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GEOMETRICAL DEPENDENCES OF SEED SHELL FOR LARGE DEFORMATION OF PEA*

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A b s t r a c t. The model describing a spherical shell during compression of a single seed between parallel plates is presented. Increase of surface area surrounding cotyledon act caused by change of seed shape is the main aim of this work. The values of the contact area with a flat plate, the barrel diameter and the lateral surface area of the barrel for circular and parabolic shape were determined as a function of strain of compressed seed. Spherical body compressed between parallel plates change its shape in to circular barell for small deformation and in to parabolic barell for large deformation.

K e y w o r d s: pea seed, seed shell, surface area, compression

INTRODUCTION

In recent years considerable research has been devoted to studying the elastic behaviour of seeds, as indicated by axial compression tests [1,2,5,7,8,10,11]. Most of available force-deformation data on mechanical properties is of such a nature that dependence of the information on such parameters as conditions of the specimen, testing procedure, probe size and analysis of the force-deformation curve needs clarification [1,7,11,12]. Many of latest studies have made use of the Bussinesq and Hertz theories in an attempt to provide better defined parameters to express resistance to mechanical damage of seed [1,7,11]. A values necessary to calculating a modulus of elasticity have been determined from the force-deformation curves. Most of the authors have made use the assumption that seeds subjected to quasi-static, uni-axial compression tests had spherical shapes [1,2,3,7,10].

The change of seed shape during compression caused increase of surface area surrounding cotyledon act and the describing of geometrical dependences of the surface area for large deformation was the main aim of this work.

GEOMETRICAL DEPENDENCES OF CONVEX BODY

Davison *et al.* [2] showed that a typical analysis of rapeseed kernel indicates mainly a content of oil, moisture and protein. In the proposed model, these elements were assumed to act in a manner similar to a fluid; that is, the contents are capable of flowing but cannot sustain a shearing stress at rest. This results suggested that the compressibility of cotyledon is low and for large deformation could be ignored. Using the assumption of a constant volume make it possible to determine the changes of the surface area of a shell surrounding cotyledon act being compressed between parallel plates.

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It is therefore necessary to calculate the surface area of the barrel which consists of the lateral surface $2B_p$ area and the surface of both barrel bottoms A:

$$S_{BP} = 2B_p + 2A \tag{1}$$

The area of bottom A is calculated using the formula of a circle with the diameter a (Fig. 1), formed as the effect of flattening the spherical cap, hence area a equals:

$$A \approx \frac{\Pi}{4}\left(2dl + \frac{1}{3}l^2\right). \tag{2}$$

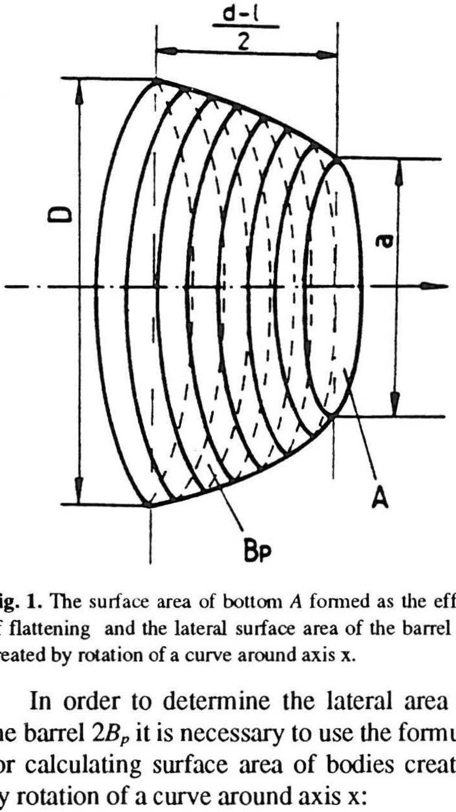

Fig. 1. The surface area of bottom A formed as the effect of flattening and the lateral surface area of the barrel Bp created by rotation of a curve around axis x.

In order to determine the lateral area of the barrel $2B_p$ it is necessary to use the formula for calculating surface area of bodies created by rotation of a curve around axis x:

$$S = 2\Pi \int_{x_1}^{x_2} y\sqrt{1 + (y')^2}\, dx . \tag{3}$$

This curve (Fig. 2) is described by the equation of the parabola:

$$y = \left(-px^2 + \frac{D}{2}\right). \tag{4}$$

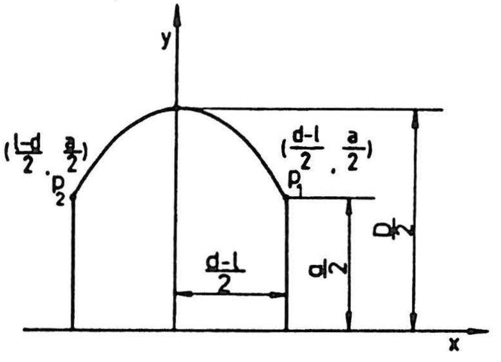

Fig. 2. The parabola describing a cross section of the lateral surface area.

Substituting into x and y coordinates of point P_1 value of p is determined and equation of parabola was defined as:

$$y = \frac{2(a-D)}{(d-l)^2}x^2 + \frac{D}{2}. \tag{5}$$

Calculating derivative:

$$y' = 4\frac{(a-D)}{(d-l)^2}x \tag{6}$$

and substituting Eqs (5) and (6) into the formula for surface areas of bodies created by rotation (3) was obtained:

$$B_p = 2\Pi \int_0^{\frac{d-l}{2}} \left(2\frac{(a-D)}{(d-l)^2}x^2 + \frac{D}{2}\right) \sqrt{l + \left(4\frac{(a-D)}{(d-l)^2}\right)^2 x^2}\, dx \tag{7}$$

Substituting $2\frac{a-D}{(d-l)^2} = Z$ and $\frac{1}{4Z^2} = z^2$ leads to:

$$B_p = 4Z^2\Pi \int_0^{\frac{d-l}{2}} x^2\sqrt{z^2 + x^2}\, dx + 2ZD\Pi \int_0^{\frac{d-l}{2}} \sqrt{z^2 + x^2}\, dx. \tag{8}$$

Using following formulas 9 and 10 for integration:

$$\int_{x_1}^{x_2} x^2 \sqrt{z^2+x^2}\, dx =$$

$$\left[\frac{2x^3+z^2x}{8}\sqrt{x^2+z^2} - \frac{z^4}{8}\ln\left(x+\sqrt{x^2+z^2}\right)\right]\left[\begin{matrix} x_2 \\ x_1 \end{matrix}\right. \tag{9}$$

and

$$\int_{x_1}^{x_2} \sqrt{z^2+x^2}\, dx =$$

$$\frac{1}{2}\left[x\sqrt{x^2+z^2} + z^2 \ln\left(x+\sqrt{x^2+z^2}\right)\right]\left[\begin{matrix} x_2 \\ x_1 \end{matrix}\right. \tag{10}$$

in limit of integration $(0, \frac{d-l}{2})$, a lateral surface area was defined as:

$$B_p = 4\Pi Z^2 \left[\frac{1}{8}\left(\frac{(d-l)^3}{4} + \frac{z^2(d-l)}{2}\right)\sqrt{\frac{(d-l)^2}{4}+z^2} + \right.$$
$$\left. -\frac{z^4}{8}\ln\left(\frac{d-l}{2} + \sqrt{\frac{(d-l)^2}{4}+z^2}\right) + \frac{z^4}{8}\ln z\right] +$$
$$\Pi\,|Z|\,D\left[\frac{d-l}{2}\sqrt{\frac{(d-l)^2}{4}+z^2} + \right.$$
$$\left. z^2 \ln\left(\frac{d-l}{2} + \sqrt{\frac{(d-l)^2}{4}+z^2}\right) - z^2 \ln z\right]. \tag{11}$$

Substituting $z = \dfrac{1}{2Z}$ and after transformation the Eq. (11) leads to:

$$B_p = \frac{1}{16}\Pi Z^2 \left\{\left[(d-l)^3 + \frac{d-l}{2Z^2}\right]\sqrt{(d-l)^2 + \frac{1}{Z^2}} + \right.$$
$$\left. -\frac{1}{2Z^4}\ln\frac{1}{2}\left(d-l+\sqrt{(d-l)^2+\frac{1}{Z^2}}\right) + \frac{1}{2Z^4}\ln\frac{1}{2Z}\right\} +$$
$$+\frac{\Pi D\,|Z|}{4}\left\{(d-l)\sqrt{(d-l)^2+\frac{1}{Z^2}} + \right.$$
$$\left. \frac{1}{Z^2}\ln\frac{1}{2}\left(d-l+\sqrt{(d-l)^2+\frac{1}{Z^2}}\right) - \frac{1}{Z^2}\ln\frac{1}{2Z}\right\}. \tag{12}$$

Substituting for $\sqrt{(d-l)^2+\frac{1}{Z^2}} = U$ was obtained:

$$B_p = \frac{\Pi Z^2}{16}\left\{\left(U^2 - \frac{1}{2Z^2}\right)(d-l)U + \right.$$
$$\left. \frac{1}{2Z^2}\left[\ln(d-l+U) + \ln Z\right]\right\} +$$
$$\frac{\Pi D\,|Z|}{4}\left\{(d-l)\,U + \frac{1}{Z^2}\left[\ln(d-l+U) + \ln Z\right]\right\}. \tag{13}$$

Assuming $\frac{1}{Z^2}[\ln(d-l+U) + \ln Z] = T$, finally form of half lateral surface area of barrel for the parabola shape was obtained:

$$B_p = \frac{\Pi Z^2}{16}\left[\left(U^2 - \frac{1}{2Z^2}\right)(d-l)U - \frac{T}{2Z^2}\right] +$$
$$\frac{\Pi D\,|Z|}{4}\left[(d-l)U + T\right], \tag{14}$$

where diameter of barrel D is unknown. It is possible to determined diameter D using the assumption of a constant volume of barrel being compressed:

$$\frac{1}{6}\Pi d^3 = \frac{1}{15}\Pi\,(d-l)\left(2D^2 + Da + \frac{3}{4}a^2\right). \tag{15}$$

After transformation quadratic equation was obtained:

$$2D^2 + aD - \frac{10d^3 + 3a^2(d-l)}{4(d-l)} = 0, \tag{16}$$

where:

$$\Delta = a^2 + \frac{20d^3 + 6a^2(d-l)}{d-l}. \tag{17}$$

For $0 < a < \sqrt{\Delta}$ positive root of an equation is equal diameter of barrel $D = D_{BP}$

$$D_{BP} = \frac{1}{4}\left(\sqrt{2dl + \frac{1}{3}l^3 + \frac{20d^3 + 6(d-l)(2dl + \frac{1}{3}l^3)}{(d-l)}} + \right.$$
$$\left. \sqrt{2dl + \frac{1}{3}l^3}\right) \tag{18}$$

Finally lateral surface area of barrel for the parabola shape leads to:

$$S_{BP} = \frac{\Pi Z^2}{8}\left[\left(U^2 - \frac{1}{2Z^2}\right)(d-l)\,U - \frac{T}{2Z^2}\right] +$$
$$\frac{\Pi D\,|Z|}{2}\left[(d-l)U + T\right] + \frac{\Pi}{2}a^2 \tag{19}$$

where:

$$T = \frac{1}{Z^2}\,[\ln(d-l+U) + \ln Z\,] \tag{20}$$

$$U = \sqrt{(d-l)^2 + \frac{1}{Z^2}} \tag{21}$$

$$Z = 2\,\frac{a-D}{(d-l)^2} \tag{22}$$

$$D = \frac{1}{4}\left(\sqrt{a^2 + \frac{20d^2 + 6(d-l)a^2}{d-l}} - a\right) \tag{23}$$

$$a = \sqrt{2dl + \frac{1}{3}l^2}\ . \tag{24}$$

In previous papers [3-6] concerning with model of large deformation of compressed rapeseed the barrel diameter D_{BK} and the lateral surface area of barrel for the circular shape S_{BK} was calculated. In order to determine the lateral surface area of the barrel for circular shape it is also necessary to use the formula for calculating surface areas of rotational bodies [3]. The surface of a barrel for circular shape is created by turn of a curve segment. This curve is described by the Eq. (25) of the circle of radius R and the coordinates of the circumcentre (0, s').

$$y = \sqrt{R^2 - x^2} + s'\ . \tag{25}$$

Finally, the surface area of a barrel for circular shape S_{BK} is represented by the formula:

$$S_{BK} = 2\Pi\left[2\left(\frac{D}{2} - R\right)\arcsin\frac{d-l}{2R} + R\,(d-l) + \frac{1}{4}\left(2dl + \frac{1}{3}\,l^2\right)\right], \tag{26}$$

where Eqs (27) and (28) should be substituted for D and R.

$$D_{BK} = \sqrt{\frac{d^3}{d-l} - ld - \frac{1}{6}\,l^2} \tag{27}$$

$$R = \frac{(d-l)^2}{4(D-a)} + \frac{D-a}{4}\ . \tag{28}$$

The expressions for the contact area with a flat plate A (Eq. (2)), the barrel diameter D_{BP} (Eq. (18)) and the lateral surface area of barrel for parabolic shape S_{BP} (Eq. (19)) allow to determine the above value as a function of strain of compressed seed. Changes of these values with respect to strain as so the values of barrel diameter D_{BK} and the lateral surface area of barrel for the circular shape S_{BK} determined in the previous paper are presented in Fig. 3.

A values of the barrel diameters D_{BP} and D_{BK} are similar for each strain of compressed seed. However, the body created by turn of parabola curve is not convex in its shape before compression and value of diameter D_{BP} is insignificant higher than initial diameter of spherical seed. The barrel diameter D_{BK} for circular shape equal initial diameter of spherical seed for strain lower than 0.2 prove that for small deformation, the seeds during compression had spherical shape and most of the authors have used correct assumption.

Changes of cotyledon shape during compression caused shell tensions and increase of surface area surrounding body after deformation. The value of lateral surface area of barrel for the circular shape S_{BK} is lower than lateral surface area of barrel for the parabolic shape for $\varepsilon \leq 0.33$. For the large deformation ε over 0.4 increase of surface area of barrel for the

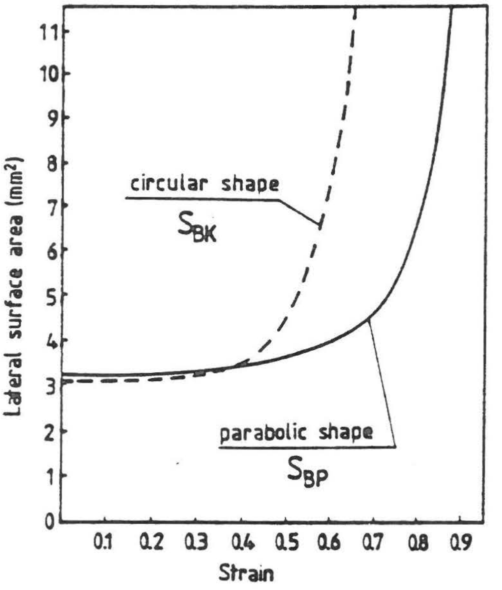

Fig. 3. A lateral surface area of barrel for parabolic shape S_{BP} and for circular shape S_{BK} related to strain of convex body during compression between parallel plates.

circular shape S_{BK} was higher than for parabolic shape S_{BP}.

DISCUSSION

The resistance of pea seeds to compression was studied for 4 pea varieties for wide range of moisture content of pea seeds (8-70 %). Every single seed was compressed between two parallel plates with the INSTRON testing machine and the force-displacement curve was registered.

It has been found [5] that the seeds at low moisture content were hard and damage occurs at a slight deformation (0.6-1 mm). The plasticity of seeds grows together with the increase of moisture content and the seed damage occurs at very large displacement of up to 7.5 mm. It was characteristic of all varieties that the values of the mechanical resistance increase together with the growth of the moisture content, but only to the level 14-18 %. Further increase of moisture content caused decrease of mechanical resistance of pea seeds.

The moisture content of 20-22 % tends to decrease which results from the increasing stress exerted on the shell by the swelling seed.

It was interesting to note [5] that the force causing the shell damage for smooth seeds is considerably smaller than for wrinkled seeds in the range of their greatest resistance to damage (12-18 %), and it was (343-418 N) for smooth seeds of variety Sześciotygodniowy while for wrinkled of variety Nike was (480-643 N). The differentiation of the resistance to damage of the smooth seeds and those which are wrinkled and only partly filled with water-despite their comparable resistance in the dry or swollen state - seems to suggest that the shell of a wrinkled seed with the same volume as the smooth one but with larger surface causes shell damage at greater deformation and strain (Fig. 4).

The surface area of barrel for the circular shape S_{BK} higher than for parabolic shape S_{BP} shows that geometrical model describing surface area of barrel for parabolic shape is adequate for wrinkled seed and for circular shape is adequate for smooth seed during compression between parallel plates.

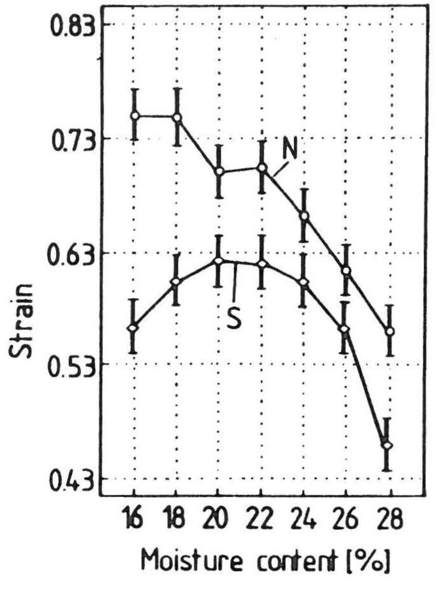

Fig. 4. Strain noticed at the break of seed shell for pea of Nike (N) and Sześciotygodniowy (S) varieties compressed between parallel plates at different moisture level.

CONCLUSIONS

1. Spherical body compressed between parallel plates change its shape into circular barell for small deformation and into parabolic barell during large deformation, for strain over 0.4.

2. Geometrical model describing surface area of barrel for parabolic shape is adequate for wrinkled pea seed during compression between parallel plates and surface area of barrel for circular shape is adequate for smooth pea.

3. Deformation of cotyledon while seed compressed caused the shell tension, but only for strain higher than 0.2. It suggests that for lower deformation any force caused tension stress of pea seed shell.

4. The highest values of strain $\varepsilon = 0.75$ was observed for wrinkled variety Nike at 16-18 % moisture content and for smooth variety Sześciotygodniowy strain reached value $\varepsilon = 0.62$ at 20-22 % of moisture content.

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