

# MATHEMATICAL MODELING AND COMPUTER AIDED PLANING OF COMMUNAL SEWAGE NETWORKS

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## Abstract:

*In the paper the basic questions connected with modeling of wastewater networks are presented. Methods of modeling basic sewage parameters and appropriate calculation algorithms are described. The problem concerns the gravitational networks divided by nodes into branches and sectors. The nodes are the points of connection of several network segments or branches or the points of changing network parameters as well as of location of sewage inflows to the network. The presented algorithms for networks hydraulic calculation concern sanitary or combined sewage nets. It is assumed that the segments parameters such as shape, canal dimension, bottom slope or roughness are constant. Because of these assumptions all relations considered concern the steady state conditions for the network. The calculation of flow velocities and the filling heights in the segments of the wastewater net are carried out for the known slopes and diameters of the canals.*

**Keywords:** *mathematical modeling of sewage network, hydraulic parameter of canal*

## 1. Characteristic of Sewage Systems

Taking into account a design and the operating processes we can distinguish the following sorts of sewage: housekeeping (sanitary) sewage, industrial sewage, rain wastewater, drainage sewage and ground water. The following sewage systems can be marked out depending on the kind of wastewater dump:

- a) combined sewage system
- b) separated sewage system
- c) semi-separated sewage system.

In an universal sewage system (combined system) all kinds of the wastewater are led using the common canals. At the present time the separated sewage systems are mostly used and there are two separated sewage nets to notice:

- a) a sewage net, used for the housekeeping sewage and for the industrial sewage
- b) a rainwater net, used for carrying out the rain-wastewater.

The semi-separated sewage system is a system enclosing two kinds of nets: the housekeeping net and the rainwater one. In this system the sewage net can receive a part of the rain run-off.

In this paper the following basic assumptions are made:

- Only housekeeping or combined sewage nets are considered, divided into branches and segments by nodes.
- The nodes are the points of connection of several network segments or branches or the points of changing of network parameters as well as of location of sewage inflows to the network (sink basins, rain inlets, connecting basins). In the connecting nodes the flow balance equations and the condition of levels consistence are satisfied.
- It is assumed that the segments parameters such as shape, canal dimension, bottom slope or roughness are constant. Because of these assumptions all relations concern the steady state problem.
- The nets considered are of gravitational type.

## 2. Basic Problems

Designing and analysis of sewage networks are connected with the following tasks:

1. Making hydraulic analysis of the network for known section crosses and for known canal slopes. In this case the calculation of filling heights of the canals as well as the calculation of flow velocities depending on the sewage flow rates must be done. These calculations are done for the respective net segments using the earlier received flow values.
2. Designing of new segments of the network. It concerns the case when the new segments of the network must be added to the existing ones. In this situation diameters and canals slopes must be chosen for the new canals. It is assumed that the sewage inflows are known.

## 3. Basic Hydraulic Dependences in Sewage Network

According to Manning formula the flow velocity of sewage depends on hydraulic radius  $R$  and radius  $R$  depends on the filling height  $H$ . The Manning formula for velocity  $v$  has the form:

$$v = \frac{1}{n} \cdot R^{\frac{2}{3}} \cdot J^{\frac{1}{2}} \quad (1)$$

where:  $R$  – hydraulic radius,  $J$  – canal slope,  $n$  – roughness coefficient,  $v$  – flow velocity.

The relations presented in the following concern the canals with circular section. From Manning formula and taking into account canal geometry one can obtain the following relation:

for  $H \leq 0.5d$ :

$$A = \frac{d^2}{8} \cdot (\varphi - \sin \varphi) \tag{2a}$$

$$\varphi = 2 \cdot \arccos\left(1 - 2 \cdot \frac{H}{d}\right) \tag{2b}$$

$$R = \frac{1}{4} d \left(1 - \frac{\sin \varphi}{\varphi}\right) \tag{2c}$$

for  $H > 0.5d$ :

$$A = \frac{\pi d^2}{4} - \frac{d^2}{8} \cdot (\varphi - \sin \varphi) \tag{3a}$$

$$\varphi = 2 \cdot \arccos\left(2 \cdot \frac{H}{d} - 1\right) \tag{3b}$$

$$R = \frac{d}{4} + \frac{d}{8} \cdot \frac{\sin \varphi}{\pi - 0.5\varphi} \tag{3c}$$

where: A – cross-section area, H – filling height, r – radius of circular canal,  $\varphi$  – central angle, d – canal inside diameter.

From the above expressions one can see that for circular canals the cross-section area A and the hydraulic radius R depend on the canal filling height H and as a result the sewage flow velocity depends also on the canal filling height H when canal slope J and diameter d are given.

We define for the following the canal filling degree in form of relation H/d. In Figures 1 and 2 the relations between A and H/d and between R and H/d for different diameters values d are shown. The figures show that section area A increases monotonically with growing canal filling degree H/d. For greater diameter values the increase of section area is faster and its values are greater. The greatest value of A is in the case of total canal filling and it equals  $\pi d^2/4$ . Hydraulic radius R increases from zero and achieves its maximum for the filling ratio of 81.3% and then it decreases to the value equal to half of the canal height. For the total filling and for the half canal filling the value of radius is d/4. For greater diameters d also

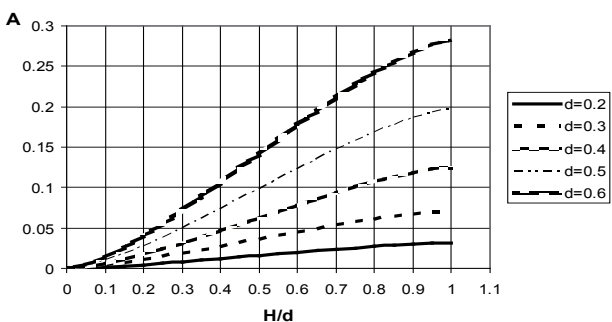


Fig. 1. Dependences between the cross-section area and canal filling degree for different diameter values

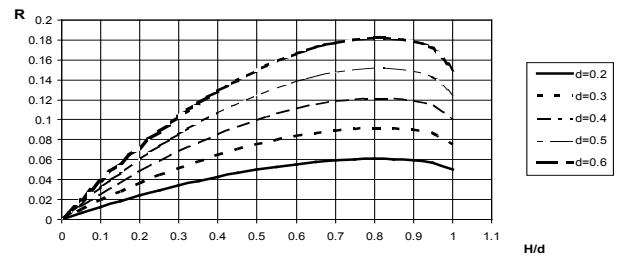


Fig. 2. Dependences between the hydraulic radius and canal filling degree for different diameter values

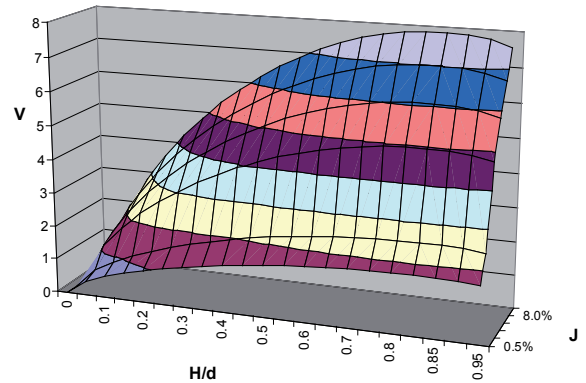


Fig. 3. Dependences between flow velocity v, canal filling degree H/d and canal slope J for roughness coefficient  $n=0.013$  and for canal diameter  $d=0.6$

the hydraulic radius grows but the shape of the curves does not depend on d. The sewage velocity depends on the canal parameters like diameter, canal slope and roughness coefficient and on the canal filling degree (Fig. 3).

The sections of the surface from Fig. 3 using planes  $J=\text{const.}$  are presented in Fig. 4. It shows that the function describing velocity v depending on filling degree H/d has the shape similar to the function describing hydraulic radius R. The sewage velocity increases from zero and achieves its maximum for the filling degree of 81.3% and then it decreases to the value equal to half of the canal height. Greater diameters d

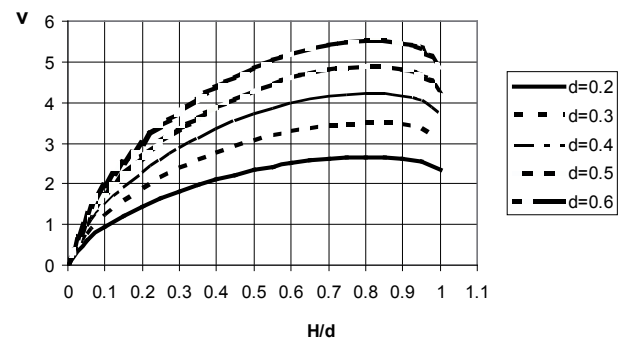
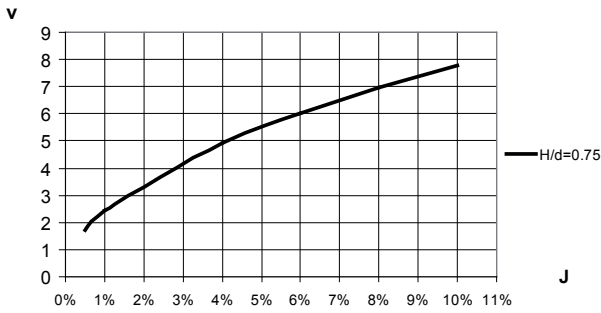
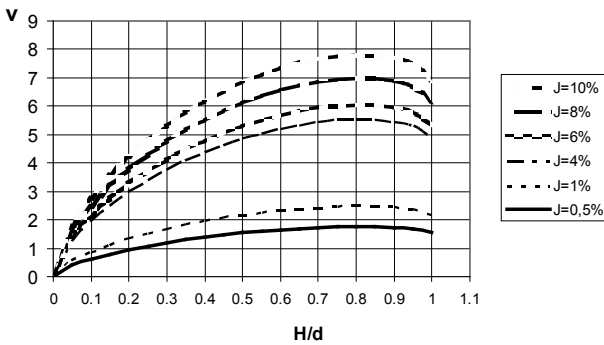


Fig. 4. Dependences between flow velocity v and canal filling degree H/d for roughness coefficient  $n=0.013$  and for canal slope  $J=5\%$  by different diameter values d



**Fig. 5. Dependences between flow velocity  $v$  and canal slope  $J$  for given canal filling degree  $H/d$**



**Fig. 6. Dependences between flow velocity  $v$  and canal filling degree  $H/d$  for roughness coefficient  $n=0.013$  and for diameter value  $d=0.6$  for different canal slopes  $J$**

increase only velocities  $v$  but the shape of the curves presented does not depend on  $d$ .

The section of the surface from Fig. 3 using plane  $H/d=\text{const}$  is shown in Fig.5. It shows that the flow velocity increases monotonically with the growing canal slope for the given filling degree.

Fig. 6 shows the relation between flow velocity  $v$  and canal filling degree  $H/d$  for different slope values  $J$ . One can see from Fig. 6 that greater values of canal slope increase only velocity values and they do not influence the shape of the curves presented.

#### 4. Algorithm for the Calculation of Wastewater Networks

##### 4.1 The algorithm for Calculation of Canal Filling Heights and Flow Velocities

The algorithm presented requires the following data for its calculation:

- type of the network– housekeeping sewage net or combined sewage net
- structure of the network – numbers of segments and nodes and type of nodes
- maximal sewage inflow into the network and the corresponding node number
- slopes of canal bottoms and the canal dimensions.

The task of the algorithm is to determine the following values for given values of rate inflows  $Q_i$ :

- filling heights in each wastewater network segment,
- flow velocity for each network segment.

The calculation scheme presented below is for the canals with circular section. The algorithm consists of the following steps:

**Step 1.** Entering the network structure and input data, i.e. number of nodes  $NW$ , number of segments  $NO$ , set of nodes  $W=\{j=1,\dots,NW\}$ , set of segments  $U=\{i=1,\dots,NO\}$ , set of diameters  $\{d_i\}$ , set of slopes for segments  $J_i$ ,  $i=1,\dots,NO$ , roughness coefficients  $n_i$ .

**Step 2.** Calculating the inflow rates for network input nodes; they are calculated depending on the kind of sewage. For the housekeeping and industrial sewages the maximal hour inflow  $Q$  for given network segment can be calculated according to the relation: [1], [4], [7], [8]

$$Q_{h\max} = \frac{N_{h\max} M q}{24} \quad (4)$$

where:  $M$  – number of residents for the given segment of the net,  $q_{sr}$  – average wastewater amount for average housekeeping unit,  $N_{h\max}$  – rate of irregularity for twenty four hours.

For the rain the wastewater inflow can be expressed as follows: [1], [4], [7], [8]

$$Q = q_d \cdot \psi \cdot F \cdot \varphi \quad (5)$$

where:  $Q$  – rain wastewater inflow caused by infiltration [ $\text{dm}^3/\text{s}$ ],  $F$  – area of drainage basin for the canal segment considered [ $\text{ha}$ ],  $\psi$  – ratio between the rain wastewater amount passing into canals and the rain wastewater amount coming from the whole area given,  $\varphi$  – rate of delay between the rain time and the time of infiltration result,  $q_d$  – rain intensity.

**Step 3.** For given rate inflows  $Q_i$  in segments  $i=1,\dots,NO$  one can determine the following values: filling heights  $H_i$ , hydraulic radius values  $R_i$  and flow velocities  $v_i$ .

1. From the Manning formula and taking into account the canal geometry one can obtain the

following relations with  $x = \frac{H}{d}$ :

For  $H/d \leq 0.5$

$$\beta \cdot F_1(x) - Q = 0 \quad (6a)$$

$$F_1(x) = \frac{(\varphi_1(x) - \sin(\varphi_1(x)))^5}{\varphi_1(x)^3} \quad (6b)$$

$$\varphi_1(x) = 2 \cdot \arccos(1 - 2 \cdot x) \quad (6c)$$

For  $H/d > 0.5$

$$\beta \cdot F_2(x) - Q = 0 \quad (7a)$$

$$F_2(x) = 2 \cdot \frac{(\pi - 0.5 \cdot \varphi_2(x) + 0.5 \cdot \sin(\varphi_2(x)))^{\frac{5}{3}}}{(\pi - 0.5 \cdot \varphi_2(x))^{\frac{2}{3}}} \quad (7b)$$

$$\varphi_2(x) = 2 \cdot \arccos(2 \cdot x - 1) \quad (7c)$$

$$\beta = 0.5 \cdot \frac{1}{n} \cdot (d)^{\frac{8}{3}} \cdot \left(\frac{1}{4}\right)^{\frac{5}{3}} \cdot J^{\frac{1}{2}} \quad (8)$$

where: H – filling height,  $\varphi$  – central angle, d – inside canal diameter, J – canal slope, n – roughness coefficient, Q – rate inflow, H/d - canal filling degree.

The  $\beta$  parameter in (8) depends on canal diameter d and on canal slope J and for the fixed diameter values and canal slopes it is constant.

Solving equations (6a)–(7b) we obtain canal filling degree H/d as a function of flow rate Q.

2. For canal filling degree H/d calculated above the hydraulic radius  $R_1$  should be determined according to the formula:

For H/d ≤ 0.5:

$$R = \frac{1}{4} d \left(1 - \frac{\sin \varphi}{\varphi}\right) \quad (9a)$$

$$\varphi = 2 \cdot \arccos\left(1 - 2 \cdot \frac{H}{d}\right) \quad (9b)$$

For H/d > 0.5:

$$R = \frac{d}{4} \left(\frac{\pi - 0.5\varphi + 0.5\sin(\varphi)}{\pi - 0.5\varphi}\right) \quad (10a)$$

$$\varphi = 2 \cdot \arccos\left(2 \cdot \frac{H}{d} - 1\right) \quad (10b)$$

3. The flow velocity should be calculated from:

$$v = \frac{1}{n} R^{\frac{2}{3}} \cdot J^{\frac{1}{2}} \quad (11)$$

Knowing the network geometry, i.e. slopes, shapes and diameters of canals as well as the wastewater inflows  $Q_i$ , one can calculate filling heights and flow velocities for each network canal. The calculations are carried out for each network segment beginning from the farthest one and going step by step to the nearest segment regarding the wastewater treatment plant.

**Step 4.** The equations of flow balances

$$\sum_{j \neq i} Q_j = 0$$

and the conditions of surface levels equality are calculated in each network node.

**Step 5.** The whole network will be calculated once again with the wastewater inflows changed. Under

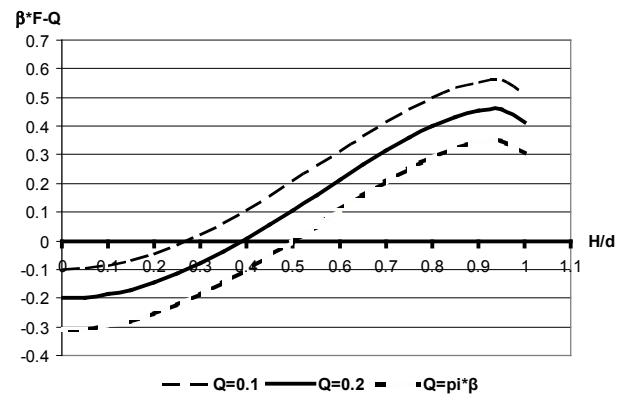
assumption of constant sewage flows in the network segments the sewage system simulation can be executed for a sequence of time steps, for a couple of hours or days; by such the calculation the change of the wastewater inflows occurring with the time must be considered.

There is to notice that the parameters analyzed in the algorithm, i.e. filling heights, hydraulic radius values and flow velocities depend on the wastewater inflows and by the rain wastewaters there is important to take into account their changes and to repeat the simulation runs according to their frequency. The algorithm presented can be considered as a part of the complex model for calculation sewage networks also under unsteady state conditions.

**4.2 Analysis of Equations (6a)–(6c) and (7a)–(7b)**

Equations (6a)–(7b) for calculating the canal filling degree are nonlinear and to solve them the standard numerical methods for solving nonlinear algebraic equations can be applied. In order to determine the equation roots some conditions for parameter  $\beta$  and sewage flow Q must be fulfilled that will be discussed in the following.

Function  $F(x) = F_1(x) + F_2(x)$  is continuous in values range (0; 1>. For  $x=1$ , i.e. for the full canal filling, there is  $F = 2\pi$  and for  $x=0.5$  we get  $F = \pi$ . In values range (0; 0.8> function  $F(x)$  is growing monotone. In values range (0.8; 1>. function reaches its maximum  $F_{max} = 6.7588$  for  $x = 0.9381$ . It is diminishing in values range (0.9381; 1>. This analysis has been done for  $d=0.6$ ,  $J=1\%$  and  $n=0.013$ . For fixed network parameters like canal diameter d and canal slope J, equation  $\beta \cdot F(x) - Q = 0$  has solutions depending on sewage flow Q (Fig. 7).



**Fig. 7. Diagrams of function  $\beta \cdot F(x) - Q$  for different values of Q in values range (0;  $\pi \cdot \beta$ )**

Equation  $\beta \cdot F(x) - Q = 0$  has the following roots:

1. For  $x \in (0; 0.5)$  there is only one root and the following inequality must be fulfilled:  $0 < Q \leq \pi \cdot \beta$ . This inequality defines a values range for sewage flows Q for fixed canal diameters d and canal slopes J.

2. For  $x \in (0.5; 1)$  equation  $\beta \cdot F(x) - Q = 0$  has the following roots:

- ◆ one root for  $x \in (0.5; 1)$  and  $\pi \cdot \beta < Q < 2\pi \cdot \beta$
- ◆ two roots for  $x \in (0.5; 1)$  and  $2\pi \cdot \beta \leq Q < \beta \cdot 6.7586936$ , whereas for  $Q = 2\pi \cdot \beta$  there are  $x_1 = 1$  and  $x_2 = 0.81963$ .

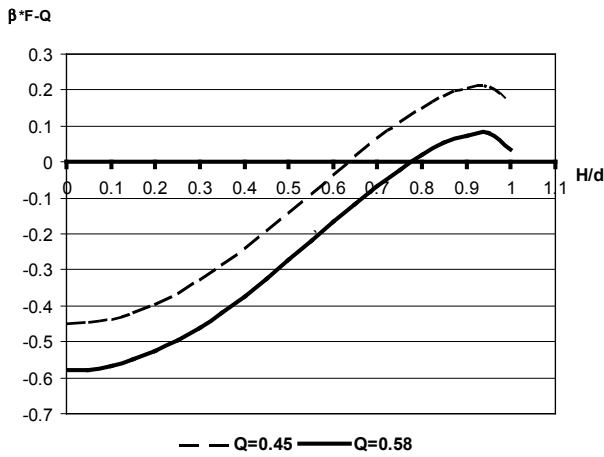


Fig. 8. Diagrams of function  $\beta \cdot F(x) - Q$  for  $\pi \cdot \beta < Q < 2\pi \cdot \beta$

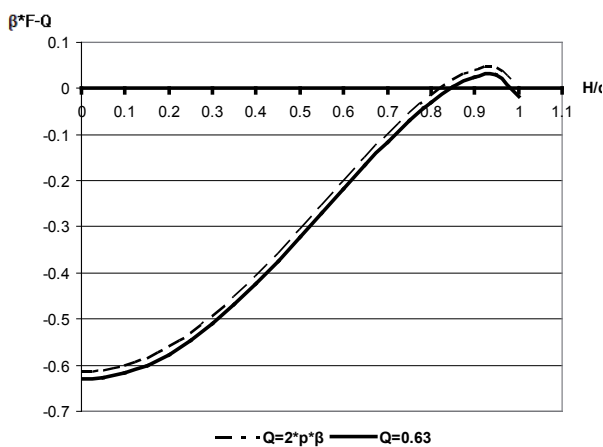


Fig. 9. Diagrams of function  $\beta \cdot F(x) - Q$  for  $2\pi \cdot \beta \leq Q < \beta \cdot F_{max}$

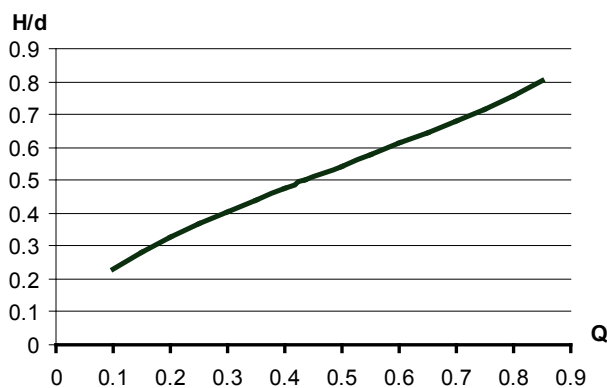


Fig. 10. Relation between the solution of  $\beta \cdot F(x) - Q = 0$  and flow  $Q$

The results of the discussion are shown in Figures 8 and in Fig. 9 the case with two roots of equation  $\beta \cdot F(x) - Q = 0$  is presented with  $Q = 2\pi \cdot \beta$  and  $Q = 0.63$  ( $Q < \beta \cdot 6.7588$ ).

For the fixed network parameters such as a canal diameter  $d$  and canal slope  $J$  the above relations let to

decide what are the solutions for the given flow  $Q$  and whether the value of  $Q$  is not greater than the upper limit  $\beta \cdot 6.7586936$ , what means the lack of solutions. In such the case a change of one or of both of the fixed network parameters  $d$  and  $J$  must be considered.

The result of the above relations is that the flow value  $Q$  depends on the parameter  $\beta$ . The parameter  $\beta$  depends on the canal diameter  $d$  and on the canal slope  $J$ . The equation describing the dependence of canal filling on the flow in the range  $(0; 2\pi \cdot \beta)$  has one solution in this range and that is why this range is relevant.

In Fig. 10 the relation between the solution of equation  $\beta \cdot F(x) - Q = 0$  and flow  $Q$  for  $d = 0.6$ ,  $J = 2\%$ ,  $n = 0.013$  and  $0 < Q < 2\pi \cdot \beta$  is graphically shown.

### 4.3 Calculation of Canal Diameters for Given Flow Values

The calculation procedure shown below concerns the following cases:

- flow  $Q$  exceeds the upper boundary of values domain for  $\beta \cdot 6.7586936$ ; then a change of values for given canal diameters  $d$  and slopes  $J$  have to be considered.
- new segments must be added to the existing network; then the diameters and slopes must be defined for the new canals under the assumption that the sewage inflows  $Q$  into the canals have been forecasted and they are known.

In both cases while calculating diameters and slopes for the new canals for given flows  $Q$  the inequality  $2\pi \cdot \beta - Q > 0$  has to be considered. The fulfilling of the inequality warrants the existence of only 1 solution of the equation describing the relation between canal slope  $J$  and canal flow  $Q$ .

The calculation procedure consists of the following steps which are realized for the forecasted and fixed flow values  $Q$ :

**Step 1.** Determination of canal slope value  $J$ . The value can be determined according to the existing technical standards or calculated regarding the relations for minimal slopes which are known from literature [7], [9], [13], [12]:

$$J = \frac{a}{d} \tag{12a}$$

where  $a$  - parameter depending on the art of sewage system, or

$$J = \frac{\tau_{min}}{\rho \cdot R} = \frac{4 \cdot \tau_{min} \cdot (\pi - 0.5 \cdot \varphi)}{\rho \cdot (\pi - 0.5 \cdot \varphi + 0.5 \cdot \sin \varphi)} \cdot \frac{1}{d} \tag{12b}$$

$$\text{with } \varphi = 2 \cdot \arccos\left(2 \cdot \frac{H}{d} - 1\right),$$

where  $J$  - minimal canal slope ensuring still the occurrence of canal self purification,  $\tau_{min}$  - tangential tension  $\text{kg/m}^2$ , when  $\tau_{min} > 0.225 \text{ kg/m}^2$  for communal and industrial wastewater,  $\rho$  - specific gravity of sewage  $\text{kg/m}^3$ ,  $R$  - hydraulic radius.

The canal slope shall be calculated for 60%–70% of the canal filling height.

**Step 2.** Solution of the following equation:

$$\zeta \cdot d^{\frac{8}{3}} - Q = 0 \quad (13)$$

$$\zeta = \frac{\pi}{n} \cdot \left(\frac{1}{4}\right)^{\frac{5}{3}} \cdot J^{\frac{1}{2}}$$

If a solution  $d_*$  of the equation exists, then inequality

$$\zeta \cdot d^{\frac{8}{3}} - Q > 0$$

is valid for all values  $d > d_*$ .

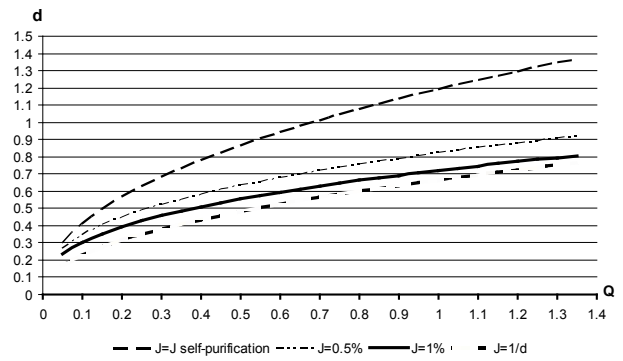
If canal slope  $J$  has been calculated from relations (12a)–(12b) and now a value  $d$  greater than  $d_*$  will be taken into account, then one shall pass to Step 1 and the canal slope must be calculated again.

If a solution of equation (13) does not exist then one shall return to Step 1, change the value  $J$  and solve once again equation (13).

In Fig. 11 the relations between the solution of equation (13) (concerning canal diameter  $d$ ) and canal flow  $Q$  for different canal slopes  $J$  are shown.

## 5. Computational Example

The considered algorithm has been tested on an exemplary housekeeping network consisting of 17 nodes connected by 16 segments. The net has got 9 input nodes ( $W_6, W_7, W_8, W_{10}, W_{11}, W_{14}, W_{15}, W_{16}, W_{17}$ ) and 1 output node  $W_1$ . Other nodes constitute the connections between different segments of the network. [11]



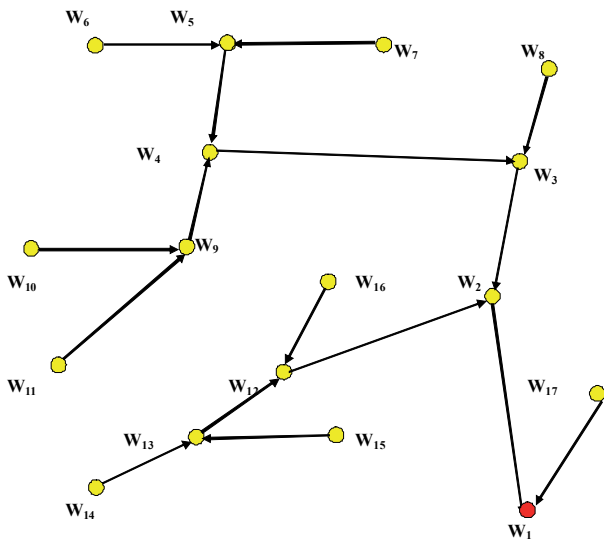
**Fig. 11.** Relations between canal diameter  $d$  and canal flow  $Q$  for different canal slopes  $J$

The arrows in Fig. 12 show the sewage flow direction. The sewage flow rates values for the input nodes are given. The flow rates in the connection nodes should be calculated according to the balance equation. For the respective segments the values of diameters  $d$  and canal slopes  $J$  are given.

For such a structure of the net the fillings  $H/d$  and the velocities of flows  $v$  in respective segments are calculated. The conclusion is that for these values of sewage rate flows and for the given values of geometric parameters (diameters and canal slopes), the heights of filling are lower than the half of canal diameters. So there is a possibility of increasing of the input flows in some sewage nodes. The calculations results are shown in Table 1.

**Table 1.** The results of hydraulic computations for the exemplary net shown in Fig. 12

Upper node	Lower node	Segment.	D [m]	Q [dm <sup>3</sup> /s]	J ‰	H/d	v [m/s]
W6	W5	1	0.2	0.53	5	10.72%	0,309
W7	W5	2	0.2	0.31	5	8.09%	0.259
W5	W4	3	0.2	1.14	5	15.08%	0.383
W10	W9	4	0.2	0.36	6	8.32%	0.289
W11	W9	5	0.2	1.13	9	13.03%	0.469
W9	W4	6	0.2	2.13	5	20.48%	0.460
W4	W3	7	0.2	3.91	5	27.78%	0.549
W8	W3	8	0.2	0.11	5	4.98%	0.189
W3	W2	9	0.2	4.12	5	28.53%	0.557
W14	W13	10	0.2	0.11	5	4.98%	0.189
W15	W13	11	0.2	0.32	5	8.22%	0.261
W13	W12	12	0.2	0.66	5	11.59%	0.325
W16	W12	13	0.2	0.24	5	7.18%	0.24
W12	W2	14	0.2	2.76	5	23.29%	0.497
W17	W1	15	0.2	6.33	5	35.70%	0.629
W2	W1	16	0.2	7.61	5	39.42%	0.661



**Fig.12. Structure of the sewage net used computational example**

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#### REFERENCES

- [1] Biedugnis S., *Metody informatyczne w wodociągach i kanalizacji*, Oficyna Wydawnicza Politechniki Warszawskiej, Warsaw 1998, in Polish.
- [2] Bogdan L., Petriczek G., *Zagadnienia modelowania sieci kanalizacyjnej dla potrzeb zarządzania przedsiębiorstwem wodociągowym*, series: Studia i Materiały Polskiego Stowarzyszenia Zarządzania Wiedzą, vol. 22, 2009, pp. 32–42, in Polish.
- [3] Błaszczak W., Stamatello H., Błaszczak P., „Kanalizacja. Sieci i pompownie”. vol. 1, publ.: Arkady, Warsaw 1983, in Polish.
- [4] Chudziński J., Sosnowski S., „Instalacje kanalizacyjne”, publ.: „Seidel-Przywecki”, Warsaw 2004 in Polish.

- [5] Jaromin K., Jilali A., Borkowski T., Widomski M., Ładóg G., „Rodzaje materiału i sposoby eksploatacji a współczynniki szorstkości w przewodach kanalizacji grawitacyjnej”. In: *Proceedings of ECOpole*, vol. 2, no. 2, 2008, in Polish.
- [6] Karnowski J.M., „Warunki transportu wleczonych części mineralnych w przewodach kołowych o dowolnym nachyleniu”. In: *Materiały Konferencji Naukowo-Technicznej PZITS*, Poznań 1973, in Polish.
- [7] Kwietniewski M., Nowakowska-Błaszczak A., „Obliczenia hydrauliczne kanałów ściekowych na podstawie krytycznych natężeń stycznych”. *Wodociągi i Kanalizacja*, 13, 1981, in Polish.
- [8] Niedzielski W., „Charakter przepływu w sieci kanalizacji deszczowej”, *Ochrona Środowiska*, no. 434/3–4, 1984, pp. 20–21, in Polish.
- [9] Puchalska E., Sowiński N., „Wymiarowanie kanałów ściekowych metodą krytycznych naprężeń stycznych”, *Ochrona Środowiska*, no. 3–4, 1984, pp. 53–62, in Polish.
- [10] Serek M., „Zastosowanie mikrokomputerów do obliczania sieci kanalizacji deszczowej”, *Ochrona Środowiska*, no. 488/1–2, 1986, pp. 27–28, in Polish.
- [11] Służalec A., *Sieć kanalizacji ściekowej – obliczenia hydrauliczne. Raport Badawczy IBS PAN*, Warsaw 2010, in Polish.
- [12] Wartalski J., „Komputerowe metody projektowania i analizy hydraulicznej sieciowych układów kanalizacyjnych”, *Ochrona Środowiska*, no. 434/3–4, 1984, pp. 20–21, in Polish.
- [13] Wartalski A., Wartalski J., „Projektowanie hydrauliczne rurociągów z tworzyw sztucznych”. *Ochrona Środowiska*, no. 1/76, 2000, pp. 19–24, in Polish.
- [14] WIŁO Polska – Producent pomp i urządzeń sanitarnych, „Podstawy odprowadzania i pompowania ścieków”. Oferta handlowa, in Polish.