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INFLUENCE OF THE IRREGULARITY COEFFICIENT OF LOADING ON CALCULATED FATIGUE LIFE

Roland Pawliczek, Krzysztof Kluger

Opole University of Technology, Department of Mechanic and Machine Design, Opole, Poland
e-mail: r.pawliczek@po.opole.pl; k.kluger@po.opole.pl

The paper presents comparison of two methods for determination of the fatigue damage degree in a material subjected to random loadings with participation of the mean loading and including the irregularity coefficient of the loading. The authors estimated effectiveness of the method where the stress history was transformed considering the global mean value of the load history and the method for which the stress mean value of the distinguished loading cycles was taken into account. When the irregularity coefficient of loading increases, we can observe significant divergences of the calculated fatigue life of the material related to the model of amplitude transformation because of the loading mean value.

Key words: fatigue, irregularity coefficient, mean stress

1. Introduction

Variable-amplitude and random loadings, so-called service loadings, are about 90% of all loadings occurring in practice. The occurrence of an additional static loading of machine elements and structures is an important element of the service loadings. The additional static value is often a result of the influence of a dead weight of a working element or a structure, it can be also an effect of pre-tension of elements transferring loadings (for example wedge belts in transmissions). Estimation of the fatigue life under such loadings is difficult – it requires selection of a suitable estimation model; there are also many difficulties connected with time-consuming calculations, etc.

In the case of service loadings, the widely applied algorithm for fatigue life determination includes the following steps (Macha *et al.*, 2005, 2009; Kluger and Łagoda, 2007; Łagoda *et al.*, 2008):

1. Determination of loading history in form of histories of strain, stress, forces or moments;
2. Determination of amplitudes and mean values of cycles and half-cycles with the rain flow method;
3. Determination of equivalent values (stress, strain) according to chosen criteria of multiaxial fatigue;
4. Determination of the damage degree according to the chosen hypothesis of damage accumulation;
5. Calculation of fatigue life of the element.

The stress models are the most often applied models of fatigue life estimation, including influence of static loading. The models proposed by Goodman, Gerber and Morow (Kluger and Łagoda, 2007; Łagoda *et al.*, 2008; Macha *et al.*, 2009) are the most popular because they are very simple, and it is easy to apply them under amplitude-constant loadings. In the case of their application for service loadings, calculations require much calculation power and they are time-consuming as simple transformations of amplitudes must be repeated many times because of the mean values.

This paper presents test results including the influence of factors resulting from the parameters of time histories not included in the general algorithm of fatigue life determination.

2. Calculation algorithm

Łagoda *et al.* (2001) compared effectiveness of the algorithm for fatigue life calculation according to some procedures including the loading mean value in a different way. This paper concerns loadings with mean values, so the fatigue life is determined according to two methods: method 1 includes the global mean stress history approach, and method 2 is the local mean stress of the selected cycle approach (see Fig. 1).

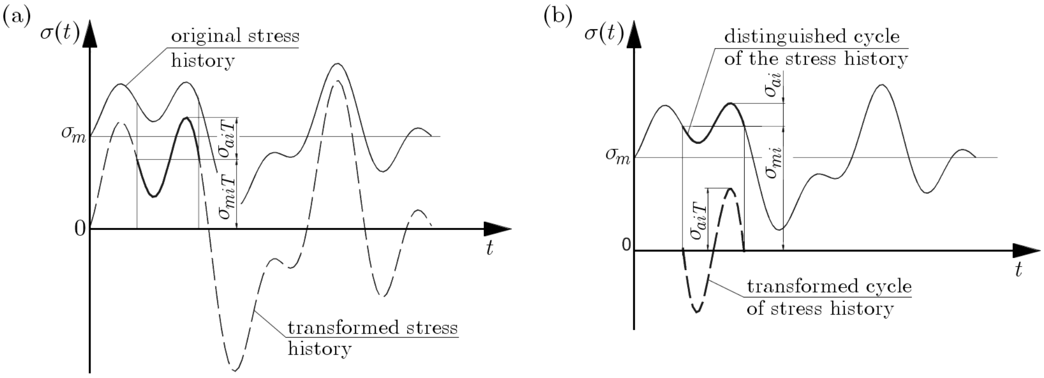

Fig. 1. Graphical interpretation of the procedure of stress transformation amplitude; (a) method 1 – global approach, (b) method 2 – local approach

In the global approach, the transformation includes the global mean value of the stress σ_m. The transformation is performed by history scaling with the coefficient K dependent on the assumed transformation model

$$K = \frac{1}{1 - \left(\frac{\sigma_m}{A}\right)^c} \tag{2.1}$$

where: $A = R_m$ and $c = 1$ for the Goodman relationship, $A = R_m$ and $c = 2$ for the Gerber relationship, and $A = \sigma'_f$ and $c = 1$ for the Morrow relationship, R_m – tensile strength of the material, σ'_f – fatigue strength coefficient.

In a consequence, we obtain the transformed history with the zero mean value of the stress and higher instantaneous values. At the next step, when cycles of the transformed history in the distinguished cycle with the amplitude σ_{aiT} are counted, the mean value σ_{miT} occurs (Fig. 1a) – however, it is not taken into account because it causes double transformation.

In the case of the local approach, at first cycles with the given amplitude, σ_{ai} and the local mean value σ_{mi} are distinguished. Next, each distinguished cycle is transformed (Fig. 1b) and in a consequence we obtain a cycle with the zero mean value $\sigma_{miT} = 0$ and the amplitude σ_{aiT} enlarged with the coefficient K_i

$$K_i = \frac{1}{1 - \left(\frac{\sigma_{mi}}{A}\right)^c} \tag{2.2}$$

where A and c are defined like in Eq. (2.1). Let us note that the coefficient value is not constant and it changes for each distinguished cycle.

The damage degree $S(T_O)$ at the observation time T_O of the stress history is calculated by summation of damages from the distinguished amplitudes of cycles and half-cycles σ_{ai} according to the following relationship

$$S(T_O) = \begin{cases} \displaystyle\sum_{i=1}^{k} \frac{n_i}{N_G\Big(\frac{Z_{rc}}{\sigma_{aiT}}\Big)^m} & \text{for} \quad \sigma_{aiT} \geqslant aZ_{rc} \\ 0 & \text{for} \quad \sigma_{aiT} < aZ_{rc} \end{cases} \tag{2.3}$$

where: σ_{ai} – stress amplitude of the distinguished cycle, n_i – number of cycles with the amplitude σ_{ai}, m – coefficient of the Wöhler curve slope, Z_{rc} – fatigue limit for tension-compression, N_G – limit number of cycles corresponding to Z_{rc}.

While summing up the damages, the amplitudes below the fatigue limit were also included, assuming $a = 0.5$ (Lachowicz *et al.*, 1995).

There are significant differences between the considered procedures. From Łagoda *et al.* (2001) it appears that including the influence of the loading static value in the global approach (all the loading history) gives similar results of fatigue life calculations like in the case of the local approach, i.e. within one cycle of loading change. The global approach allows one to shorten time of calculations, and this is important when the calculation model is applied for estimation of the fatigue life of elements at real time, or in the case of fatigue tests performed at the stands with participation of generated loading histories.

In laboratory practice, a reproduced load history should correspond to the service loading as better as possible. That leads to use some standardized load histories (e.g. Shutz *et al.*, 1990) or generation of load histories with the use of the Markov matrix (Shutz *et al.*, 1990). Additionally, generation methods must take into account different statistical parameters of the reproduced loads (Macha and Pawliczek, 2007, 2010). However, there are no clear reasons to say that other parameters of service courses, not resulting from the considered calculation procedure, influence its results.

In the case of fatigue behavior of materials, also the coefficient of loading irregularity is an important parameter. It is defined as

$$I = \frac{N^+_{\sigma_m}}{N^+_{\sigma_{max}}} \tag{2.4}$$

where: $N^+_{\sigma_m}$ is the number of transitions through a level of the expected loading value with a positive slope at the time unit, $N^+_{\sigma_{max}}$ – number of local peaks at the time unit.

Figure 2a presents fragments of variable-amplitude histories with the same mean values and the history variance but with different values of the irregularity factor ($I = 0.4$ and $I = 0.99$ respectively). Both histories were generated based on the same Markov matrix – basic statistical parameters are the same for both histories. The histograms received for analyzed courses are presented in Fig. 2b. It can be seen that distribution of the stress values is almost the same for $I = 0.4$ and $I = 0.99$. However, there is a visible difference between the time history shapes. Such an effect influences the parameters of the cycles distinguished, when the rain flow algorithm is applied. For higher values of factor I, cycles counted in the rain flow method characterizes with lower mean stress values and higher amplitudes. When the irregularity coefficient decreases, the rain flow method detects cycles with higher values of the mean stress and smaller amplitudes. Such a situation can strongly influence a degree of the accumulated damages in the considered histories, especially if the mean load is present.

Different values of coefficient I lead to a different type of the load. For $I = 1$, the cyclic process can be observed – only one dominant frequency is detected. Decreasing the value of the irregularity coefficient, we can observe a narrow band random process and a wide band random process for lower values of this coefficient (Agreskov and Nielsen, 1999; Łagoda *et al.*,

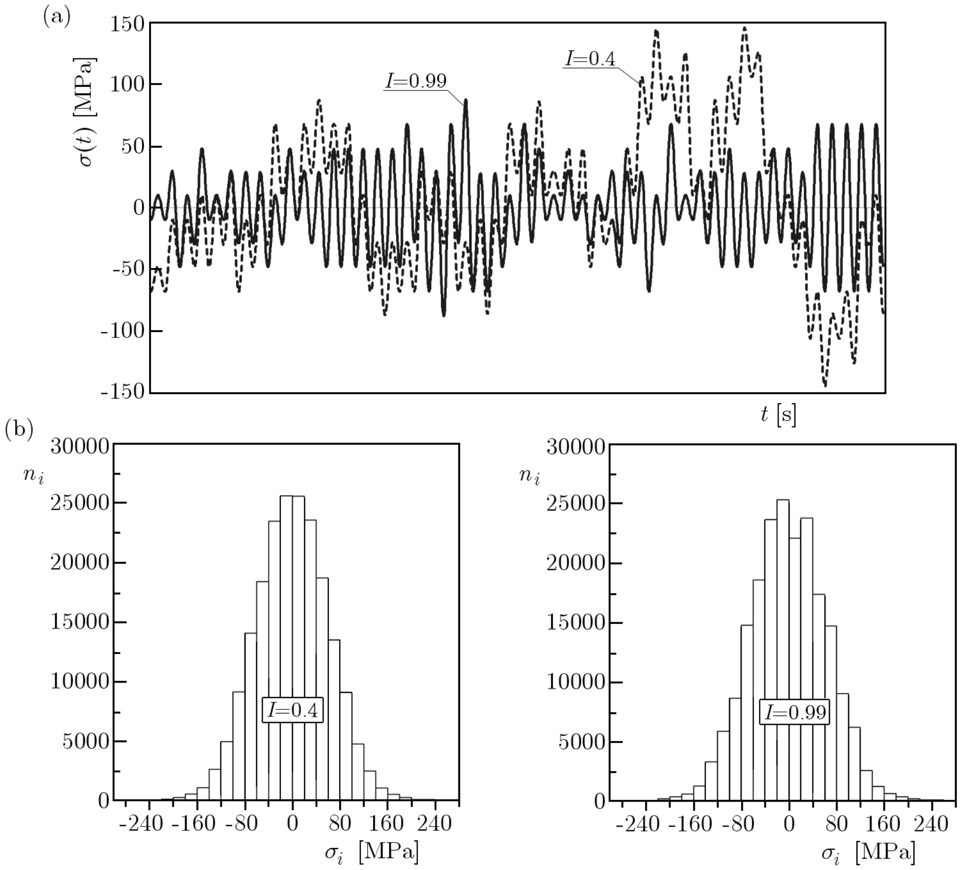

Fig. 2. Stress histories with coefficients of irregularity $I = 0.4$ and $I = 0.99$; (a) example of the time courses of the stresses, (b) histograms of the stress values

2005b; Petrucci and Zuccarello, 2004). Using the power spectrum density of the random process, it is possible to estimate irregularity factor I and the spectral method of the fatigue damage accumulation analysis can be defined based on the frequency domain (Łagoda *et al.*, 2005a,b; Petrucci and Zuccarello). The authors shown that the spectral method can be effectively used to estimate fatigue life of the component with similar or better accuracy comparing to the time domain method. In Petrucci and Zuccarello (2004), relationships between the moments of the probability density function of the fatigue cycles of the equivalent stress, two spectral parameters, the common irregularity factor of the stress process and a bandwidth parameter were determined. In the paper by Łagoda *et al.* (2005a) the authors present results of fatigue tests of specimens subjected to multiaxial random fatigue for which the spectral based algorithm for fatigue life estimation was used. A loading with Gaussian distribution and narrow- and broad-band frequency spectra were chosen for tests and the spectral method based on the assumption of the linear Serensen-Kogayev hypothesis was used. It was shown that under a multiaxial random loading, the results of fatigue life calculated according to the considered algorithms in frequency and time domains are well correlated with the results of experiments.

The linear hypothesis used in the spectral methods allow analyzis of the fatigue behavior of materials only for the high cycle fatigue range – for time domain methods, it is easier to use a selected elasto-plastic theory to estimate the low cycle fatigue stress history.

It has to be indicated, that the authors of the papers mentioned above used the irregularity factor as a parameter which characterizes the investigated load histories, however there is no clear suggestion how it affects the calculations. There is no answer for the accuracy problem in generation of the load history according to the irregularity coefficient of the load history – how small differences between the irregularity factors for the service load and load simulated in a fatigue test influences the results of fatigue life estimation.

3. Simulation tests

Simulations were performed in order to determine the influence of the coefficient of loading irregularity combined with the mean load value on the damage accumulation degree. The tests were realized for histories with different values of the coefficient of irregularity and different expected values.

Stress histories with the following expected values were generated: $\sigma_m = 0, 150, 200$ and 225 MPa. The coefficients of loading irregularity $I = 0.1, 0.2, \ldots, 0.99$ were assumed for each level of the expected value. The damage degree was calculated according to procedures of the global and local approaches to the mean stress value. Table 1 contains some strength and fatigue properties of 10HNAP steel applied in the calculations (Łagoda *et al.*, 2001).

Table 1. Strength and fatigue properties of 10HNAP steel

R_e [MPa]	R_m [MPa]	A_{10} [%]	Z [%]	E [GPa]	ν [–]	σ'_f [MPa]	Z_{rc} [MPa]	N_G [cycle]
418	566	30.7	36.5	215	0.29	746	252.3	$1.28 \cdot 10^6$

The influence of the stress expected value on the accumulated damage degree was included with three models described with Eqs. (2.1) and (2.2) for the global and local approach, respectively. Table 2 presents the results of calculations of damage accumulation degree for the considered stress histories.

The relation between the accumulated damage degree and the coefficient of history irregularity is shown in Fig. 3. When the stress mean value is $\sigma_m = 0$ (Fig. 3a), the calculation results are identical independently of the chosen transformation relationship. In all cases, it is observed that for each investigated relationship (Goodman, Gerber and Morrow) both methods, local and global approach, give the same results for damage degree accumulation. Generally, the damage degree increases as the coefficient of irregularity increases by some orders. An increase in the mean loading causes a significant increase in the accumulated damage degree for the Goodman model comparing to the Gerber and Morrow, where the Gerber and Morrow transformation models give similar results (Fig. 3b-d). Additionally, the mean stress $sigma_m = 150$ MPa (Fig. 3b) leads to a smaller value of the accumulation damage degree by a few orders comparing to $\sigma_m = 200$ MPa (Fig. 3c) and $\sigma_m = 225$ MPa (Fig. 3d).

It can be observed, that for lower factor I $(I < 0.5)$ the accumulated damages do not change so strong like for a higher value of the irregularity factor $(I > 0.5)$.

4. Conclusions

The presented tests proved that both, global and local approaches to the mean value influence, give similar values of the accumulated damage degree independently of the value of the coefficient of history irregularity. On the other hand, there are some other aspects which have not been taken into account so far:

- an increase in the coefficient of irregularity causes an increase in the degree of the accumulated fatigue damages by some orders,
- an increase in the loading mean value causes occurrence of significant differences between the obtained damage degrees for particular transformation relationships. The Goodman relationship shows the greatest differences.

Table 2. Accumulation degree of fatigue damages

σ_m [MPa]	I	$S(T_O)\cdot 10^{-6}$					
		Local approach			Global approach		
		Goodman	Gerber	Morrow	Goodman	Gerber	Morrow
0	0.10	0.01	0.01	0.01	0.01	0.01	0.01
	0.20	0.54	0.52	0.52	0.51	0.51	0.51
	0.30	1.14	1.12	1.12	1.12	1.12	1.12
	0.40	1.50	1.48	1.48	1.48	1.48	1.48
	0.50	1.90	1.89	1.89	1.89	1.89	1.89
	0.60	2.60	2.58	2.59	2.58	2.58	2.58
	0.70	3.47	3.42	3.44	3.42	3.42	3.42
	0.80	5.06	4.99	5.00	4.97	4.97	4.97
	0.90	8.32	8.26	8.26	8.24	8.24	8.24
	0.99	13.99	13.90	13.92	13.89	13.89	13.89
150	0.10	0.02	0.00	0.00	0.02	0.00	0.00
	0.20	0.74	0.04	0.08	0.70	0.04	0.08
	0.30	1.56	0.08	0.18	1.53	0.08	0.18
	0.40	2.02	0.10	0.25	2.00	0.10	0.25
	0.50	2.55	0.13	0.30	2.54	0.13	0.30
	0.60	3.47	0.18	0.41	3.44	0.18	0.41
	0.70	4.66	0.24	0.55	4.58	0.23	0.55
	0.80	6.82	0.35	0.81	6.70	0.34	0.81
	0.90	11.21	0.57	1.33	11.10	0.57	1.33
	0.99	18.53	0.94	2.21	18.39	0.94	2.21
200	0.10	13.84	1.36	3.23	13.68	1.36	3.21
	0.20	695.50	59.76	140.06	562.86	55.95	132.25
	0.30	1400.00	125.73	294.71	1200.00	122.69	290.07
	0.40	1700.00	165.02	390.25	1600.00	163.65	391.24
	0.50	2100.00	206.97	488.10	2100.00	205.58	486.01
	0.60	3000.00	285.98	674.06	2800.00	282.93	668.89
	0.70	4000.00	380.86	897.82	3800.00	373.40	882.63
	0.80	5900.00	555.69	1300.00	5500.00	544.86	1300.00
	0.90	9600.00	919.68	2200.00	9100.00	907.30	2100.00
	0.99	15700.00	1500.00	3600.00	15300.00	1500.00	3600.00
225	0.10	15.75	0.58	1.27	15.56	0.58	1.27
	0.20	807.33	26.67	55.57	648.44	24.23	52.87
	0.30	1600.00	54.82	116.52	1400.00	52.76	115.07
	0.40	2050.00	122.24	261.22	2000.00	117.98	257.38
	0.50	2500.00	123.50	195.83	2400.00	89.49	195.21
	0.60	3400.00	125.02	270.03	3300.00	123.04	268.38
	0.70	4800.00	170.62	367.00	4400.00	165.77	361.72
	0.80	7000.00	251.76	540.80	6500.00	244.58	533.62
	0.90	11500.00	414.95	893.46	10900.00	406.50	886.88
	0.99	18300.00	673.77	1500.00	17800.00	665.68	1500.00

Taking into account that the load histories generated in the laboratories should correspond to the service loads, the irregularity factor must be analyzed, especially for higher values of this coefficient ($I > 0.5$) because the presented algorithm for fatigue life estimation is sensitive to this factor.

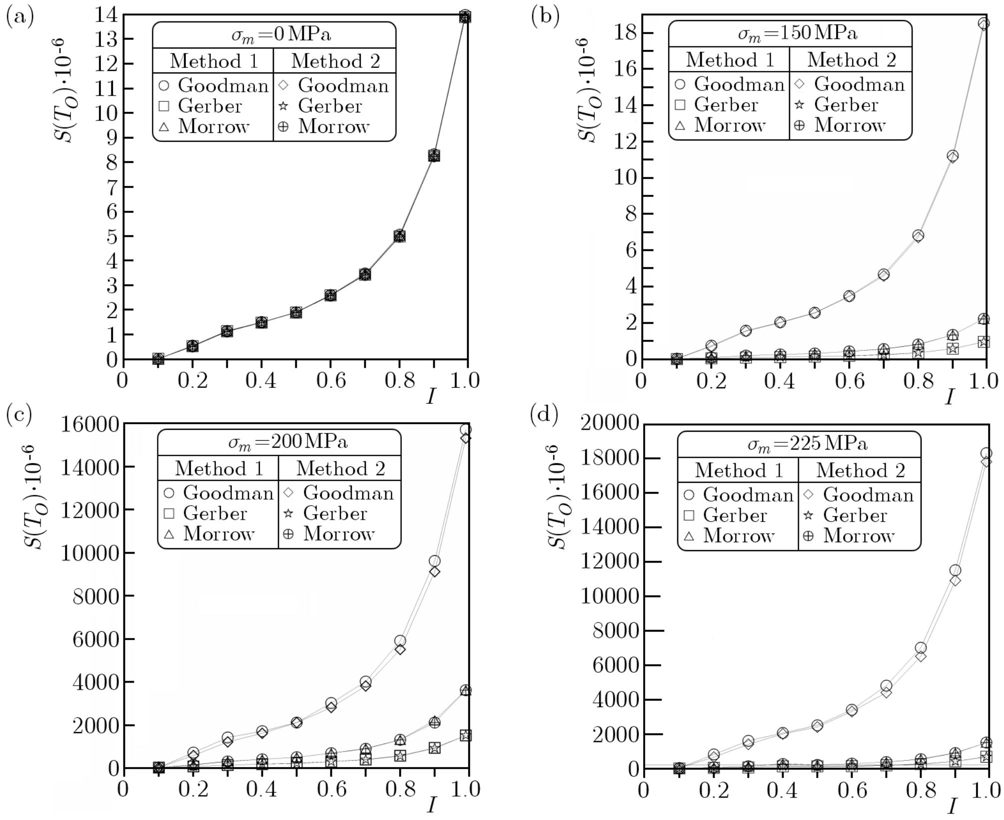

Fig. 3. Changes of the damage accumulation degree $S(T_O)$ versus irregularity factor I for different values of the mean stress

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