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On a formula finding fractal dimension

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ABSTRACT

Purpose: The article is devoted to the determination of the fractal dimension of cellular concrete, in particular foam concrete, and the further clarification of the relationship between fractal dimension and porosity and average density of cellular concrete.

Design/methodology/approach: In the theoretical description of disordered systems, the fundamentals of fractal theory are actively used, which allow obtaining statistical indicators of chaotic natural and artificially disordered systems, which include cellular concrete. The parameters of the pore structure are difficult to quantify by conventional methods because of the complexity and irregularity of the pore structure due to their random distribution.

Findings: Formulas for calculating the fractal dimension and average density of highly porous material are calculated and proposed. The formula for calculating the average density takes into account the density of the material between the pore walls.

Research limitations/implications: The calculation of the fractal dimension is one of the main factors affecting the practical application of the theory of fractals, a natural problem arises on a theoretical basis to justify these calculations.

Practical implications: The formulas proposed in this work for calculating the fractal dimension and density of a highly porous structure improve research on methods for producing substances with a controlled fractal structure, which will help create materials with unusual mechanical properties, density, and porosity.

Originality/value: The formula for calculating the fractal dimension obtained in the work improves the well-known Hausdorff-Bezikovich formula. On the other hand, it makes it possible to obtain a highly porous structure with a given density of the material under study.

Keywords: Fractal dimension, Cellular concrete, Structure, Mineral binder, Porosity, Density

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

Aerated concrete mixtures – foam and aerated concrete – are multiphase systems in which the dispersion medium is an aqueous solution of a surfactant, and the disperse medium is solid particles and gas inclusions [1]. highly porous structure is one of the indispensable characteristics of these concretes. as is known [1-5], porosity is one of the most important factors that determines the mechanical properties of concrete on a mineral binder. when developing cellular concrete, they always try to obtain a material with a homogeneous structure, in which the pores had the required shape and size, and also were located in those coordinates of the space where they are able to provide the material with the specified physical and mechanical properties. it is known that high porosity in foam concrete is created by adding additional foam in the preparation of the foam concrete mixture. in reality, these pores are polydisperse, i.e., having different dimensional and geometric factors, they are distributed throughout the volume of the material. in this regard, the pore structure parameters are difficult to quantify using conventional methods because of the complexity and irregularity of the pore structure due to their random distribution.

Currently, in the theoretical description of disordered systems, the fundamentals of fractal theory are actively used, which allow obtaining statistical indicators of chaotic natural and artificially disordered systems [6].

An important property of fractal systems is their self-similarity [7,8]. This means that the structure of a fractal on one scale is similar to its structure on another, larger scale, that is, by increasing by a certain number of times any element of the fractal structure, we obtain an element of the structure of the same fractal.

The theory of fractals arose due to the impossibility of describing natural objects using the classical mathematical apparatus based on the linear geometry of Euclid. As Mandelbrot himself notes [7], the existing classical geometry is moving away from nature, falling into the realm of abstractions, and is increasingly losing touch with the real definition of reality in its categories that do not correspond to anything that surrounds us.

The priority problems and tasks that must be solved, first of all, include the development of methods for producing substances with a controlled fractal structure, which will help create materials with unusual mechanical properties, density, porosity, etc.

The calculation of fractal dimension is one of the main factors affecting the practical application of fractal theory. There are various methods for calculating the fractal dimension, such as counting boxes, the Brownian motion measurement method, the spectral number method, the Hausdorff measurement method, the capacity measurement method, the Cantor measurement method and others [8-19]. The fractal dimensions of the pore surface, pore volume and pore axis are calculated by these methods.

2. Calculation of fractal dimension

We divide the side of the unit square (Fig. 1) into equal n parts so that the equality

$$(a+b)n+a=1, \tag{1}$$

where:
a – is the thickness of the septum;
b – is the length of the side of the pore, i.e., $\delta = b$.

Assume that the thickness of the septum and the length of the side of the pore are related as $k=\frac{a}{b}$. Then from $\big((k+1)n++k\big)b=1$ we find

$$\delta = b = \frac{1}{(n+1)k+n} = \frac{1}{(k+1)n+k}. \tag{2}$$

It is clear that the number of pores is $N=n^2$. Find the fractal dimension

$$D=\frac{\ln N}{\ln\frac{1}{\delta}}=\frac{2\ln n}{\ln\big((k+1)n+k\big)}. \tag{3}$$

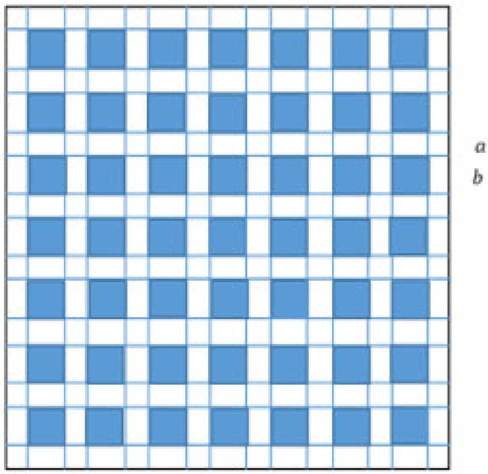

Fig. 1. Theoretical case

An interesting fact is that regardless of the ratio $k=\frac{a}{b}$ with an infinite division of a segment into parts, the fractal dimension is 2:

$$D_\infty = \lim_{n\to\infty}\frac{2\ln n}{\ln\big((k+1)n+k\big)} = \lim_{n\to\infty}\frac{\frac{2}{n}}{\frac{k+1}{(k+1)n+k}} =$$

$$\frac{2}{k+1}\lim_{n\to\infty}\frac{(k+1)n+k}{n} = \frac{2}{k+1}\lim_{n\to\infty}\left(k+1+\frac{k}{n}\right)=2. \tag{4}$$

We calculate the density of the porous substance determined above by the above method. Let ρ_1 be the density of the substance filling the septum, ρ_2 be the density of the pore. Then the density of the mixture $\rho_{N,b}$ is determined by the formula:

$$\rho_{N,b} = \frac{m_{n,k}}{V} = \frac{m_1+m_2}{V_1+V_2} = \frac{\rho_1(V-n^3b^3)+\rho_2 n^3 b^3}{(V-n^3b^3)+n^3b^3} = \frac{\rho_1 V-(\rho_1-\rho_2)\sqrt{N^3}b^3}{V}. \quad (5)$$

If we assume that $V = 1\ m^3$, then the dependence of the density ρ_1 on the values of N and b looks like this

$$\rho_{N,b} = \rho_1\left(1 - b^3\sqrt{N^3}\right). \quad (6)$$

Since $b = \delta$, then

$$\rho_{N,\delta} = \rho_1\left(1 - \delta^3 \cdot \sqrt{N^3}\right). \quad (7)$$

Let $n = 10^6$, $k = \frac{1}{10}$. Then

$$N = n^2 = 10^{12}, \quad (8)$$

$$\delta = \frac{1}{kn+k+n} \approx \frac{10}{11} \cdot 10^{-6}\,. \quad (9)$$

Therefore

$$\rho_{N,\delta} = \rho_1 \cdot \frac{331}{1331} \approx 0.2486\rho_1\,. \quad (10)$$

In this case fractal dimension

$$D = \frac{2\ln n}{\ln((k+1)n+k)} \approx 1.9862. \quad (11)$$

If $n = 10^6$, $k = \frac{1}{20}$, then

$$N = n^2 = 10^{12}, \quad (12)$$

$$\delta = \frac{1}{kn+k+n} \approx \frac{20}{21} \cdot 10^{-6}\,. \quad (13)$$

Therefore

$$\rho_{N,\delta} = \rho_1 \cdot \frac{1261}{9261} \approx 0.1361\,\rho_1. \quad (14)$$

The fractal dimension in this case is equal to

$$D = \frac{2\ln n}{\ln((k+1)n+k)} \approx 1.9929. \quad (15)$$

We now turn to the presentation of the general case (Fig. 2). Let the experimental results be as follows: N is the number of pores in the material, δ is the average value of the pore diameter, a is the average value of the thickness of the partition.

Then $n = \sqrt{N}$, $k = \frac{a}{\delta}$, and therefore, the fractal dimension is calculated by the formula

$$D = \frac{\ln N}{\ln\left((k+1)\sqrt{N}+\frac{a}{\delta}\right)}, \quad (16)$$

and the density of the substance is according to the formula

$$\rho_{N,b} = \rho_1\left(1 - \frac{\pi}{6} \cdot \delta^3 \cdot \sqrt{N^3}\right). \quad (17)$$

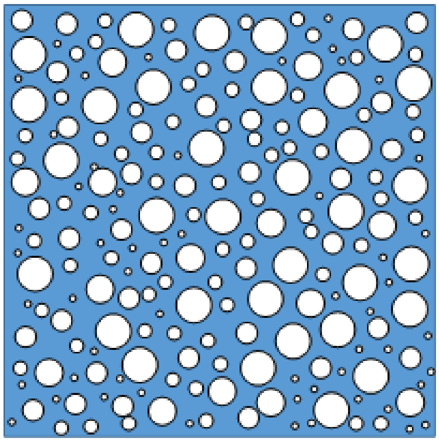

Fig. 2. General case

Remarks:

1) If $a = 0$, then$k = 0$. As a result, we get

$$D = \frac{\ln N}{\ln\sqrt{N}} = 2. \quad (18)$$

It is clear that the equality $a = 0$ means that the partition in the body is not present and the body is sometimes filled. Therefore, in this case, the fractal dimension and the topological dimension coincide.

2) If $a = 1 - \delta(k+1)\sqrt{N}$, then from (16) we obtain the well-known Hausdorff-Bezikovich formula:

$$D = \frac{\ln N}{\ln\frac{1}{\delta}}. \quad (19)$$

3. Conclusions

Since the calculation of fractal dimension is one of the main factors affecting the practical application of the theory of fractals, a natural problem arises of a theoretical basis for substantiating these calculations.

The formulas proposed in this paper for calculating the fractal dimension and density of a highly porous structure improve research on methods for producing substances with a controlled fractal structure, which will help create materials with unusual mechanical properties, density, and porosity. The above remark was shown that the formula for calculating the fractal dimension obtained in the work improves the well-known Hausdorff-Bezikovich formula.

On the other hand, it makes it possible to obtain a highly porous structure with a given density of the material under study.

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