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Application of the Nonhomogeneous Poisson Process for Counting Earthquakes

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ABSTRACT

Nonhomogeneous Poisson processes are Poisson processes with parameters that depend on time and not constant from time to time, also mutually independent. In addition, the probability of no occurrence in the initial state is one and the probability of the *n* event in the initial state is zero. In this study, nonhomogeneous Poisson processes were applied to predict and count the number of earthquake events in Indonesia. Because earthquakes that occur in Indonesia from one month to the next do not affect each other and the numbers are not the same, by not paying attention to the geophysical cause of the earthquake. The data used in the study is the occurrence of earthquakes in Indonesia from January 2016 to July 2018, sourced from the Meteorology, Climatology and Geophysics Agency obtained online. The results were obtained that the odds of predicting dan counting the occurrence of earthquakes in Indonesia in the first week of December 2018 were around 184 times with a standard deviation of around 14 times.

***Keywords*:** Nonhomogeneous Poisson processes, earthquakes, simple linear regression

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1. INTRODUCTION

The Poisson process is a counting process for the number of events that occur at a certain time, with a parameter λ. The Poisson process is a special activity of the process of counting where intervals of events are mutually independent, have free increases (do not need to be stationary) and all are exponentially distributed. If the exponential distribution has the same parameter value then it is called a homogeneous Poisson process. However, if it is not the same, it is called a nonhomogeneous Poisson process. Nonhomogeneous Poisson processes with for each are ordinary Poisson processes.

The following are previous studies conducted in the development and expansion of the theory of Poisson processes. According to Yu (2015), catastrophe option pricing model with double jump processes: (i) stock process of insurance companies that sell catastrophe options are explained by an exponential jump-diffusion process, and (ii) all jump terms are modeled with two compound Poisson processes [1]. In his research, Rao (2015) introduced a process class called filtered fractional Poisson process and studied its properties [2], while Kataria and Vellaisamy (2017) obtained probabilities for various fractional versions of the classic homogeneous Poisson process using the Adomian decomposition method [3]. Elizar (2016) analyzed the Poisson Aggregation process which is a stochastic model in which random collections of random balls are stacked over a general metric space [4]. Cholaquidis et al. (2017) tackle the problem of supervised classification of the Poisson process with values in a general metric space via the classical *k*-nearest neighbor and Bayes rule [5]. Fu and Feng (2018) showed that the sample autocovariance and autocorrelation of the increments of a mixed Poisson process converge to zero were almost certain as the sample size goes to infinity so that the sample autocovariance or autocorrelation could not be used in the moment method for estimating mixed Poisson process parameters [6].

Nonhomogeneous Poisson processes are Poisson processes with parameters that depend on time. This means that the probability of no event in the initial state is one and the probability of *n* event in the initial state is zero. Nonhomogeneous Poisson processes can be applied to everyday life, for example measuring daily ozone gas [7], model of noise exposure [8], new approaches to improving software reliability models [9], [10] and descriptions the different kinds of accident number [11], [12]. Fathi-Vajargah and Khoskar-Foshtomi (2014) study the general nonhomogeneous Poisson point process based on the function of intensity and algorithm to produce it [13]. Asfaw and Lindqvist (2015) studied the possible consequences of heterogeneity in the failure intensity of repairable system where the basic model used is a nonhomogeneous Poisson process with power-law intensity function [14].

Cifuentes-Amado and Cepeda-Cuervo (2016) show how the modifications applied to the intensity function of the Poisson nonhomogeneous process specifically improve the analysis and suitability of hospital admissions every day because of dengue fever at Ribeirão Petro, Brazil [15]. Slimacek and Lindqvist (2016) develop methods for parameter estimation of nonhomogeneous Poisson processes with unobserved heterogeneity that do not require parametric assumptions about heterogeneity [16]. Leonenko et al. (2017) introduced a nonhomogeneous fractional Poisson process by replacing time variables in the fractional Poisson process to obtain the distribution of arrival times [17]. Cebrián et al. (2015) made it easier for researchers interested in modeling data using a nonhomogeneous Poisson process by introducing R packages for modeling nonhomogeneous Poisson processes in one dimension called **NHPoisson** [18].

Indonesia is grieving again, has not finished recovery after the earthquake at Lombok on July 29th, 2018. Two months later, Indonesia was again rocked by a massive earthquake reaching 7.4 SR, which was soon followed by a five-meter high tsunami that devastated the city of Palu and its surroundings on September 28th, 2018.

It was even noted that after the shock of the large magnitude earthquake it was followed by other shocks measuring 1.0 to 5.0 SR. Actually the occurrence of earthquakes in Indonesia is not a rare occurrence, it has been noted that in the last 15 years there have been many earthquakes with a magnitude of considerable magnitude that have rocked Indonesia, such as in Aceh (2004, 2012), Nias (2005), Pangandaran (2006), Bengkulu (2007), Mentawai (2016), Lombok (2018), then Palu and Donggala (2018).

Indonesia is geographically located between the plates of Australia, Eurasia and the Pacific so that the movement of friction between plates can cause earthquakes from small to large forces. In addition, Indonesia is also included in the Pacific fire ring which is a group of volcanoes in the world. This causes Indonesia to have frequent earthquakes, both tectonic and volcanic. The Meteorology, Climatology and Geophysics Agency noted that over a period of a month an earthquake that occurred in Indonesia could reach more than 300 times. The consequences of earthquakes are often followed by other disasters, such as shocks and landslides and tsunamis. Large-scale earthquakes, usually more than 5.0 SR, can cause losses, such as loss of life, damage to homes and settlements, damage to public infrastructure (roads, schools and hospitals), and even fire.

Based on the background of the problem described, this study applied a nonhomogeneous Poisson process to count the number of earthquakes occurring in Indonesia, and to predict their probability.

The following are previous studies of earthquakes both mathematically and generally. Purwanto and Oyama (2014) quantitatively examined trends in past natural disasters, especially earthquakes and tsunamis in Japan and Indonesia by applying mathematical policy analysis techniques in natural disaster risk analysis and assessment to develop policies to reduce casualties caused by both of these natural disasters [19].

Ikram and Qamar (2015) proposed an expert system for predicting earthquakes from previous data by applying mining association rules [20]. Florido et al. (2015) proposed and analyzed seismic time series from four of the most active zones in Chile to find a preliminary pattern of large earthquakes [21].

The system for predicting earthquakes used by Asencio-Cortés et al. (2016) is that different parameters for input in the supervised learning algorithm have been thoroughly analyzed using a new methodology [22], then the Asencio-Cortés et al. (2018) study reverted to using a combination of algorithms regression with ensemble learning explored in the context of big data [23].

Martín-González (2018) provides information about the parameters and sources of historical and archaeological seismic events that are used to evaluate seismic hazards in a region to be better [24]. Xu et al. (2018) proposed a method of predicting earthquakes in California based on minimum edge weight [25]. In particular, Kurasaki et al. (2019) investigated the effect of earthquakes on patients with Parkinson's disease [26].

Furthermore, the systematic writing in this paper includes: Section 2 discusses the basic theory of nonhomogeneous Poisson processes, Section 3 describes the data and research methods used, Section 4 presents the results and discussion of applying nonhomogeneous Poisson processes to count earthquake events, and Section 5 presents the conclusion.

2. NONHOMOGENEOUS POISSON PROCESSES

Experiments that produce values for a random variable *X*, namely the number of experiments that occur during a certain interval or in a certain area are called Poisson experiments. Poisson experiments have the following characteristics [27]:

1) The number of results of experiments that occur at intervals or certain regions does not depend on the number of experimental results that occur at intervals or other regions.
2) Probabilities for the occurrence of one experimental result during a very short interval or a very small area, proportional to the length of the time interval or the size of the area and does not depend on the number of experimental results that occur outside the time interval or other regions.
3) Probabilities that more than one result of an experiment will occur in a short time interval or a small area, can be ignored.

Suppose *X*, which states the number of results of a Poisson experiment, is called a Poisson random variable and its probability distribution is called a Poisson distribution. Poisson distribution is a discrete probability distribution that states the probability of the number of events over a period of time if the average inter-time event of that time is mutually independent. This distribution is usually applied to events with a large number of possibilities and is quite rare.

Definition 1. The probability distribution for Poisson random variables *X*, which states the number of experimental results that occur during a certain interval or area, namely:

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!};\ x = 1, 2, 3, \ldots$$

where: is the average number of results of the experiment that occurs during the specified time interval or area and $e = 2.71828\ldots$

Definition 2. [28] The stochastic process $\{N(t); t \geq 0\}$ is said to be the counting process if $N(t)$ represents the total number of events that have occurred from time to time or in the time interval *t*, if it satisfies:

i. $N(t) \geq 0$,
ii. $N(t)$ is integer number,
iii. if $s < t$ then $N(s) \leq N(t)$,
iv. $s < t$, $N(s) - N(t)$ states the number of events that occur at time intervals $(s, t]$.

The counting process is called an independent increment process if the number of events that occur at intervals is mutually independent. For example, the number of events at time *t* is $N(t)$, independent of the number of events that occur at the time between *t* and $t + s$ that is $N(t+s) - N(t)$.

The counting process is called a process with stationary increments if the distribution of the number of events that occur at certain time intervals depends only on the length of the interval, not depending on that location. For example, the number of events at the time interval $(t_1+s, t_2+s]$ is $N(t_2+s)-N(t_1+s)$ has the same distribution with the number of events at time intervals $(t_1, t_2]$ is $N(t_2)-N(t_1)$ for all $t_1 < t_2, s > 0$.

Definition 3. [28] The Poisson process is a process of calculating with parameters $\lambda > 0$, it is satisfies:

i. $N(0)=0$,
ii. the process has an independent increment,
iii. the number of events at time intervals during t follows the Poisson distribution with averages λ_t for all s and $t \geq 0$

$$P\{N(t+s)-N(s)=n\} = e^{-\lambda t}\frac{(\lambda t)^n}{n!}; \quad n = 0, 1, 2, ...$$

Definition 4. [11] Counting process $\{N(t); t \geq 0\}$ is called nonhomogeneous Poisson process with the intensity function $\lambda(t)$, $t \geq 0$, if:

i. $P\big(N(0)=0\big)=1$,
ii. the counting process $\{N(t); t \geq 0\}$ is is a stochastic process with independent increment,
iii. $P\{N(t+s)-N(t)=k\} = \dfrac{\left(\int_t^{t+s}\lambda(x)dx\right)^k}{k!} e^{-\int_t^{t+s}\lambda(x)dx}$.

Based on the definition above, then the chance of no event in the initial state is one and the number of events that occur at a time interval with the next time interval is mutually independent.

Suppose

$$\Lambda(t) = \int_0^t \lambda(x)dx$$

then

$$P\{N(t+s)-N(t)=k\} = \frac{e^{-(\Lambda(t+s)-\Lambda(t))}(\Lambda(t+s)-\Lambda(t))^k}{k!}$$

with $n \geq 0$. Thus $N(t+s)-N(t)$ Poisson distributed with an average value $\Lambda(t+s)-\Lambda(t)$. Therefore for $n=0$, then obtained $P_0(s) = e^{-(\Lambda(t+s)-\Lambda(t))}$. The nonhomogeneous Poisson process can also be illustrated in Figure 1, as follows:

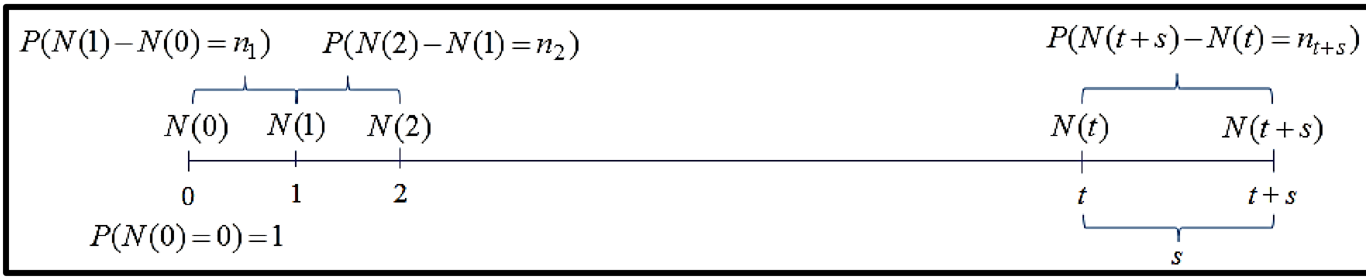

Figure 1. Illustration of nonhomogeneous Poisson processes

Moreover, the distribution of nonhomogeneous Poisson processes is as follows:

$$P\{N(t)=k\}=\frac{\left(\int_0^t \lambda(x)dx\right)^k}{k!}e^{-\int_0^t \lambda(x)dx}. \quad (1)$$

Expectations:

$$\Lambda(t)=E[N(t)]=\int_0^t \lambda(x)dx, \qquad t\geq 0. \quad (2)$$

Variance:

$$\mathrm{V}(t)=V[N(t)]=\int_0^t \lambda(x)dx, \qquad t\geq 0. \quad (3)$$

Standard derivation:

$$\mathrm{D}(t)=\sqrt{V[N(t)]}=\sqrt{\int_0^t \lambda(x)dx}, \qquad t\geq 0. \quad (4)$$

Expected value from time interval increase, $N(t+s)-N(t)$:

$$\Delta(t,s)=E\big(N(t+s)-N(t)\big)=\int_t^{t+s} \lambda(x)dx. \quad (5)$$

Standard derivation:

$$\sigma(t,s)=\sigma\big(N(t+s)-N(t)\big)=\sqrt{\Delta(t,s)}=\sqrt{\int_t^{t+s} \lambda(x)dx}. \quad (6)$$

Theorem 5. The number of *n* events is independent of the nonhomogeneous Poisson process, where the addition of random variables is Poisson distribution, i.e. $N_1(t), N_2(t), \ldots, N_n(t)$ with each parameter $\lambda_1(t), \lambda_2(t), \ldots, \lambda_n(t)$ is a nonhomogeneous Poisson process with parameters $\lambda(t)=\lambda_1(t)+\lambda_2(t)+\ldots+\lambda_n(t)$, $t\geq 0$.

In other words, the counting process $\{N(t);t\geq 0\}$, $N(t)=N_1(t)+N_2(t)+\ldots+N_n(t)$ is a nonhomogeneous Poisson process. This process has the following distribution of probabilities:

$$P\{N(t)=k\}=\frac{\left(\int_0^t \lambda(x)dx\right)^k}{k!}e^{-\int_0^t \lambda(x)dx}, \qquad k=1,2,3,\ldots$$

where $\lambda(x)=\lambda_1(x)+\lambda_2(x)+\ldots+\lambda_n(x)$, $x\geq 0$.

3. DATA DAN METHOD

Earthquakes are the events of vibrating or shaking of the earth due to the movement or sudden shift of rock layers on the earth's skin due to the movement of tectonic plates or volcanic activity [29]. Indonesia is an area with high earthquake activity because it is geographically located between the plates of Australia, Eurasia and the Pacific, so that the movement of friction between plates can cause earthquakes from small to large forces. In addition, Indonesia is also included in the Pacific fire ring which is a group of volcanoes in the world. The Meteorology, Climatology and Geophysics Agency noted that over a period of a month an earthquake that occurred in Indonesia could reach more than 300 times. The consequences of earthquakes are often followed by other disasters, such as shocks and landslides, and tsunamis.

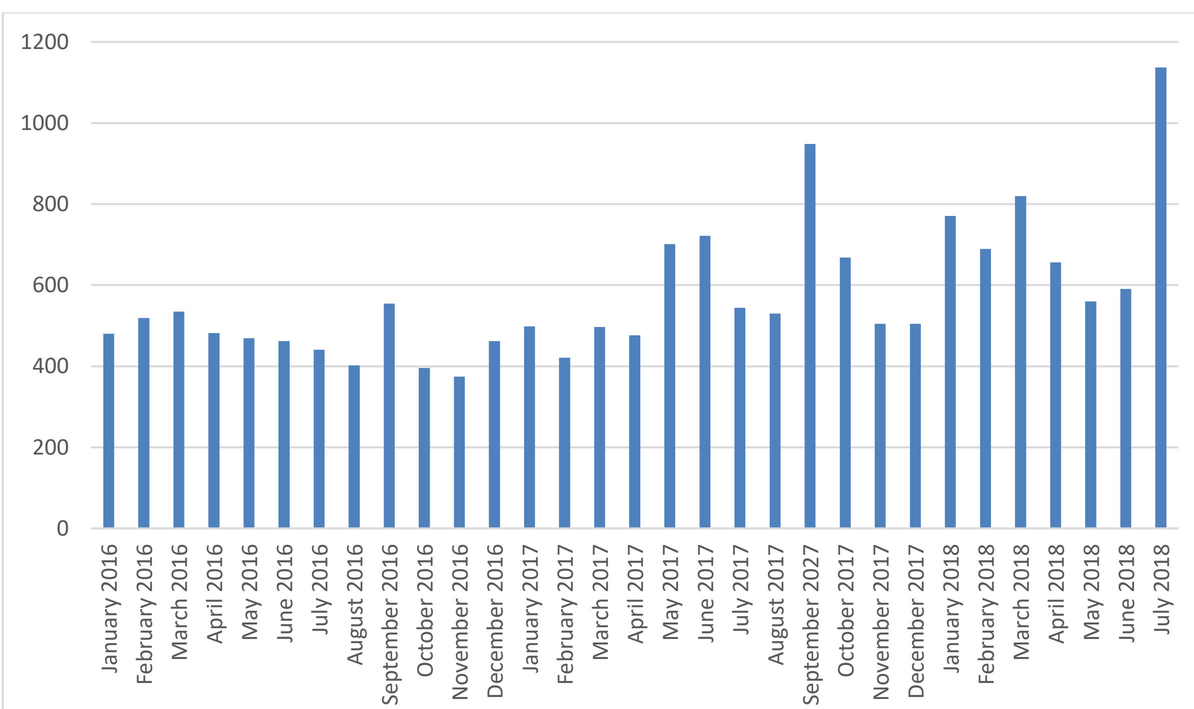

Figure 2. The number of earthquakes that occurred in Indonesia from January 2016 to July 2018

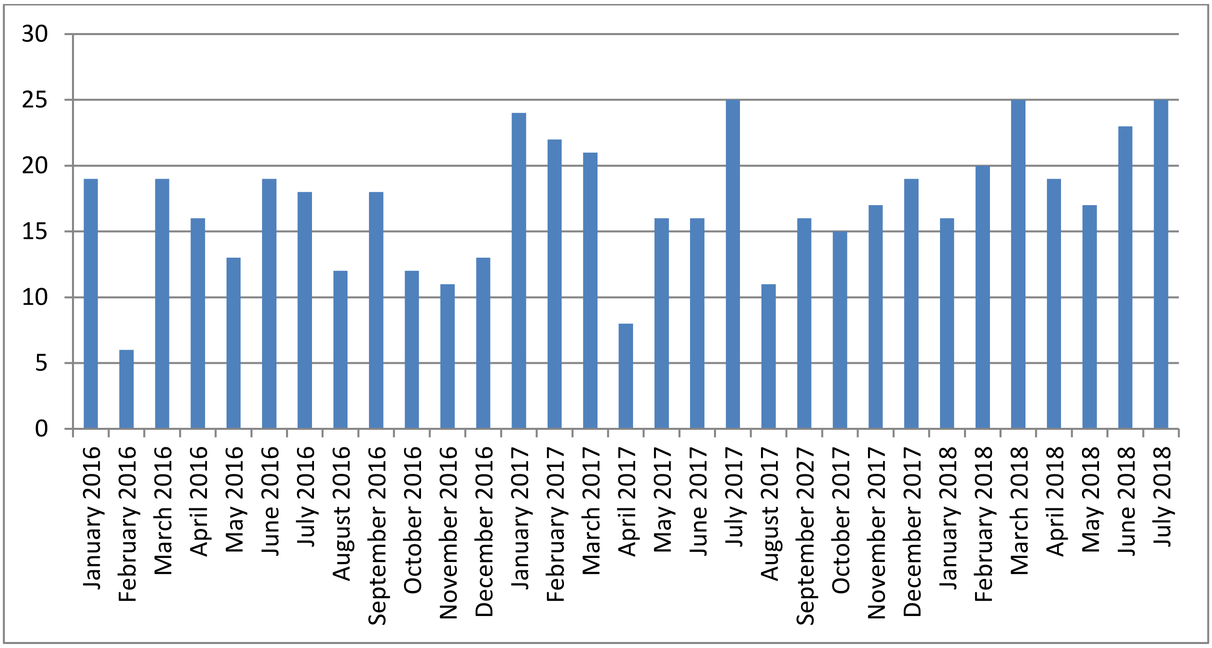

Figure 3. The number of earthquakes that occurred in Indonesia from January 2016 to July 2018 with a magnitude of more than 5.0 SR.

Large-scale earthquakes, usually more than 5.0 SR, can cause losses, such as loss of life, damage to homes and settlements, damage to public infrastructure (roads, schools and hospitals), and even fire. In this study used earthquake data in Indonesia obtained online sourced from the Meteorology, Climatology and Geophysics Agency from January 2016 to July 2018.

It can be seen in Figures 2 and 3, that earthquakes that occur from one month to the next month do not affect each other and the numbers are not the same, not paying attention to the geophysical cause of the earthquake. So that the number of earthquakes can be assumed as a nonhomogeneous Poisson process.

Simple Linear Regression

Basically, simple linear regression is a functional influence or relationship between one independent variable (X) with one non-independent variable (Y) [30]. In this study, linear regression was used to connect the independent variables of time (per day) and the non-independent variables of the number of earthquake events in Indonesia. Here is a simple general linear regression equation:

$$\hat{Y} = \beta_0 + \beta_1 X \tag{7}$$

with

$\hat{Y}$: the number of non-free variables estimated

β_0 : value Y if $X = 0$

β_1 : regression coefficient

X : independent variable

where the value of a and b can be searched by the formula as follows:

$$\beta_0 = \frac{\left(\sum Y_i\right)\left(\sum X_i^2\right) - \left(\sum X_i\right)\left(\sum X_i Y_i\right)}{n\sum X_i^2 - \left(\sum X_i\right)^2}, \tag{8}$$

$$\beta_1 = \frac{n\sum X_i Y_i - \left(\sum X_i\right)\left(\sum Y_i\right)}{n\sum X_i^2 - \left(\sum X_i\right)^2}. \tag{9}$$

4. DISCUSS AND RESULT

Note the data as presented in Figure 2, by dividing the number of earthquakes every month with 28, 29, 30 or 31 days, the intensity of the earthquake per day will be obtained, as shown in the table below.

Table 1. The empirical intensity of earthquakes in Indonesia from January 2016 to July 2018.

Month	Interval	Median Interval	Number of Earthquakes	Intensity per day
January 2016	[0, 31)	15.50	480	15.48387
February 2016	[31, 60)	45.50	519	17.89655
March 2016	[60, 91)	75.50	535	17.25806
April 2016	[91, 121)	106.0	482	16.06667
May 2016	[121, 152)	136.5	469	15.12903
June 2016	[152, 182)	167.0	462	15.40000
July 2016	[182, 213)	197.5	441	14.22581
August 2016	[213, 244)	228.5	402	12.96774
September 2016	[244, 274)	259.0	554	18.46667
October 2016	[274, 305)	289.5	396	12.77419
November 2016	[305, 335)	320.0	374	12.46667
December 2016	[335, 366)	350.5	462	14.90323
January 2017	[366, 397)	381.5	498	16.06452
February 2017	[397, 425)	411.0	421	15.03571

March 2017	[425, 456)	440.5	497	16.03226
April 2017	[456, 486)	471.0	476	15.86667
May 2017	[486, 517)	501.5	701	22.61290
June 2017	[517, 547)	532.0	722	24.06667
July 2017	[547, 578)	562.5	544	17.54839
August 2017	[578, 609)	593.5	530	17.09677
September 2027	[609, 639)	624.0	948	31.60000
October 2017	[639, 670)	654.5	668	21.54839
November 2017	[670, 700)	685.0	505	16.83333
December 2017	[700, 731)	715.5	505	16.29032
January 2018	[731, 762)	746.5	771	24.87097
February 2018	[762, 790)	776.0	689	24.60714
March 2018	[790, 821)	805.5	820	26.45161
April 2018	[821, 851)	836.0	656	21.86667
May 2018	[851, 882)	866.5	560	18.06452
June 2018	[882, 912)	897.0	591	19.70000
July 2018	[912, 943)	927.5	1137	36.67742

Using equations (7), (8) and (9) for the data in Table 1 above, a simple linear regression approach is obtained for the number of earthquakes each day, as follows:

$$\hat{\lambda}(x) = 13.0444 + 0.01242x. \qquad (10)$$

Then through equation (2), obtained:

$$\Lambda(t) = \int_0^t (13.0444 + 0.01242x)\,dx = 13.0444t + 0.00621t^2. \qquad (11)$$

Based on equation (1), (2) and in equation (11), a one-dimensional distribution is obtained for nonhomogeneous Poisson processes on earthquake problems in Indonesia, namely

$$P\{N(t) = k\} = \frac{(\Lambda(t))^k}{k!} e^{-\Lambda(t)} = \frac{(13.0444t + 0.00621t^2)^k}{k!} e^{-13.0444t + 0.00621t^2}. \qquad (12)$$

So modeling the number of earthquakes in Indonesia is a nonhomogeneous Poisson process with parameters $\Lambda(t) = 13.0444\,t + 0.00621\,t^2$, where $t \geq 0$.

Then from Definition 4, obtained

$$P\{N(t+s) - N(t) = k\} = \frac{(\Lambda(t+s) - \Lambda(t))^k}{k!} e^{-(\Lambda(t+s)-\Lambda(t))}. \quad (13)$$

This means that we can anticipate the number of earthquakes at a time interval with the length of the interval *s*, with the expected value of the time interval increase $N(t+s) - N(t)$, as in equation (5). Suppose we will predict the number of earthquakes occurring from 1 to 8 December 2018, So that we get a time interval [1065, 1072), where the length of the interval is *s* = 7 and *t* = 1065. Using equation (5), we get

$$\Delta(1065{,}7) = E(N(1072) - N(1065)) = \int_{1065}^{1072} (13.0444 + 0.01242x)\,dx = 184.20619,$$

and

$$\sigma(1065{,}7) = \sigma(N(1072) - N(1065)) = \sqrt{\Delta(1065.7)} = \sqrt{184.20619} = 13.5722581.$$

This means that the prediction of the average number of earthquakes in Indonesia from December 1 to 8 2018 is around 184 times the earthquake with a standard deviation of around 14. Furthermore, if the earthquake event is grouped based on its magnitude scale as shown in Figure 4. Then the nonhomogeneous Poisson process can be represented as the sum of $N(t) = N_1(t) + N_2(t) + N_3(t) + N_4(t)$, where $N_1(t)$ is the number of earthquakes with magnitude less than 3.0 SR, $N_2(t)$ is the number of earthquakes with magnitude 3.0-3.9 SR, $N_3(t)$ is the number of earthquakes with a magnitude scale of 4.0-4.9 SR and $N_4(t)$ is the number of earthquakes with magnitude scales more than 5.0 SR, Which matches the process parameters from Figure 4 below:

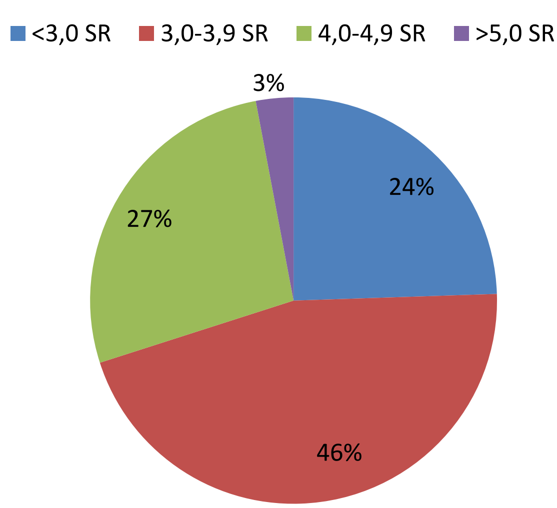

Figure 4. The number of earthquakes in Indonesia based on their magnitude scale.

An earthquake with a magnitude of more than 5.0 SR has begun to be felt by many people and causes minor to severe damage to a building, regardless of the depth of the epicenter. Based on equations (11) and (14), process expectations or estimates $N_4(t)$ illustrate the number of earthquakes occurring in Indonesia with magnitudes more than 5.0 SR, namely:

$$\Lambda_4(t) = 0.02981\left(13.0444\,t + 0.00621\,t^2\right) = 0.388853564\ \ t + 0.0001851\ \ 201t^2. \qquad (15)$$

Then, suppose we will predict the number of earthquakes in Indonesia with magnitude scales of more than 5.0 SR from 1 to 8 December 2018. So that time intervals are obtained [1065, 1072). Then the expectations or estimates obtained are as follows:

$$\Delta_4(1065,7) = E\left(N(1072) - N(1065)\right) = 5.491186524\ ,$$

and

$$\sigma_4(1065.7) = \sigma\left(N(1072) - N(1065)\right) = \sqrt{5.491186524} = 2.343328087.$$

This means that the prediction of the average number of earthquakes in Indonesia with a magnitude of more than 5.0 SR on 1 to 8 December 2018 is about 5 times the earthquake with a standard deviation of about 2 times.

5. CONCLUSION

Indonesia is one of the countries with a high potential for earthquakes because it is geographically located between three plates, namely the Australian, Eurasian and Pacific plates. In addition, Indonesia is also included in the Pacific fire ring which is a group of volcanoes in the world. This causes Indonesia to have quite frequent earthquakes both tectonic and volcanic. The earthquakes that occurred in Indonesia from one month to the next did not affect each other and the numbers were not the same, not paying attention to the geophysical causes of the earthquake. So that the number of earthquakes can be assumed as a nonhomogeneous Poisson process.

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