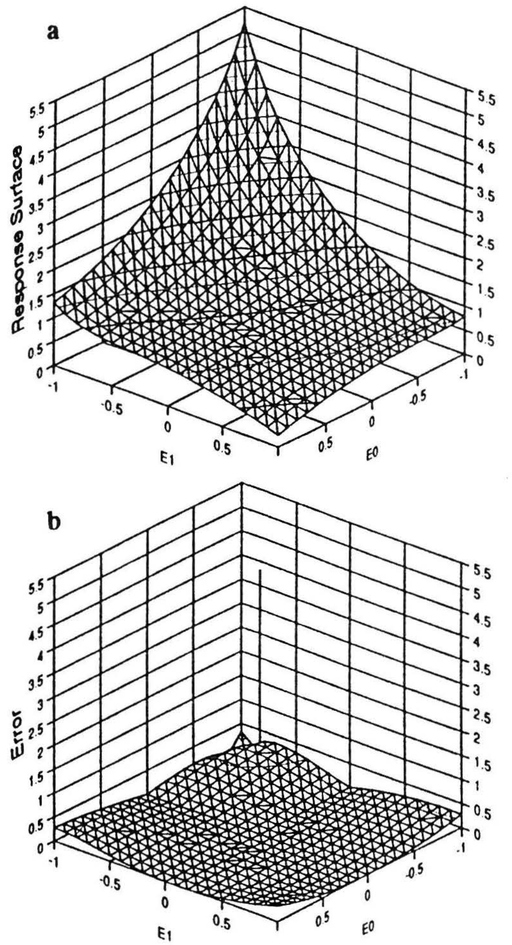
In order to determine the material parameters (E_0 and E_1), seven distinct response surfaces were created using seven different applied stresses. The stress values used were 344.7 kPa (50 psi), 517.1 kPa (75 psi), 689.5 kPa (100 psi), 861.8 kPa (125 psi), 1034.2 kPa (150 psi), 11206.6 kPa (175 psi) and 1379.0 kPa (200 psi). The adequacy of the method is illustrated by 13 examples. One example is based on a randomly selected values of parameters, E_0 and E_1. The other twelve examples were based on parameter values along the diagonal, $E'_0=E'_1$. The Eq. (27) was used to calculate the real response. These values were used instead of the experimental values in the optimization step. Since there is no experimental error in this case, we should, in principle, get exact values of the assumed parameters back. Inaccuracy in the results is solely due to the inadequacy of the response surface. Five different initial guess values of the parameters were considered in the nonlinear optimization process for each one of these examples. The first initial guess values were the mid point values of the parameters, three others were selected randomly and the fifth one was the exact solution.

The transformation equation was estimated using 50 random points of the real surface using Eq. (24) as a basis for regression. The results of the simulations are listed in Table 1. These examples clearly show that transformation of experimental data is necessary to obtain good estimates of the parameters, especially in the region where the real and response surfaces depart from each other significantly. Additional simulations were conducted along the diagonal line ($E'_0=E'_1$) along which maximum error is expected to occur. The maximum error of the prediction of each of the parameters in the whole range in all the 13 cases studied did not exceed 3 %, if the transformation technique was employed with second order correction as necessary. It should be emphasized that even in the worst case (at and near the minimum values of E_0 and E_1), the error in parameter estimation was no more than 3 %. Moreover, nonuniqueness of the solution was not a concern in this two parameter case.

Effect of including the third order correction (TOC) was also examined for this two parameter case. The form of the third order correction is:

$$E_i=e_{i1}E_0E_1+e_{i2}E_0^2E_1+e_{i3}E_0E_1^2 \quad (31)$$

Figure 3a shows the effect of including third order correction on the response surface. Inclusion of third order correction further reduces the error in the zone of high error (i.e., in the 3 % region where E_0 and E_1 values are near or at their minimum). Figure 3b is a plot of error when the third order correction is included. Comparison of this figure with Figs 1c and 1d shows that



although the response surface which includes third order correction reduces the error in the zone of high error, the error in other regions does not necessarily decrease. In fact, the error increases slightly in some areas. A plot of real surface versus response surface shown in Fig. 4 indicates that all points are in the vicinity of 1:1 line. A comparison of Figs 4 with Figs 2a and 2b reveals that the third order correction is mainly beneficial in the region where the response was much greater than center point response (1.5). In the region where the normalized response is less than 1.5, inclusion of third order correction leads to

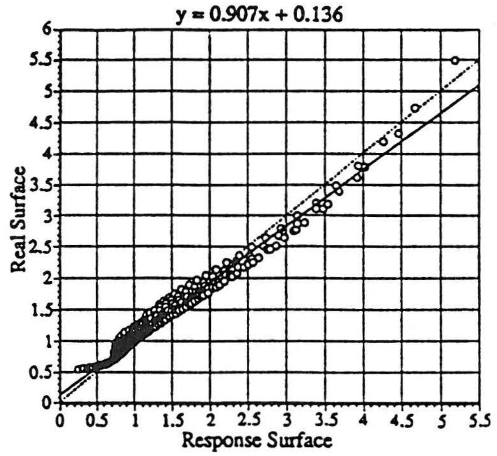


Fig. 3. Plot of orthogonal response surface (a) and error (b) in E_0-E_1 space when a third order correction was included.

Fig. 4. Plot of real surface versus response surface when a third order correction was included for a two parameter hypo-elastic model for soil.

Table 1. Results of simulation studies

Sim. No.	E_0 (kPa) Actual	E_0 (kPa) Predicted	E_0 (kPa) % Error	E_1 Actual	E_1 Predicted	E_1 % Error	Remarks
1	5666	5585	1.34	61.8	61.84	0.07	Random points
2	5666	6453	13.9	61.8	62.21	0.66	SOC*
3	5666	5617	0.86	61.8	61.65	0.24	With transformation
4	1310	751	42.7	19	20	5.32	$E'_0=E'_1=0.8$
5	1310	1066	18.6	19	20	5.34	SOC*
6	1310	1290	1.5	19	18.3	3.9	With transformation
7	1310	1347	2.8	19	19.02	0.09	SOC*and transformation

*Second order correction.

less error than inclusion of second order correction. However, uncorrected response surface predicts the real surface better than the case in which the third order correction is included. Since even in the worst situation the error in estimating the parameters is less than 3 %, we can conclude that inclusion of third order correction generally improves the response surface and reduces the possibility of large error. It should be noted that this particular soil model shows large increases in response when E_0 and E_1 values are very small compared to all other values of E_0 and E_1. This makes it difficult to come of with a response surface which is good everywhere in the region. In fact, even very complicated models do not show such singularities or large changes in a small region as we will see with a five parameter Drucker-Prager model described below.

A five parameter nonlinear elastic soil model with extended Druker-Prager yield criteria

The elasto-plastic constitutive material model with Druker-Prager yield criteria is widely used in geomechanics [6]. Here we explore the feasibility of identifying material parameters of this fairly complex material model using the proposed methodology. Actual field data will not be considered. Analysis of the field data to identify material parameters will be dealt in a later study. A commercial finite element program, ABAQUS was used in this study to obtain model response. The orthogonal response surfaces were built using six parameters logarithmic bulk modulus, κ; Poison's ratio, ν; yield surface shape factor, K; cohesion, C; internal friction angle, Φ and void ratio, e. The last parameter, void ratio, is not a material parameter and is usually known from field tests.

Two plates of diameters 50 mm (2 in.) and 100 mm (4 in.) were simulated, with applied load ranging from 137.9 kPa (20 psi) to 1034.2 kPa (150 psi) in increments of 68.9 kPa (10 psi), a total of 14 tests for each plate. A response surface was built for each of those tests in the following parameter range:

	Min	Max	Mid point	No. of points
κ	0.01	0.05	0.03	9
ν	0.05	0.37	0.21	9
K	0.60	1.0	0.8	11
C	9.0	21.0	15.0	9
Φ	22.5	47.5	35.0	11
e	0.6	1.6	1.1	11

Figure 5 is a plot of nondimensional plate sinkage versus the nondimensional values of parameter Φ (angle of internal friction) for a 100 mm (4 in.) plate subjected to 1034.3 kPa (150 psi) pressure. Note that all other parameters are held at their respective mid-point values. This response curve was obtained using an orthogonal regression technique. The very high R^2 value (coefficient of multiple determination) indicates that this curve represents the real response very accurately. Figure 5 is typical of all response curves obtained by orthogonal regression analysis. In all cases the orthogonal response curve for any parameter (κ, m, K, C or Φ) was an excellent representation of the real response curve for that parameter. The graph of the real surface versus the orthogonal response surface without any correction for a 100 mm (4 in.)

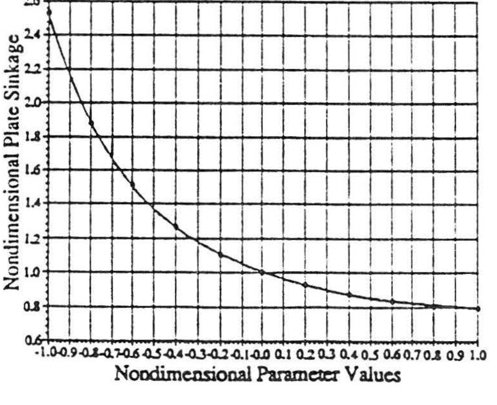


Fig. 5. Nondimensional values of plate sinkage versus nondimensional values of parameter φ for a 100 mm (4 in.) plate subjected to 551.6 kPa (80 psi) pressure.

plate subjected to an applied load of 551.6 kPa (80 psi) is shown in Fig. 6a. This graph consists of 55 orthogonal points and an additional 60 random points. Figure 6a reveals that under high load the orthogonal response

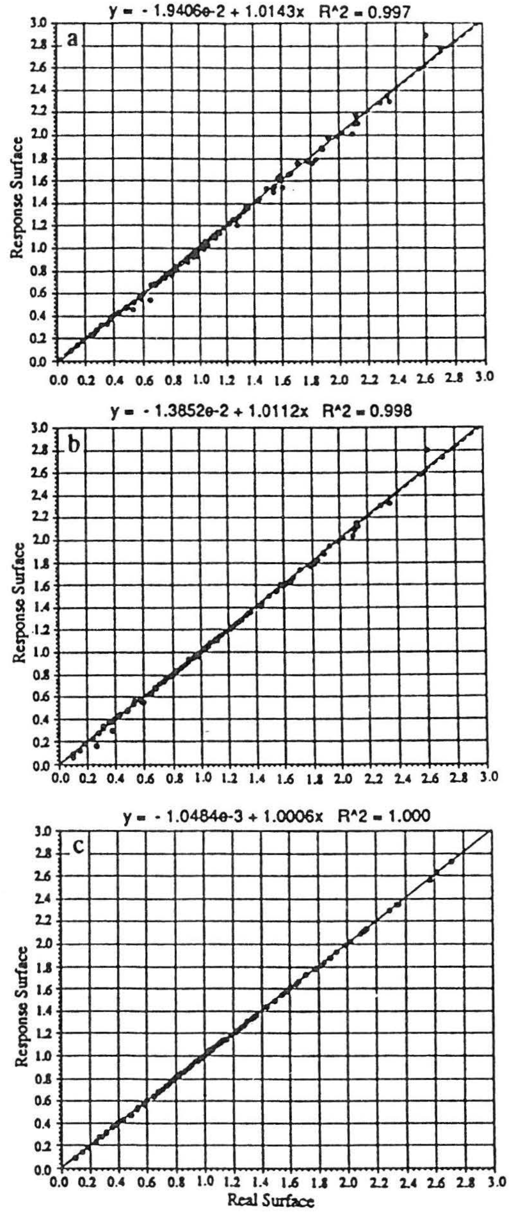


Fig. 6. Orthogonal response surface versus real surface for a 100 mm (4 in.) plate subjected to 551.6 kPa (80 psi) pressure without higher correction (a) when a second (b) and third (c) order correction was employed.

surface begins to depart from the real surface when nondimensionalized displacements exceed about 1.4. Figures 6b is similar to Fig. 6a except that a second order correction has been added to the orthogonal response surface. This figure indicates only marginal improvement over Fig. 6a. Perhaps a higher order correction is beneficial especially at high plate loads. Figure 6c is similar to Figs 6a and 6b, except that a third order correction has also been added to the orthogonal response surface. Inclusion of third order correction has resulted in an orthogonal response surface which is almost identical to the real surface even at high loads. This indicates that an orthogonal response surface with third order correction can be used reliably to predict the real response without having to resort to FEM analysis.

Since we are dealing with a nonlinear problem the solution is not necessarily unique. Following recommendations may be used as a guide for selecting the best solution from several optimum solutions resulting from the presence of 'local minimums':

1. Discard all solutions that have a significantly high SSR (cf. Eq. (23)).

2. Use more than one geometry (i.e., 50 mm and 100 mm diameter plates) and look for optimum for each of the geometries and also the combination of all the geometries. Accept those solutions which are approximately same in all cases. From a practical point of view two plates will be sufficient.

3. Reject any solution in which more than one parameter hit the bounds of the search domain. The probability of more than one parameter hitting the bounds simultaneously is low. If in fact, if this really is the case for several initial guesses, and the above two criteria will be met.

4. In spite of these steps, if more than one solutions are obtained we recommend using Eq. (23) to compute SSR. Unfortunately, this requires the use of model evaluation at these competing optimal solutions. But number of computations needed are

only a few (i.e. once for each plate for each competing solution).

To explore the suitability of this method to identify the material parameters of this complex constitutive equation for soil, we selected five different random sets of parameters and tried to re-predict those parameters using the response surface methodology. The random sets of parameters selected are:

Point	κ	ν	K	C	Φ	e
1	0.027	0.238	0.954	17.333	24.717	1.139
2	0.017	0.246	0.823	19.160	23.932	1.436
3	0.013	0.309	0.967	18.745	30.589	0.905
4	0.020	0.175	0.935	19.759	30.548	1.048
5	0.021	0.176	0.960	12.563	46.666	1.379

Two sets of orthogonal response surfaces were built - one for a 50 mm plate and the other is the for a 100 mm plate. The parameters were back calculated using the response surfaces for the 50 mm plate by itself, and by combining the response surfaces of both 50 mm and 100 mm plate together. The transformation technique was employed for all cases (Eq. (25)). Seven different initial guess values were used: one is the exact solution, the second is the 'mid point' values of the parameters and the remaining five are the following random points:

Point	κ	ν	K	C	Φ
6	0.018	0.200	0.754	9.948	41.311
7	0.011	0.167	0.833	20.006	38.263
8	0.034	0.223	0.795	10.173	41.889
9	0.017	0.072	0.642	9.523	24.854
10	0.020	0.214	0.651	11.842	24.163

A quasi-Newton optimization technique was employed. The results of the optimization process are listed in Table 2. For the first four cases, reasonably good results were obtained. Errors in estimation of the parameters were very low in many of the parameters, but occasionally large errors up to about 25 % did occur. The results for the case of random point 5 was, however, very discouraging. Very high SSR values were obtained for all the seven initial guess values. Two or three parameters hit the bounds of the search region. Criteria number 1 and 3 lead to the rejection of all solutions obtained. However, the selection criteria listed above clearly indicated that the true optimum was not obtained. A higher order correction to the orthogonal response surface may be beneficial in this case.

The second order correction based on 20 random points increased the accuracy of the results only with point 1 in this study. Table 3 lists the effect of including the second order correction on point 1. Significantly

Table 2. Predicted results using the orthogonal response surface without second order correction

Point	Plate size	κ error	ν error	K error	C error	Φ error	SSE
		%					
1	50 mm	1.27	3.88	3.76	21.16	1.20	0.0480
	50 and 100 mm	10.52	8.28	1.01	19.23	0.68	0.0550
2	50 mm	3.88	3.44	10.61	9.60	1.13	0.0329
	50 and 100 mm	0.08	9.44	13.29	9.60	2.01	0.0362
3	50 mm	9.32	5.78	2.70	11.24	8.29	0.0323
	50 and 100 mm	17.85	18.59	3.41	4.60	11.78	0.0836
4	50 mm	3.61	11.07	6.04	4.84	1.85	0.0199
	50 and 100 mm	13.77	25.03	3.41	1.51	1.73	0.0879

better results were obtained when a second order correction was included. Error in estimating C went down from 21.16 % to 0.54 % while other parameters were predicted with about the same accuracy as before. However, second order correction was not beneficial in general (i.e., for point 2 to 4). Perhaps a third order correction to the orthogonal response surface is necessary. In order to explore the effect of random noise on the optimization procedure, a random noise derived from a normal distribution with mean zero and variance 0.01 was added to the numerical displacement values (non-dimensionalized) derived from the FEM analysis. This technique of adding a random, normal noise was thought to simulate the experimental noise in real data. Results of this analysis are given in Tables 3 and 4. Interestingly enough, the presence of noise did not have an adverse effect on the parameters identified. Although some of the parameters were less accurate, others were more accurately predicted in the presence of noise. The range of error in the estimated parameters were about the same as in the case where the true model response was used.

Effect of including third order correction to the orthogonal response surface was investigated using 50 mm (2 in.) plate only. Following five sets of random parameter values were selected:

Point	κ	ν	K	C	Φ	e
1	0.027	0.238	0.954	17.333	24.717	1.139
2	0.017	0.246	0.823	19.160	23.932	1.436
3	0.012	0.177	0.960	16.702	27.858	0.606
4	0.047	0.151	0.712	15.712	26.949	1.881
5	0.036	0.261	0.981	14.980	44.539	1.145

The first two points were same as before. The last three points were chosen from random sets of points to cover the parameter range better. Seven different initial guesses were used to seek the optimum solution. These initial guesses values were the same as the ones selected before. Table 5 lists the

Table 3. Predicted results for point 1 when second order correction to the orthogonal response surface was employed

Plate size	κ error	ν error	K error	C error	Φ error	SSE
			%			
50 mm	2.57	1.92	3.45	0.54	2.68	0.0030
50 mm + random noise	0.65	4.82	4.82	3.05	4.13	0.0073

Table 4. Predicted results for the case when random noise was added to the model results to simulated field data

Point	Plate size	κ error	ν error	K error	C error	Φ error	SSE
				%			
1	50 mm	5.30	13.37	4.82	15.92	1.51	0.0480
	50 and 100 mm	3.51	1.16	4.82	14.11	0.85	0.0237
2	50 mm	5.52	16.63	21.51	4.60	1.52	0.0793
	50 and 100 mm	5.72	17.49	15.80	8.60	1.81	0.0665
3	50 mm	17.85	18.59	3.41	4.60	11.78	0.0836
	50 and 100 mm	18.47	15.96	4.55	12.03	1.58	0.0764
4	50 mm	14.27	25.04	6.95	0.57	7.48	0.0935
	50 and 100 mm	9.44	12.94	6.95	1.55	1.68	0.0310

results of optimization. In general, the results look very promising. Some of the reasonable solutions have 20 to 30 % error in parameters K and C (point 1 and 5). However, if SSE and SSR are used as a guide to select the best solution, in general a solution with very small errors in parameter values gets selected. Note that the maximum error is 18.17 % for parameter K corresponding to point 3. We have found in our simulation studies that the real response is not very sensitive to parameter K if it is greater than 0.6. As K changes from 0.6 to 1.0 (which is the range of interest), response changes by less than 10 %. In the neighborhood of K=1.0, the response is very flat. In view of this, we feel that this is a reasonable solution.

During the course of this study it was realized that the range of κ selected (0.01 to 0.05) in building the response surface as explained above was not wide enough to simulate observed field response. It was necessary to widen the range of κ (0.01 to 0.15) and reconstruct the response surfaces based on this new wider range of κ. We conducted simulation runs to verify the methodology using this wider range of κ also. Since third order correction (TOC) to the orthogonal response surface appears to be necessary to obtain reasonable results, we will only explore the situation in which TOC is added to the orthogonal response surface. Once again we chose five different random sets of parameter values to be re-predicted using the optimization technique. These random sets of points are as follows:

Point	κ	ν	K	C	Φ	e
1	0.127	0.132	0.938	19.406	26.984	0.630
2	0.011	0.343	0.611	16.813	31.305	0.755
3	0.056	0.146	0.946	16.512	28.985	1.095
4	0.022	0.093	0.776	13.893	41.297	1.049
5	0.083	0.233	0.884	16.867	34.856	1.123

The initial guess values selected were the 'exact solution', 'mid point values' and a set of five randomly selected parameter values listed below:

Point	κ	ν	K	C	Φ	e
6	0.356	0.238	0.889	10.791	24.717	34.579
7	0.131	0.297	0.996	45.693	23.932	23.932
8	0.039	0.116	0.889	16.271	27.858	30.728
9	0.110	0.088	0.610	20.472	26.949	23.566
10	0.014	0.360	0.907	16.979	44.539	23.778

The re-pridiction process was carried out using 50 mm (2 in.) plate, 100 mm (4 in.) plate and a combination of 50 mm (2 in.) and 100 mm (4 in.) plates. For each case we used the exact solution and the mid point parameter values as initial guesses. The five randomly selected guess points were used only for point 1 for both plates, and also the combination of plates. However, for the 100 mm (4 in.) plate all seven initial guess

Table 5. Predicted results using the orthogonal response surface developed for narrow range of κ when third order correction was employed

Point	Plate size	Reasonable solution	κ error	ν error	K error	C error	φ error	SSE	SSR
			%						
1	2 in	1	0.95	0.22	0.70	2.27	0.05	0.00066	0.00491
		2	5.60	8.54	22.31	21.16	4.67	0.10715	0.00055
		3	10.05	10.56	10.73	4.87	1.38	0.03531	0.00121
2	2 in	1	0.16	1.11	0.25	0.63	0.42	0.00019	0.00033
3	2 in	1	2.01	6.44	18.17	2.03	6.93	0.04280	0.00012
4	2 in	1	0.28	1.11	0.53	0.96	0.16	0.00025	0.00049
5	2 in	1	6.15	5.32	1.94	1.25	2.34	0.00770	0.00050
		2	6.60	6.79	3.42	22.34	6.21	0.06390	0.00011
		3	1.80	1.03	17.45	29.66	1.65	0.11915	0.00043

values were use to seek the optimum solution for each of the five random set of parameter values.

The results of this analysis are listed in Table 6. An examination of the results indicates that the reasonable solution with minimum SSE usually leads to good solution except for point 4. Point 4 results in very large errors for both parameters κ and μ. However, an examination of SSE indicates that none of the solutions are reasonable. Our suspicion is that for this values of κ and μ, the soil is extremely hard and deforms very little. Under these circumstances, the nonlinear elastic model for soil with Drucker-Prager yield criteria is perhaps inappropriate.

CONCLUSIONS

Based on this study we reached the following conclusions:

1. A response surface methodology based on an orthogonal regression in the parameter space has been developed to 'identify', or 'calibrate' engineering properties of any material based on *in situ* tests. The orthogonal response surface was created from an analytical or numerical(such as FEM) solution to the underlying differential equation of the system which utilizes these engineering properties in a constitutive equation. A transformation technique was developed to map the model response or experimental data on to the response surface.

Table 6. Predicted results using the orthogonal response surface developed for wide range of κ when third order correction was employed

Point	Plate size	Reasonable solution	κ error (%)	ν error (%)	K error (%)	C error (%)	φ error (%)	SSE	SSR
1	2 in	1	0.82	3.77	10.07	6.82	1.00	0.01639	0.00001
		2	3.93	9.01	8.33	6.26	1.38	0.02071	0.000004
	4 in	1	4.93	14.83	3.28	14.39	3.07	0.03531	0.00001
		2	3.04	10.70	10.56	10.93	1.00	0.03557	0.00004
		3	0.72	0.84	8.39	4.79	0.21	0.00947	0.00004
	2 and 4 in	1	0.11	1.58	5.13	6.43	1.86	0.00737	0.00011
		2	0.09	1.51	5.20	5.11	1.54	0.00578	0.00011
2	2 in	1	6.60	6.79	3.42	22.34	6.21	0.06390	0.00011
	4 in	1	4.96	15.51	0.92	2.17	0.76	0.02713	0.00003
		2	3.34	15.88	14.55	30.12	16.90	0.16681	0.00075
	2 and 4 in	1	0.37	0.98	2.04	3.01	0.76	0.00149	0.00183
3	2 in	1	9.09	2.62	1.69	38.73	1.14	0.15936	0.00974
	4 in	1	9.09	5.14	3.63	6.56	2.65	0.01722	0.00239
		2	9.09	6.32	33.64	4.41	32.12	0.23056	0.00065
	2 and 4 in	1	9.09	2.05	3.80	15.13	28.45	0.11396	0.00570
4	2 in	1	30.51	79.76	1.61	0.55	11.84	0.74363	0.00012
	4 in	1	54.55	121.88	10.00	15.90	13.18	1.83600	0.00095
	2 and 4 in	1	43.34	147.46	0.09	2.17	11.84	2.37638	0.00012
5	2 in	1	4.90	6.10	2.60	3.42	0.24	0.00797	0.00001
	4 in	1	1.11	1.37	2.24	1.93	0.47	0.00121	0.000003
		2	15.12	19.22	8.39	4.64	2.43	0.06957	0.000004
		3	9.06	11.54	5.26	4.10	0.83	0.00149	0.000004
	2 and 4 in	1	5.40	6.78	2.14	3.50	0.06	0.00919	0.00002

2. The proposed methodology worked very well (i.e. very little error) in the case of a two parameter hypo-elastic model for soil. When the second order correction was included with a transformation of data very small errors resulted in parameter estimation. Inclusion of third order correction to the orthogonal response surface reduced the chance of large error in parameter values.

3. When this technique was used in the presence of random noise, the predicted parameters were found to be insensitive to the noise.

4. When this methodology was applied to a complex five parameter model for soil (nonlinear elastic behavior with Drucker-Prager yield criteria and associated plastic flow upon yield), it appeared to work reasonably well. A third order correction to the orthogonal response surface appears to be necessary to obtain reasonably good solution. When both the logarithmic bulk modulus (κ) and Poisson's ratio (μ) are low, soil becomes very rigid and the methodology will not yield a good solution. Under such circumstances, perhaps the soil model chosen is inappropriate.

ACKNOWLEDGEMENTS

The authors acknowledge with gratitude the financial support received from the Goodyear Tire and Rubber Co. and the Bi-National (United States-Israel) Agricultural Research Development (BARD) agency for this project.

REFERENCES

1. ABAQUS, Thoery Manual. Version 4.8, Hibbitt, Karlsson and Sorensen, Inc., 1989.
2. **Anandarajah A., Agarwal D.:** Computer-aided calibration of a soil plasticity model. Int. J. Numerical and Analytical Methods in Geomechanics, 15, 835-856, 1991.
3. **Chen W.F., Baladi G.Y.:** Soil Plasticity-Theory and Implementation, Developments in Geotechnical Engineering: 38. ELSEVIER, NY, 1985.
4. **Desai C.S.:** Some aspects of constitutive models for geologic media. Third Int. Conf. 'Numerical Methods in Geomechanics'. Achen, 2-6, 1979.
5. **Desai C.S., Siriwardane H.J.:** Constitutive Laws for Engineering Materials with Emphasis on Geologic Materials. Prentice Hall Inc., Englewood Cliffs, NJ, 1984.
6. **Drucker D.C., Prager W.:** Soil mechanics and plastic analysis or limit design. Quarterly Appl. Mech., 10, 157-165, 1952.
7. **Gill W.R., Van den Berg G.E.:** Soil Dynamics in Tillage and Traction. Agriculture Handbook, 316, U.S. Govt. Printing Office, Washington, D.C., 1968.
8. **Gioda G.:** Some remarks on back analysis and characterization problems in geomechanics. Fifth Int. Conf. on Geomechanics, Nagya, 47-61, 1985.
9. **Glancey J.L., Upadhyaya S.K.:** A testing procedure for agricultural implements. ASAE Paper, No. 90-1542, ASAE, St. Joseph, MI 49085, 1990.
10. **Gupta S.C., Allmaras R.R.:** Model to assess the susceptibility of soils to excessive compaction. In: Adv. Soil Sci., Springer Verlag, NY, 56-100, 1987.
11. LAWR - Cooperative Extension. Water penetration in California soils. Tech. Rep., Joint Infiltration Committee, Dept. of LAWR, Univ. Cal., Davis, CA, 1984.
12. **Sakurai S., Takeuchi K.:** Back analysis of measured displacement of tunnels. Rock Mechanics and Rock Engineering, 16, 173-180, 1983.
13. **Schafer R.L., Bailey A.C., Johnson C.E., Raper R.L.:** A rationale for modeling soil compaction behavior: An engineering mechanics approach. Trans. ASAE, 34(4), 1609-1617, 1991.
14. **Snedecor G.W., Cochran W.G.:** Curvilinear regression. In: Statistical Methods. Sixth ed., 447-471, 1967.
15. **Upadhyaya S.K., Rubinstein D., Sime M.:** An inverse solution technique to determine engineering properties of material. ASAE Paper, No. 93-1084, ASAE, St. Joseph, MI 49085, 1993.