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A QUANTITATIVE MANAGEMENT SUPPORT MODEL OF A CERTAIN PRODUCTION-SUPPLY SYSTEM – BOUNDARY CONDITIONS

The paper is dedicated to constructing a method for the probabilistic analysis of the functioning a certain production-supply system. Previously a set of partial differential equations has been derived satisfied by the joint density function of the state of a three-dimensional process characterizing the functioning of the system. The operation of the system at the boundaries of the stock levels is analyzed. Two sets of differential equations have been derived, one describing the operation of the system when the stock level is zero and one describing the operation of the system when the stocks are full.

Keywords: *production-supply system, process, boundary state, system of differential equations*

1. Introduction

Supply systems are the s of analyses in various publications (e.g. [1, 3–7, 10, 11, 13–19]). This paper is a continuation of the research carried out previously [3–8, 10, 11, 13, 17, 18]. More specifically, in the paper, we further develop new probabilistic methods of analyzing the operation of a production-supply system presented elsewhere [8]. It contains a description of the operation of such a system, theoretical char-

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acteristics of the functioning of the system and a set of partial differential equations, satisfied by the joint density function of the state of a three-dimensional process describing the operation of the system when stock levels are neither empty nor full.

Continuing the research, we will carry out a probabilistic analysis of the operation of the system at the boundaries of the stock levels, that is when stock levels are either empty or full.

2. When stock levels are empty

The analysis of the system considered is split into three cases ([8]):

Intermediate stock levels

$$0 < z(t) < V \tag{1}$$

The lower boundary (no stock)

$$z(t) = 0 \tag{2}$$

The upper boundary (stock levels full)

$$z(t) = V \tag{3}$$

These cases should be dealt with separately, because they correspond to different operating conditions. The analysis of the case (1) was carried elsewhere [8]. The results, assumptions and conclusions presented in the paper [8] will be used for the analysis of the system in the final two cases.

The operation of the system considered is described by a three-dimensional process, $(y_1(t), y_2(t), z(t))$, where $y_1(t)$ and $y_2(t)$ denote the streams of production (products) delivered through the transport subsystems T_1 and T_2 to the store M, that supplies the recipient E at a constant rate a, while $z(t)$ denotes the state of the subsystem M (the level of stocks) at time t. Let us denote possible states of the Markov process $y_1(t)$ by $y_{11}, y_{12}, ..., y_{1n}$, and of the Markov process $y_2(t)$ by: $y_{21}, y_{22}, ..., y_{2m}$. The intensity of transitions between these states (the rate of product supply) of the delivery streams $y_1(t)$ and $y_2(t)$ will be denoted by $\pi_{jk}^{(1)}$ and $\pi_{si}^{(2)}$, respectively, which are schematically given in the form:

$$y_{1j} \xrightarrow{\pi_{jk}^{(1)}} y_{1k} \quad \text{for} \quad j \neq k \tag{4}$$

$$y_{2s} \xrightarrow{\pi_{si}^{(2)}} y_{2i} \quad \text{for} \quad s \neq i \tag{5}$$

The authors [8] derived a system of partial differential equations satisfied by the density functions $f_{x_{2i}}^{x_{1k}}(z,t)$ which determines probabilities of the form:

$$P\left(a_1 \le z(t) \le b_1, \quad x_1(t) = x_{1k}, \quad x_2(t) = x_{2i}\right) = \int_{a_1}^{b_1} f_{x_{2i}}^{x_{1k}}(z,t)\,dz \tag{6}$$

where $0 \le a_1 < b_1 \le V$, x_{1k} is the k-th state of the process $x_1(t) = y_1(t) - a$ ($x_{1k} = y_{1k} - a$, $k = 1, 2, ..., n$), while x_{2i} denotes the i-th state of the process $x_2(t) = y_2(t) - a$ ($x_{2i} = y_{2i} - a$, $i = 1, 2, ..., m$).

Next, we introduce systems of differential equations, which are satisfied by the probabilities:

$$P\left(z(t) = 0, \quad x_1(t) = x_{1k}, \quad x_2(t) = x_{2i}\right) \tag{7}$$

$$P\left(z(t) = V, \quad x_1(t) = x_{1k}, \quad x_2(t) = x_{2i}\right) \tag{8}$$

These probabilities will be denoted by $Q_{x_{2i}}^{x_{1k}}(\{0\},t)$, $Q_{x_{2i}}^{x_{1k}}(\{V\},t)$. These systems of equations will be derived analytically (cf. [8]).

Using the operating conditions of the system considered, for case (2) we obtain the equations:

$$Q_{x_{2i}}^{x_{1k}}(\{0\},t+\tau) = P\left(z(t+\tau) = 0, x_1(t+\tau) = x_{1k}, x_2(t+\tau) = x_{2i}\right)$$

$$\approx Q_{x_{2i}}^{x_{1k}}(\{0\},t)\left[1 - \left(\pi_k^{(1)} + \pi_i^{(2)}\right)\tau\right]$$

$$+P\left(0 < z(t) < -\left(x_{1k} + x_{2i} + a\right)\tau, x_1(t) = x_{1k}, x_2(t) = x_{2i}\right) + [1 - (\pi_k^{(1)} + \pi_i^{(2)})\tau]$$

$$+ \sum_{\substack{k' \ne k \\ -(x_{1k'} + x_{2i} + a) \ge 0}} P(z(t) = 0, x_1(t) = x_{1k'}, x_2(t) = x_{2i})\pi_{k'k}^{(1)}\tau(1 - \pi_i^{(2)}\tau)$$

$$+ \sum_{\substack{k' \ne k \\ -(x_{1k'} + x_{2i} + a) \ge 0}} P(0 < z(t) < -(x_{1k'} + x_{2i} + a)\tau, x_1(t) = x_{1k'}, x_2(t) = x_{2i})\pi_{k'k}^{(1)}\tau(1 - \pi_i^{(2)}\tau)$$

$$+ \sum_{\substack{i' \ne i \\ -(x_{1k} + x_{2i'} + a) \ge 0}} P(0 < z(t) < -(x_{1k} + x_{2i'} + a)\tau, x_1(t) = x_{1k}, x_2(t) = x_{2i'})\pi_{i'i}^{(2)}\tau(1 - \pi_k^{(1)}\tau)$$

$$+ \sum_{\substack{i' \ne i \\ -(x_{1k} + x_{2i'} + a) \ge 0}} P(z(t) = 0, x_1(t) = x_{1k}, x_2(t) = x_{2i'})\pi_{i'i}^{(2)}\tau(1 - \pi_k^{(1)}\tau)$$

$$+ \sum_{\substack{i'\neq i \\ k'\neq k \\ -(x_{1k'}+x_{2i'}+a)\geq 0}} P(0 < z(t) < -(x_{1k'} + x_{2i'} + a)\tau,\, x_1(t) = x_{1k'},\, x_2(t) = x_{2i'})\pi_{i'i}^{(2)}\tau\pi_{k'k}^{(1)}\tau$$

$$+ \sum_{\substack{i'\neq i \\ k'\neq k \\ -(x_{1k'}+x_{2i'}+a)\geq 0}} P(z(t) = 0,\, x_1(t) = x_{1k'},\, x_2(t) = x_{2i'})\pi_{i'i}^{(2)}\tau\pi_{k'\neq k}^{(1)}\tau) \tag{9}$$

where:

$$\pi_k^{(1)} = \sum_{l\neq k}\pi_{kl}^{(1)}, \qquad \pi_i^{(2)} = \sum_{l\neq i}\pi_{il}^{(2)} \tag{10}$$

The interpretation of Equation (9) is as follows: the first element of the equation indicates the probability of remaining in the state ($z = 0$, $x_1 = x_{1k}$, $x_2 = x_{2i}$). This probability is one minus the sum of the output intensities from the state ($x_1 = x_{1k}$, $x_2 = x_{2i}$) (see, e.g. [2], [9]). In our case, from definitions (4) and (5), the output intensity from state x_{1k} is $\pi_k^{(1)}$ (see formula (10)), and the output intensity from state x_{2i} is $\pi_i^{(2)}$ (see Eq. (10)).

But the level $z = 0$ may also be achieved at time $t + \tau$, if at time t the state is (x_{1k}, x_{2i}) and the store is partially filled: $0 \leq z(t) < -(x_{1k} + x_{2i} + a)\tau$. When the stock level at time t is $(a - y_{1k} - y_{2i})\tau = (-x_{1k} - x_{2i} - a)\tau \geq 0$, then the stock level reaches zero at time $t + \tau$ if the supply rates do not change. This fact takes into account the second part of Equation (9). The content of the other elements in this formula can be explained similarly. Note that we use the fact that in "simple" processes the probability of double state changes are of order higher than τ. The asymptotic equality ≈ takes this into account, which means that we omit expressions of order $o(\tau)$, which satisfy the condition

$$\lim_{\tau\to 0}\frac{o(\tau)}{\tau} = 0 \tag{11}$$

After applying Equation (6), Equation (9) takes the form:

$$Q_{x_{2i}}^{x_{1k}}(\{0\}, t+\tau) \approx Q_{x_{2i}}^{x_{1k}}(\{0\}, t)[1 - (\pi_k^{(1)} + \pi_i^{(2)})\tau]$$

$$+ \int_0^{-(x_{1k}+x_{2i}+a)\tau} f_{x_{2i}}^{x_{1k}}(z, t) - \int_0^{-(x_{1k}+x_{2i}+a)\tau} f_{x_{2i}}^{x_{1k}}(z, t)dz[\pi_k^{(1)} + \pi_i^{(2)}]\tau$$

$$+ \sum_{\substack{k'\neq k \\ -(x_{1k'}+x_{2i}+a)\geq 0}} Q_{x_{2i}}^{x_{1k'}}(\{0\}, t)\pi_{k'k}^{(1)}\tau(1 - \pi_i^{(2)}\tau)$$

$$+\sum_{\substack{k'\neq k\\ -(x_{1k'}+x_{2i}+a)\geq 0}} \int_0^{-(x_{1k'}+x_{2i}+a)\tau} f_{x_{2i}}^{x_{1k'}}(z,t)dz\pi_{k'k}^{(1)}\tau(1-\pi_i^{(2)}\tau)$$

$$+\sum_{\substack{i'\neq i\\ -(x_{1k}+x_{2i'}+a)\geq 0}} \int_0^{-(x_{1k}+x_{2i'}+a)\tau} f_{x_{2i''}}^{x_{1k}}(z,t)dz\pi_{i'i}^{(2)}\tau(1-\pi_k^{(1)}\tau)$$

$$+\sum_{\substack{i'\neq i\\ -(x_{1k}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k}}(\{0\},t)\pi_{i'i}^{(2)}\tau(1-\pi_k^{(1)}\tau)$$

$$+\sum_{\substack{i'\neq i\\ k'\neq k\\ -(x_{1k'}+x_{2i'}+a)\geq 0}} \int_0^{-(x_{1k'}+x_{2i'}+a)\tau} f_{x_{2i''}}^{x_{1k}}(z,t)dz\pi_{i'i}^{(1)}\tau\pi_{k'k}^{(2)}\tau$$

$$+\sum_{\substack{i'\neq i\\ k'\neq k\\ -(x_{1k'}+x_{2i'}+a)\geq 0}} Q_{x_{2i''}}^{x_{1k'}}(\{0\},t)\pi_{i'i}^{(2)}\tau\pi_{k'k}^{(1)}\tau \tag{12}$$

The probabilities in Equation (9), where the store is partially filled, were calculated according to Equation (12) using Equation (6), e.g.

$$P(0\leq z(t)\leq -(x_{1k}+x_{2i}+a)\tau,\, x_1(t)=x_{1k},\, x_2(t)=x_{2i})$$

$$=\int_0^{-(x_{1k}+x_{2i}+a)\tau} f_{x_{2i}}^{x_{1k}}(z,t)dz \approx -(x_{1k}+x_{2i}+a)\tau\, f_{x_{2i}}^{x_{1k}}(0,t) \tag{13}$$

Asymptotic equality is achieved here based on the mean value theorem for integrals of continuous functions (see, e.g. [12]).

After using Equation (13), equation (12) takes the form:

$$Q_{2i}^{1k}(\{0\},t+\tau)\approx Q_{x_{2i}}^{x_{1k}}(\{0\},t)[1-(\pi_k^{(1)}+\pi_i^{(2)})\tau]-f_{x_{2i}}^{x_{1k}}(0,t)(x_{1k}+x_{2i}+a)\tau$$

$$+f_{x_{2i}}^{x_{1i}}(0,t)(x_{1k}+x_{2i}+a)\tau\,[\pi_k^{(1)}+\pi_i^{(2)}]\tau+\sum_{\substack{k'\neq k\\ -(x_{1k'}+x_{2i}+a)\geq 0}} Q_{x_{2i}}^{x_{1k'}}(\{0\},t)\pi_{k'k}^{(1)}\tau(1-\pi_i^{(2)}\tau)$$

$$+\sum_{\substack{k'\neq k\\ -(x_{1k'}+x_{2i}+a)\geq 0}} f_{x_{2i}}^{x_{1k''}}(0,t)\,[-(x_{1k'}+x_{2i}+a]\tau\cdot\pi_{k'k}^{(1)}\,\tau(1-\pi_i^{(2)}\tau)$$

$$+ \sum_{\substack{i' \neq i \\ -(x_{1k}+x_{2i'}+a)\geq 0}} f_{x_{2i'}}^{x_{1k}}(0,t)\,[-(x_{1k}+x_{2i'}+a)]\tau\,\pi_{i'i}^{(2)}\tau(1-\pi_k^{(1)}\tau)$$

$$+ \sum_{\substack{i' \neq i \\ -(x_{1k}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k}}(\{0\},t)\pi_{i'i}^{(2)}\tau(1-\pi_k^{(1)}\tau)$$

$$+ \sum_{\substack{i' \neq i,\, k' \neq k \\ -(x_{1k'}+x_{2i'}+a)\geq 0}} f_{x_{2i'}}^{x_{1k'}}(0,t)[-(x_{1k'}+x_{2i'}+a)]\tau\pi_{i'i}^{(2)}\tau\pi_{k'k}^{(1)}\tau$$

$$+ \sum_{\substack{i' \neq i,\, k' \neq k \\ -(x_{1k'}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k'}}(\{0\},t)\pi_{i'i}^{(2)}\tau\pi_{k'k}^{(1)}\tau \qquad (14)$$

Now we carry out, in sequence, the following operations on Equation (14) for $-(x_{1k} + x_{2i} + a) \geq 0$:

– move the function $Q_{x_{2i}}^{x_{1k}}(\{0\},t)$ to the left hand side of Equation (14),

– divide both sides of the equation obtained by τ,

– take the boundary as $\tau \to 0$ on both sides.

As a result of these operations, the asymptotic equality obtained, similarly as in [8], takes the form of a simple equality:

$$\frac{\partial Q_{x_{2i}}^{x_{1k}}(\{0\},t)}{\partial t} = -(\pi_k^{(1)} + \pi_i^{(2)})Q_{x_{2i}}^{x_{1k}}(\{0\},t) + f_{x_{2i}}^{x_{1k}}(0,t)(x_{1k} + x_{2i} + a)$$

$$+ \sum_{\substack{k' \neq k \\ -(x_{1k'}+x_{2i}+a)\geq 0}} Q_{x_{2i}}^{x_{1k'}}(\{0\},t)\pi_{k'k}^{(1)}$$

$$+ \sum_{\substack{i' \neq i \\ -(x_{1k'}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k}}(\{0\},t)\pi_{i'i}^{(2)} \quad \text{for} \quad -(x_{1k} + x_{2i} + a) \geq 0 \qquad (15)$$

Obviously, for $-(x_{1k} + x_{2i} + a) < 0$, the following equality holds:

$$Q_{x_{2i}}^{x_{1k}}(\{0\},t) = 0 \qquad (16)$$

because when $(x_{1k} + x_{2i} + a) > 0$, i.e. $y_{1k} + y_{2i} > a$, the lower barrier cannot be achieved with positive probability.

3. When stock levels are full

We now analyze the operation of the system when stock levels reach the upper barrier, i.e. the state $z(t) = V$.

In this case, different operating conditions than in the previous two cases arise. Arguing similarly as in the case where stock levels are empty, we obtain an equation for the probability $Q_{x_{2i}}^{x_{1k}}(\{V\},t+\tau)$ in the following form:

$$Q_{x_{2i}}^{x_{1k}}(\{V\},t+\tau)=P(z(t+\tau)=V,\,x_1(t+\tau)=x_{1k},\,x_2(t+\tau)=x_{2i})$$

$$\approx Q_{x_{2i}}^{x_{1k}}(\{V\},t)[1-(\pi_k^{(1)}+\pi_i^{(2)})\tau]$$

$$+P(V-(x_{1k}+x_{2i}+a)\tau<z(t)<V,\,x_1(t)=x_{1k},\,x_2(t)=x_{2i})[1-(\pi_k^{(1)}+\pi_i^{(2)})\tau]$$

$$+\sum_{\substack{k'\neq k\\(x_{1k'}+x_{2i}+a)\geq 0}} Q_{x_{2i}}^{x_{1k'}}(\{V\},t)\pi_{k'k}^{(1)}\tau(1-\pi_i^{(2)}\tau)$$

$$+\sum_{\substack{i'\neq i\\(x_{1k}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k}}(\{V\},t)\pi_{i'i}^{(2)}\tau(1-\pi_k^{(1)}\tau)+\sum_{\substack{k'\neq k,i'\neq i\\(x_{1k'}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k''}}(\{V\},t)\pi_{i'i}^{(2)}\tau\pi_{k'k}^{(1)}\tau$$

$$+\sum_{\substack{k'\neq k\\(x_{1k'}+x_{2i}+a)\geq 0}} P(V-(x_{1k'}+x_{2i}+a)\tau<z(t)<V,\,x_1(t)=x_{1k'},\,x_2(t)=x_{2i})\pi_{k'k}^{(1)}\tau(1-\pi_i^{(2)}\tau)$$

$$+\sum_{\substack{i'\neq i\\(x_{1k}+x_{2i'}+a)\geq 0}} P(V-(x_{1k}+x_{2i'}+a)\tau<z(t)<V,\,x_1(t)=x_{1k},\,x_2(t)=x_{2i'})\pi_{i'i}^{(2)}\tau(1-\pi_k^{(1)}\tau)$$

$$+\sum_{\substack{k'\neq k,i'\neq i\\(x_{1k'}+x_{2i'}+a)>0}} P(V-(x_{1k'}+x_{2i'}+a)\tau<z<V,\,x_1(t)=x_{1k'},\,x_2(t)=x_{2i'})\pi_{k'k}^{(1)}\tau\pi_{i'i}^{(2)}\tau \qquad (17)$$

Applying Equation (6) and the mean value theorem for the integrals to Equation (17) (cf. Chapter 2), we obtain:

$$Q_{x_{2i}}^{x_{1k}}(\{V\},t+\tau)\approx Q_{x_{2i}}^{x_{1k}}(\{V\},t)[1-(\pi_k^{(1)}+\pi_i^{(2)})\tau]$$

$$+f_{x_{2i}}^{x_{1i}}(V,t)\,[(x_{1k}+x_{2i}+a)\tau]\,[1-(\pi_k^{(1)}+\pi_i^{(2)})\tau]$$

$$+\sum_{\substack{k'\neq k\\(x_{1k'}+x_{2i}+a)\geq 0}} Q_{x_{2i}}^{x_{1k'}}(\{V\},t)\pi_{k'k}^{(1)}\tau(1-\pi_i^{(2)}\tau)$$

$$+\sum_{\substack{i'\neq i\\(x_{1k}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k}}(\{V\},t)\pi_{i'i}^{(2)}\tau(1-\pi_k^{(1)}\tau)+\sum_{\substack{k'\neq k,i'\neq i\\(x_{1k'}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k''}}(\{V\},t)\pi_{i'i}^{(2)}\tau\pi_{k'k}^{(1)}\tau$$

$$+\sum_{\substack{k'\neq k\\(x_{1k'}+x_{2i}+a)\geq 0}} f_{x_{2i}}^{x_{1k'}}(V,t)[(x_{1k'}+x_{2i}+a)]\tau\pi_{k'k}^{(1)}\tau(1-\pi_{i}^{(2)}\tau)]$$

$$+\sum_{\substack{i'\neq i\\(x_{1k}+x_{2i'}+a)\geq 0}} f_{x_{2i'}}^{x_{1k}}(V,t)[(x_{1k}+x_{2i'}+a)\tau]\pi_{i'i}^{(2)}\tau(1-\pi_{k}^{(1)}\tau)$$

$$+\sum_{\substack{k'\neq k,\, i'\neq i\\(x_{1k'}+x_{2i'}+a)\geq 0}} f_{x_{2i'}}^{x_{1k'}}(V,t)[(x_{1k'}+x_{2i'}+a)\tau]\pi_{k'k}^{(1)}\tau\pi_{i'i}^{(2)}\tau \tag{18}$$

We apply, in sequence, the following operations to the Equation (18) for $x_{1k} + x_{2i} + a \geq 0$:

– move the function $Q_{x_{2i}}^{x_{1k}}(\{V\},t)$ to the left hand side of Equation (18),

– divide both sides of the equation obtained by τ,

– take the limit as $\tau \to 0$ on both sides.

As a result of these operations, the asymptotic equality obtained, as in [8] and Chapter 2, takes the form of a simple equation:

$$\frac{\partial Q_{x_{2i}}^{x_{1k}}(\{V\},t)}{\partial t} = -(\pi_{k}^{(1)}+\pi_{i}^{(2)})Q_{x_{2i}}^{x_{1k}}(\{V\},t) + \sum_{\substack{k'\neq k\\(x_{1k'}+x_{2i}+a)\geq 0}} Q_{x_{2i}}^{x_{1k}}(\{V\},t)\pi_{k'k}^{(1)}$$

$$+\sum_{\substack{i'\neq i\\(x_{1k}+x_{2i'}+a)\geq 0}} Q_{x_{2i'}}^{x_{1k}}(\{V\},t)\pi_{i'i}^{(2)} + f_{x_{2i}}^{x_{1k}}(V,t)(x_{1k}+x_{2i}+a) \tag{19}$$

for $x_{1k} + x_{2i} + a \geq 0$.

However, for $(x_{1k} + x_{2i} + a) < 0$

$$Q_{x_{2i}}^{x_{1k}}(\{V\},t) = 0 \tag{20}$$

because when $(x_{1k} + x_{2i} + a) < 0$, i.e. $y_{1k} + y_{2i} < a$, the upper barrier cannot be achieved with positive probability.

The systems of Equations (15), (16) and (19), (20) describing the operation of the system at extreme values of the stock levels, together with the results obtained in [8], will enable us to obtain measures supporting the management process of the production-supply system considered.

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