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MODELING OF CONTROL IN A HIERARCHICAL ACTIVE SYSTEM ON THE BASIS OF ENTROPY PARADIGM OF SUBJECTIVE ANALYSIS

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Abstract

Authors presented and considered a example and algorithms of modeling of control in an active system. The examples analyzed two of subjects and two alternatives. The calculated an entropies and ratings of the subjects. Shaw in investigating the role of the parameter "subjective temperature" a changes in the phase portraits.

Keywords: *active systems, subjective analysis, entropy, preferences.*

INTRODUCTION

The presented paper is a continuation of the previous paper "Control in a hierarchical active system on the basis of entropy paradigm of subjective analysis".

Theoretical materials related to a control of conflicts in a transient (two-level) system are represented in that paper.

Tasks of modeling of control in a hierarchical system

In this paper it is considered a rough example and algorithms of modeling of control in an active system and represented some results of numerical modeling.

The tasks of the paper are to search a behavior a hierarchical system of control in an active system.

The fundamental principle is the "subjective entropy maximum principle", the mathematical formulation of which coincides with the formulation of the principle by Jaynes-Gibbs.

A CASE OF TWO SUBJECTS AND TWO ALTERNATIVES

In accordance to the theory [(1)-(21)], for the two subjects *A* and *B* and the two alternatives 1 and 2, for the given data, initial preferences:

$$\pi_0^{(A1)} = 0.3;\ \ \pi_0^{(A2)} = 0.7;\ \ \pi_0^{(B1)} = 0.8;\ \ \pi_0^{(B2)} = 0.2.$$

$$\pi_0^{(A1|B1)} = 0.3;\ \pi_0^{(A1|B2)} = 0.8;\ \pi_0^{(A2|B1)} = 0.7;\ \pi_0^{(A2|B2)} = 0.2.$$

$$\pi_0^{(B1|A1)} = 0.15;\ \pi_0^{(B2|A1)} = 0.85;\ \pi_0^{(B1|A2)} = 0.4;\ \pi_0^{(B2|A2)} = 0.6.$$

$$\pi_0^{\Sigma(A1|B1)} = 0.4;\ \pi_0^{\Sigma(A2|B1)} = 0.3;\ \pi_0^{\Sigma(A1|B2)} = 0.8;\ \pi_0^{\Sigma(A2|B2)} = 0.5.$$

$$\pi_0^{\Sigma(B1|A1)} = 0.8;\ \pi_0^{\Sigma(B2|A1)} = 0.34;\ \pi_0^{\Sigma(B1|A2)} = 0.11;\ \pi_0^{\Sigma(B2|A2)} = 0.89.$$

our simplified and rough modeling brings the following results:

1. The initial simplified iteration system for the calculation of the preferences of the type of [(20), (21); (18), (19); (11), (12)]; and rough primer ratings of the kind of [(27)-(30)]:

$$\left\{\begin{array}{l}
\pi_{t+1}^{(A1)} = \pi_t^{(B1)}\pi_t^{(A1|B1)} + \pi_t^{(B2)}\pi_t^{(A1|B2)} \\
\pi_{t+1}^{(A2)} = \pi_t^{(B1)}\pi_t^{(A2|B1)} + \pi_t^{(B2)}\pi_t^{(A2|B2)} \\
\pi_{t+1}^{(B1)} = \pi_t^{(A1)}\pi_t^{(B1|A1)} + \pi_t^{(A2)}\pi_t^{(B1|A2)} \\
\pi_{t+1}^{(B2)} = \pi_t^{(A1)}\pi_t^{(B2|A1)} + \pi_t^{(A2)}\pi_t^{(B2|A2)} \\
\pi_{t+1}^{(A1|B1)} = \dfrac{\pi_t^{\Sigma(A1|B1)}F^{(A1|B1)}e^{\beta F^{(A1|B1)}}}{\pi_t^{\Sigma(A1|B1)}F^{(A1|B1)}e^{\beta F^{(A1|B1)}} + \pi_t^{\Sigma(A2|B1)}F^{(A2|B1)}e^{\beta F^{(A2|B1)}}} \\
\pi_{t+1}^{(A2|B1)} = \dfrac{\pi_t^{\Sigma(A2|B1)}F^{(A2|B1)}e^{\beta F^{(A2|B1)}}}{\pi_t^{\Sigma(A1|B1)}F^{(A1|B1)}e^{\beta F^{(A1|B1)}} + \pi_t^{\Sigma(A2|B1)}F^{(A2|B1)}e^{\beta F^{(A2|B1)}}} \\
\pi_{t+1}^{(A1|B2)} = \dfrac{\pi_t^{\Sigma(A1|B2)}F^{(A1|B2)}e^{\beta F^{(A1|B2)}}}{\pi_t^{\Sigma(A1|B2)}F^{(A1|B2)}e^{\beta F^{(A1|B2)}} + \pi_t^{\Sigma(A2|B2)}F^{(A2|B2)}e^{\beta F^{(A2|B2)}}} \\
\pi_{t+1}^{(A2|B2)} = \dfrac{\pi_t^{\Sigma(A2|B2)}F^{(A2|B2)}e^{\beta F^{(A2|B2)}}}{\pi_t^{\Sigma(A1|B2)}F^{(A1|B2)}e^{\beta F^{(A1|B2)}} + \pi_t^{\Sigma(A2|B2)}F^{(A2|B2)}e^{\beta F^{(A2|B2)}}} \\
\pi_{t+1}^{(B1|A1)} = \dfrac{\pi_t^{\Sigma(B1|A1)}F^{(B1|A1)}e^{\beta F^{(B1|A1)}}}{\pi_t^{\Sigma(B1|A1)}F^{(B1|A1)}e^{\beta F^{(B1|A1)}} + \pi_t^{\Sigma(B2|A1)}F^{(B2|A1)}e^{\beta F^{(B2|A1)}}} \\
\pi_{t+1}^{(B2|A1)} = \dfrac{\pi_t^{\Sigma(B2|A1)}F^{(B2|A1)}e^{\beta F^{(B2|A1)}}}{\pi_t^{\Sigma(B1|A1)}F^{(B1|A1)}e^{\beta F^{(B1|A1)}} + \pi_t^{\Sigma(B2|A1)}F^{(B2|A1)}e^{\beta F^{(B2|A1)}}} \\
\pi_{t+1}^{(B1|A2)} = \dfrac{\pi_t^{\Sigma(B1|A2)}F^{(B1|A2)}e^{\beta F^{(B1|A2)}}}{\pi_t^{\Sigma(B1|A2)}F^{(B1|A2)}e^{\beta F^{(B1|A2)}} + \pi_t^{\Sigma(B2|A2)}F^{(B2|A2)}e^{\beta F^{(B2|A2)}}} \\
\pi_{t+1}^{(B2|A2)} = \dfrac{\pi_t^{\Sigma(B2|A2)}F^{(B2|A2)}e^{\beta F^{(B2|A2)}}}{\pi_t^{\Sigma(B1|A2)}F^{(B1|A2)}e^{\beta F^{(B1|A2)}} + \pi_t^{\Sigma(B2|A2)}F^{(B2|A2)}e^{\beta F^{(B2|A2)}}}
\end{array}\right\} \quad (31')$$

$$\left\{\begin{array}{l}
\pi_{t+1}^{\Sigma(A1|B1)} = \pi_t^{(A1)}\xi_t^{(A|A\wedge A1|B1)} + \pi_t^{(B1)}\xi_t^{(B|A\wedge A1|B1)} \\
\pi_{t+1}^{\Sigma(A2|B1)} = \pi_t^{(A2)}\xi_t^{(A|A\wedge A2|B1)} + \pi_t^{(B1)}\xi_t^{(B|A\wedge A2|B1)} \\
\pi_{t+1}^{\Sigma(A1|B2)} = \pi_t^{(A1)}\xi_t^{(A|A\wedge A1|B2)} + \pi_t^{(B2)}\xi_t^{(B|A\wedge A1|B2)} \\
\pi_{t+1}^{\Sigma(A2|B2)} = \pi_t^{(A2)}\xi_t^{(A|A\wedge A2|B2)} + \pi_t^{(B2)}\xi_t^{(B|A\wedge A2|B2)} \\
\pi_{t+1}^{\Sigma(B1|A1)} = \pi_t^{(B1)}\xi_t^{(B|B\wedge B1|A1)} + \pi_t^{(A1)}\xi_t^{(A|B\wedge B1|A1)} \\
\pi_{t+1}^{\Sigma(B2|A1)} = \pi_t^{(B2)}\xi_t^{(B|B\wedge B2|A1)} + \pi_t^{(A1)}\xi_t^{(A|B\wedge B2|A1)}
\end{array}\right\}, \qquad \text{(continued 31')}$$

the system (31’) to be continued
the continuation of the system (31”)

$$\left\{\begin{array}{l}
\pi_{t+1}^{\Sigma(B1|A2)} = \pi_t^{(B1)}\xi_t^{(B|B\wedge B1|A2)} + \pi_t^{(A2)}\xi_t^{(A|B\wedge B1|A2)} \\
\pi_{t+1}^{\Sigma(B2|A2)} = \pi_t^{(B2)}\xi_t^{(B|B\wedge B2|A2)} + \pi_t^{(A2)}\xi_t^{(A|B\wedge B2|A2)} \\
\xi_{t+1}^{(A|A\wedge A1|B1)} = \xi_t^{(A|A\wedge A1|B1)} \\
\xi_{t+1}^{(B|A\wedge A1|B1)} = \xi_t^{(B|A\wedge A1|B1)} \\
\xi_{t+1}^{(A|A\wedge A2|B1)} = \xi_t^{(A|A\wedge A2|B1)} \\
\xi_{t+1}^{(B|A\wedge A2|B1)} = \xi_t^{(B|A\wedge A2|B1)} \\
\xi_{t+1}^{(A|A\wedge A1|B2)} = \xi_t^{(A|A\wedge A1|B2)} \\
\xi_{t+1}^{(B|A\wedge A1|B2)} = \xi_t^{(B|A\wedge A1|B2)} \\
\xi_{t+1}^{(A|A\wedge A2|B2)} = \xi_t^{(A|A\wedge A2|B2)} \\
\xi_{t+1}^{(B|A\wedge A2|B2)} = \xi_t^{(B|A\wedge A2|B2)} \\
\xi_{t+1}^{(B|B\wedge B1|A1)} = \xi_t^{(B|B\wedge B1|A1)} \\
\xi_{t+1}^{(A|B\wedge B1|A1)} = \xi_t^{(A|B\wedge B1|A1)} \\
\xi_{t+1}^{(B|B\wedge B2|A1)} = \xi_t^{(B|B\wedge B2|A1)} \\
\xi_{t+1}^{(A|B\wedge B2|A1)} = \xi_t^{(A|B\wedge B2|A1)} \\
\xi_{t+1}^{(B|B\wedge B1|A2)} = \xi_t^{(B|B\wedge B1|A2)} \\
\xi_{t+1}^{(A|B\wedge B1|A2)} = \xi_t^{(A|B\wedge B1|A2)} \\
\xi_{t+1}^{(B|B\wedge B2|A2)} = \xi_t^{(B|B\wedge B2|A2)} \\
\xi_{t+1}^{(A|B\wedge B2|A2)} = \xi_t^{(A|B\wedge B2|A2)}
\end{array}\right\}. \qquad \text{(31'')}$$

The initial ratings:

$$\begin{cases} \xi_0^{(A|A\wedge A1|B1)} = 0.3 \\ \xi_0^{(B|A\wedge A1|B1)} = 0.7 \\ \xi_0^{(A|A\wedge A2|B1)} = 0.1 \\ \xi_0^{(B|A\wedge A2|B1)} = 0.9 \\ \xi_0^{(A|A\wedge A1|B2)} = 0.2 \\ \xi_0^{(B|A\wedge A1|B2)} = 0.8 \\ \xi_0^{(A|A\wedge A2|B2)} = 0.4 \\ \xi_0^{(B|A\wedge A2|B2)} = 0.6 \\ \xi_0^{(B|B\wedge B1|A1)} = 0.5 \\ \xi_0^{(A|B\wedge B1|A1)} = 0.5 \\ \xi_0^{(B|B\wedge B2|A1)} = 0.25 \\ \xi_0^{(A|B\wedge B2|A1)} = 0.75 \\ \xi_0^{(B|B\wedge B1|A2)} = 0.6 \\ \xi_0^{(A|B\wedge B1|A2)} = 0.4 \\ \xi_0^{(B|B\wedge B2|A2)} = 0.7 \\ \xi_0^{(A|B\wedge B2|A2)} = 0.3 \end{cases}$$

For the variant of $\beta = 7;\ \alpha_\xi = 0.6;\ \beta_\xi = 0.6;$

$$\overline{U}^{(A|A\wedge A1|B1)} = \overline{U}^{(B|B\wedge B1|A1)} = 3;\ \overline{U}^{(B|A\wedge A1|B1)} = \overline{U}^{(A|B\wedge B1|A1)} = 2;$$

$$\overline{U}^{(A|A\wedge A2|B1)} = \overline{U}^{(B|B\wedge B2|A1)} = 4;\ \overline{U}^{(B|A\wedge A2|B1)} = \overline{U}^{(A|B\wedge B2|A1)} = 3;$$

$$\overline{U}^{(A|A\wedge A1|B2)} = \overline{U}^{(B|B\wedge B1|A2)} = 5;\ \overline{U}^{(B|A\wedge A1|B2)} = \overline{U}^{(A|B\wedge B1|A2)} = 6;$$

$$\overline{U}^{(A|A\wedge A2|B2)} = \overline{U}^{(B|B\wedge B2|A2)} = 1;\ \overline{U}^{(B|A\wedge A2|B2)} = \overline{U}^{(A|B\wedge B2|A2)} = 2.$$

The preferences functions of the corresponding subjects accordingly to the alternatives are shown in Fig. 1.

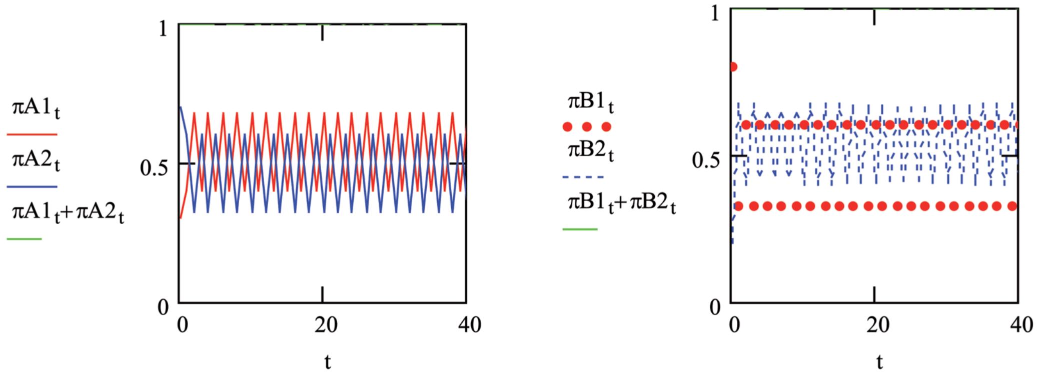


Fig. 1. Preferences of the subjects accordingly to the alternatives
[Andrey V. Goncharenko, 2014]

Phase portraits of the preferences see Fig. 2.

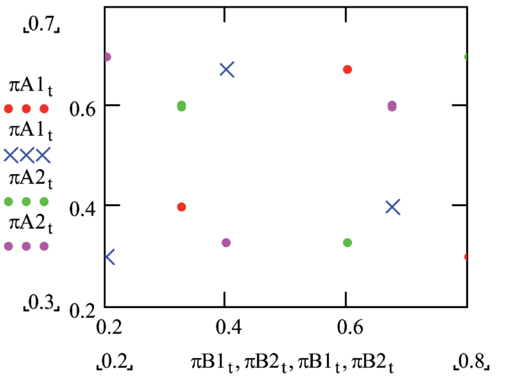


Fig. 2. Phase portraits of the preferences
[Andrey V. Goncharenko, 2014]

The entropies of the subjects' individual preferences of the two given alternatives are demonstrated in Fig. 3.

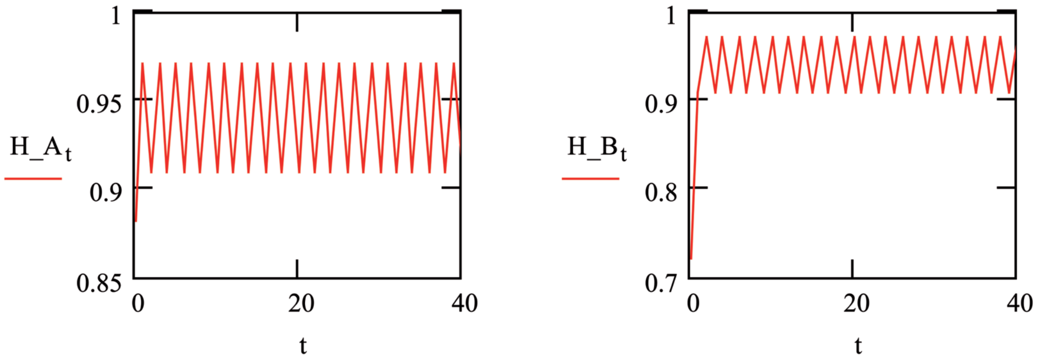


Fig. 3. The entropies of the subjects' individual preferences
[Andrey V. Goncharenko, 2014]

2. The coefficient of correlation:

$$\rho_{Ai|Bj} = \frac{\sum_{t=0}^{N}\left(\pi_t^{(Ai)} - \frac{\sum_{t=0}^{N}\pi_t^{(Ai)}}{N+1}\right)\left(\pi_t^{(Bj)} - \frac{\sum_{t=0}^{N}\pi_t^{(Bj)}}{N+1}\right)}{\sqrt{\sum_{t=0}^{N}\left(\pi_t^{(Ai)} - \frac{\sum_{t=0}^{N}\pi_t^{(Ai)}}{N+1}\right)^2 \sum_{t=0}^{N}\left(\pi_t^{(Bj)} - \frac{\sum_{t=0}^{N}\pi_t^{(Bj)}}{N+1}\right)^2}}, \tag{32}$$

where N − number of discrete time steps.

3. The special functions of $\varphi_A, \psi_A, \varphi_B, \psi_B$

$$\phi_{Ai|Bj} = \left(1 - \left|\rho_{Ai|Bj}\right|\right)\bar{H}_A, \tag{33}$$

$$\psi_{Ai|Bj} = \left|\rho_{Ai|Bj}\right|\bar{H}_B, \tag{34}$$

$$\phi_{Bj|Ai} = \left|\rho_{Ai|Bj}\right|\bar{H}_A, \tag{35}$$

$$\psi_{Bj|Ai} = \left(1 - \left|\rho_{Ai|Bj}\right|\right)\bar{H}_B. \tag{36}$$

The calculated ratings of the subjects are illustrated in Fig. 4.

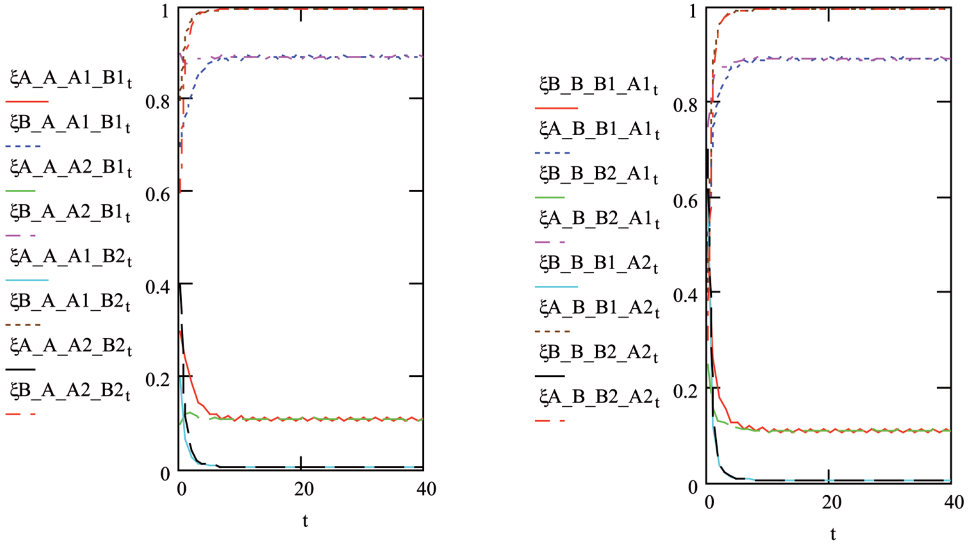
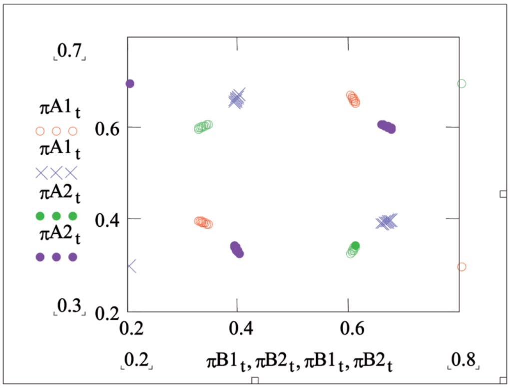
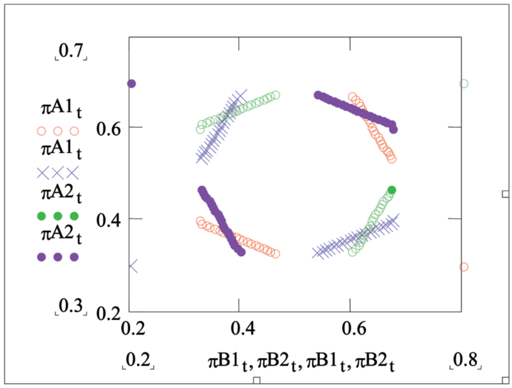


Fig. 4. The calculated ratings of the subjects [Andrey V. Goncharenko, 2014]

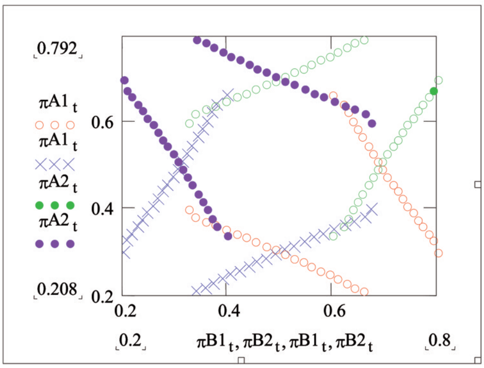
Investigating the role of the parameter β we may notice a certain changes in the phase portraits. For the variants of β = 3, β = 2, β = 1.5, β = 1, β = 0.5, β = 0.25, β = 0.125, β = 0.05, β = 0.01, β = 0.005, β = 0.001, the results are shown in a series of the diagrams in Fig. 5 a) - k).



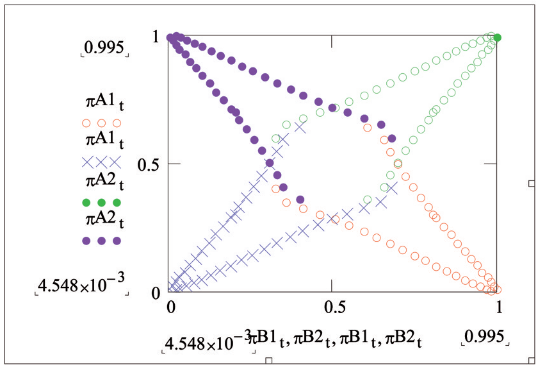
a) $\beta = 3$



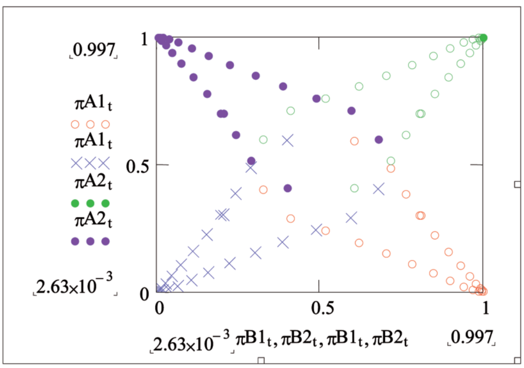
b) $\beta = 2$



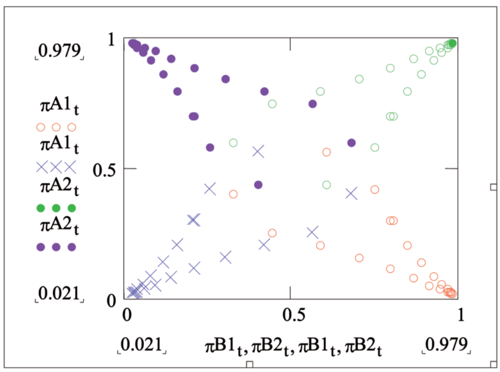
c) $\beta = 1.5$



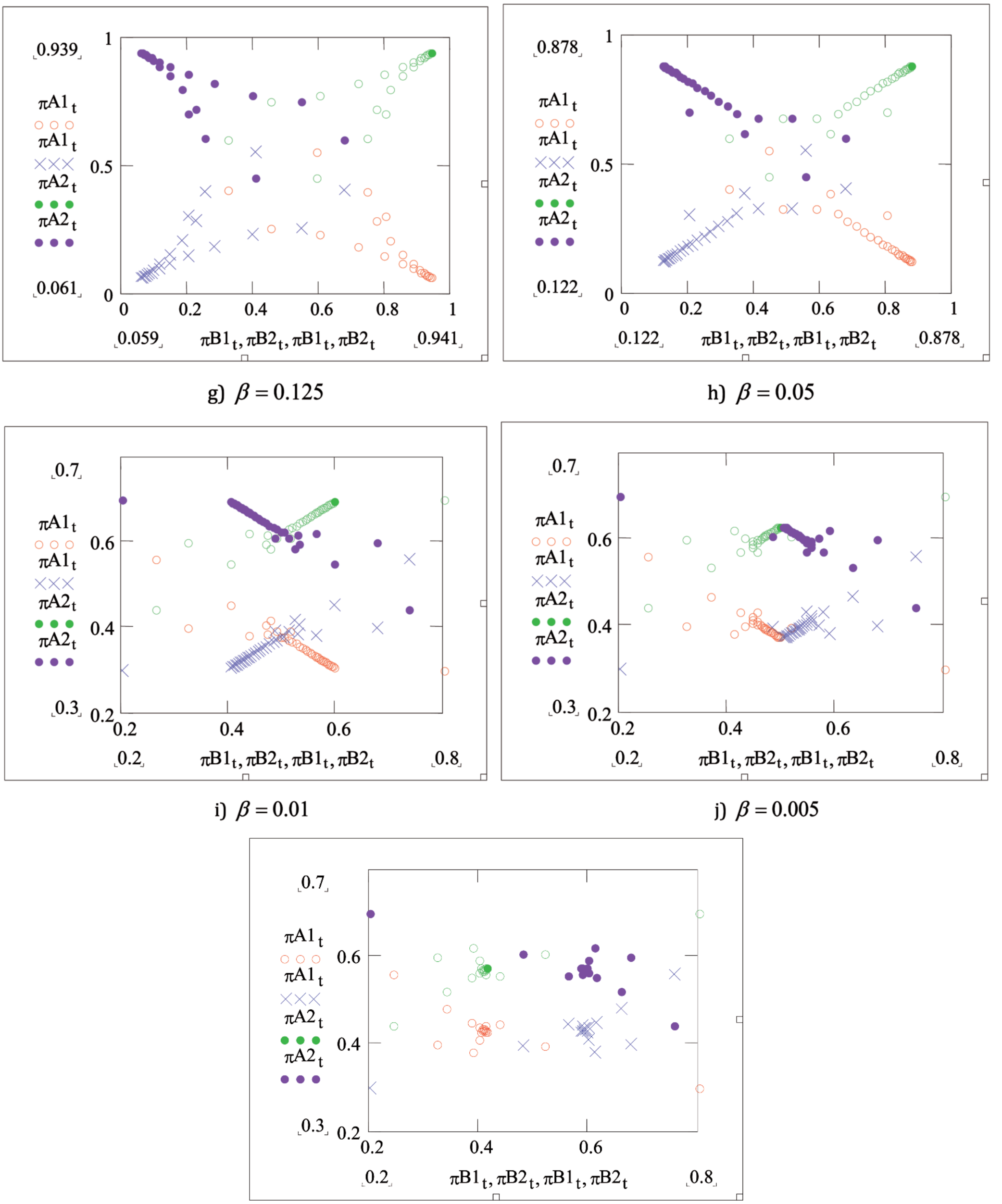
d) $\beta = 1$



e) $\beta = 0.5$



f) $\beta = 0.25$



g) $\beta = 0.125$

h) $\beta = 0.05$

i) $\beta = 0.01$

j) $\beta = 0.005$

k) $\beta = 0.001$

Fig. 5. Dynamics of the preferences for β = 3...0.001 [Andrey V. Goncharenko, 2014]

ANALYSIS OF THE PREFERENCES DYNAMICS

Analyzing the preferences dynamics (see the phase portraits Fig. 2 and Fig. 5) one can come to the following conclusions. Changes of the parameter β from 7 down to 3 merely result in the preferences, thus entropies, oscillations, which are quazy steady at β = 7, however start a slight drifting at β = 3, the entropies show the tendencies for stabilizations. The ratings are quickly stabilizing and in some cases practically absolutely contradicting (because they are almost the same for the different subjects but the same combinations of alternatives). Such a situation means conflict states, the directions, worsening, and dynamics of which are dependent upon the essence and "valuableness" of the alternatives.

Starting from $\beta = 2$ the process is slowly getting more definite. At certain times, for example, $t \approx 13...16$ for $\beta = 2$, the entropies lose the oscillation character. Their values stabilize at different relative levels within the whole diapason of possible changes. The ratings in some cases change radically.

Particular changes in the values of the reverse "subjective temperature" β reflect the dynamics of the subjects' individual attitude to the "field of conflict" and characterize the properties of their personal psych.

CONCLUSIONS AND FURTHER RESEARCHES PROSPECTS

There were two basic results obtained in the article. The first result − dependence (31) for the calculation of the preferences. In figure show the phase portraits of the preferences.

The second result is a ratios (32) for the growth rate subjective temperature and correlation with field conflict.

The next work by the authors is dedicated to the problem of further concretization of the entropy theory of conflicts and solution of the problem of conflicts control.

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MODELOWANIE STEROWANIA W HIERARCHICZNYM AKTYWNYM SYSTEMIE Z WYKORZYSTANIEM PARADYGMATU ENTROPII ANALIZY SUBIEKTYWNEJ

Streszczenie

Autorzy przedstawili i wykazali na przykładzie algorytm modelowania hierarchicznej kontroli nad aktywnym systemem. Przykłady analizują zachowanie dwóch subiektów i dwóch alternatyw. Obliczono entropię i rankingi podmiotów systemu aktywnego. W badaniu pokazano rolę parametru „subiektywnej temperatury" na zmiany portretów fazowych.

Słowa kluczowe: systemy aktywne, analiza subiektywna, entropia, preferencje.