JOURNAL OF WATER AND LAND DEVELOPMENT
ISSN 1429–7426, e-ISSN 2083-4535

Polish Academy of Sciences (PAN), Committee on Agronomic Sciences
Section of Land Reclamation and Environmental Engineering in Agriculture
Institute of Technology and Life Sciences (ITP)

JOURNAL OF WATER AND LAND DEVELOPMENT
2020, No. 45 (IV–VI): 61–69
https://DOI.org/10.24425/jwld.2020.133046

Received 23.12.2019
Reviewed 28.01.2020
Accepted 17.02.2020

The use of the sluice gate discharge equations to estimate the volumetric flow rate in the irrigation channels

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For citation: Kubrak J., Kubrak E. 2020. The use of the sluice gate discharge equations to estimate the volumetric flow rate in the irrigation channels. Journal of Water and Land Development. No. 45 (IV–VI) p. 61–69. DOI: 10.24425/jwld.2020.133046.

Abstract

The paper attempts to assess the possibility of using typical check structures equipped with sluice gates to measure the volumetric flow rate in the irrigation channels. The submerged flow through the sluice gate was considered. Experimental tests on a model of typical check structure in 1:2 scale were carried out. The conducted analyzes confirmed the possibility of using discharge equation for submerged flow through the sluice gate to estimate the water flow rate in the irrigation channels. In order to obtain accurate values of flow rate, the downstream tailwater depth should be measured at the appropriate distance from the sluice gate. For different values of gate-opening height, the downstream water depth measurement locations allowing for a correct flow estimation were indicated. This approach might be useful in calibration of other designs of sluice gates for flow measurements.

Key words: *discharge coefficient, error of discharge calculation, irrigation channels, sluice gate, submerged flow*

INTRODUCTION

Due to the increasingly frequent droughts, the management and conservation of existing water supplies becomes a necessity. One key to better water management practices is accurate water measurements [USBR 2001]. Flow measurement in irrigation channels requires the construction of additional structures equipped with expensive electronic devices [Rubicon Water 2020] or experimental calibration of existing irrigation structures [JĘDRYKA, KACA 1998; KACA 1996; LOZANO *et al.* 2009]. The latter issue is discussed in this paper, in relation to one of the simplest types of structures used in channel irrigation systems – checks. Checks or cross regulators may be fixed overflow weirs with no movable controlling device, or they may be provided with gates [KRAATZ 1975]. One of the most common opening in small hydraulic structures used for controlling the discharge is the sluice gate. Downstream flow can be free or submerged and depends on the exit flow conditions. Adjusting the gate opening allows to control the upstream water level and the flow rate. Sluice gate changes flow regime from subcritical (downstream) to supercritical (near the gate). Due to the fact that the channel slope is usually mild, hydraulic jump occurs downstream from the structure, which affects the flow conditions [KUBRAK, KUBRAK 2018]. If the hydraulic jump does not occur downstream near the sluice gate, the flow is free (unsubmerged). This takes place at a relatively large ratio of upstream depth to the gate-opening height.

A characteristic feature of a submerged flow through the sluice gate is its steadiness and low sensitivity to small changes of water levels upstream and downstream. For this reason, the sluice gates are used for flow measuring. BOITEN [1992] recommends using the sluice gate for flow measuring when $H{:}a \geq 2.70$ (H – water depth upstream gate, a – gate-opening height). The accuracy of discharge estimation with submerged flow through the sluice gate is in the range from 10 to 15%.

In this paper an attempt was made to clarify whether equations describing submerged flow through the sluice gate can be used to estimate the discharge in the channel irrigation systems. The issue of indicating the location of the cross-section at which the downstream water depth should be measured was raised. It was also examined how the position of the indicated cross-section changes while operating the gate.

STUDY METHODS

ANALYTICAL EQUATIONS DESCRIBING A FLOW THROUGH THE SLUICE GATE

For a free discharge, the formula for uniform flow velocity can be derived from the steady flow energy equation (sections 0–0 and 1–1 in Fig. 1):

$$v_1 = \varphi\sqrt{2\mathrm{g}(H_0 - h_1)} \tag{1}$$

The discharge through the sluice gate with a width of b can be calculated from:

$$Q = v_1 b h_1 = \varphi b h_1\sqrt{2\mathrm{g}(H_0 - h_1)} \tag{2}$$

Where: φ = coefficient of velocity, H_0 = total head upstream (m), h_1 = water depth near the gate downstream (m), b = width of the sluice gate, g = gravitational acceleration ($\mathrm{m \cdot s^{-2}}$).

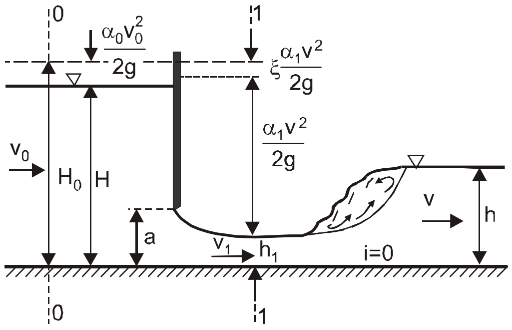

Fig. 1. Free flow through a sluice gate: a = gate opening (m), g = gravitational acceleration ($\mathrm{m \cdot s^{-2}}$), H_0 = total head upstream at section 0–0 (m), H = upstream water depth (m), h_1 = water depth at the location section 1–1 of maximum vertical contraction (m), h = downstream water depth (m), i = slope of the bottom (–), v_0 = average flow velocity upstream from gate (m:s); v_1 = average flow velocity at the location section 1–1 (m:s); v = average flow velocity downstream from gate (m:s); α_0, α_1 = velocity distribution coefficient (–), ξ = loss coefficient (–); source: own elaboration

The relation between water depth h_1 and gate-opening height a was given by ZHUKOVSKIY [1936]: $h_1 = \varepsilon a$. Assuming that the product of contraction coefficient ε and coefficient of velocity φ is the discharge coefficient μ:

$$\mu = \varepsilon\varphi \tag{3}$$

one obtains a formula for a free discharge through a sluice gate [HENDERSON 1966]:

$$Q = \mu a b\sqrt{2\mathrm{g}(H_0 - \varepsilon a)} \tag{4}$$

If the hydraulic jump is formed near the sluice gate, the flow becomes submerged. In this case, two types of the hydraulic jump can be distinguished. In the first scenario (Fig. 2a), the depth near the sluice gate h_z is clearly lower than the downstream tailwater depth h ($h_z < h$). In this situation, the downstream water surface shows undulations. In the second scenario (Fig. 2b) the depth near the sluice gate is similar to the downstream tailwater depth and the downstream water surface remains relatively smooth.

The discharge through a sluice gate in the first scenario (Fig. 2a) may be calculated on the basis of flow velocity v_1 which can be derived from the steady flow energy equation (sections 0–0 and 1–1 in Fig. 2):

$$Q = \mu a b\sqrt{2\mathrm{g}(H_0 - h_z)} \tag{5}$$

The discharge coefficient μ can be calculated from Eq. (3) as for the unsubmerged flow through the sluice gate. The values of the contraction coefficient ε were experimentally determined by ZHUKOVSKIY [1936]. The position of cross section 1–1 in which the depth h_z should be measured in order to calculate the discharge from Eq. (5) has not been clearly defined.

When the hydraulic jump is submerged, it can be assumed that the depth of h_z is the same as the downstream tailwater depth h, ie. $h_z \approx h$ (Fig. 2b).

The discharge through the sluice gate in the case of the submerged hydraulic jump can also be calculated as for the free discharge by expressing a depth in cross section 1–1 as $h_1 = \varepsilon a$ [ROUSE 1946]:

$$Q = C_d a b\sqrt{2\mathrm{g}H} \tag{6}$$

The discharge coefficient in Eq. (6) is defined as follows:

$$C = \frac{\varepsilon}{\sqrt{1+\varepsilon\frac{a}{H}}} \tag{7}$$

According to SWAMEE [1992], the type of the discharge through the sluice gate depends on the ratio of upstream depth to the gate-opening height. Submerged flow (Fig. 2 a, b) would occur for low values of this ratio. Free discharge (Fig. 1) occurs at relatively large ratio of upstream depth to gate-opening height. In this case, the water surface is smooth, whereas for a submerged flow the corresponding flow profile is rough. Free flow occurs when:

$$H > 0.81h\left(\frac{h}{a}\right)^{0.72} \tag{8}$$

Under free flow conditions the discharge coefficient increases with the increase of the H/a ratio until it reaches its maximum value of 0.611 (Fig. 3). The sluice gate closes at $H = a$ and thus for $H \leq a$ the discharge coefficient equals zero. Taking these two extreme values into account, SWAMEE [1992] obtained simple regression between values of the discharge coefficient and the H/a ratio for a free flow through the sluice gate:

$$C_d = 0.611\left(\frac{H-a}{H+15a}\right)^{0.072} \tag{9}$$

Based on the results of research carried out for a free and submerged flow through the sluice gate published by HENRY [1950] and confirmed by RAJARATNAM and SUBRAMANYA [1967], regressions for discharge coefficient C_d were derived by SWAMEE [1992]. Figure 3 shows values of discharge coefficient for a free flow through the sluice gate given by various authors.

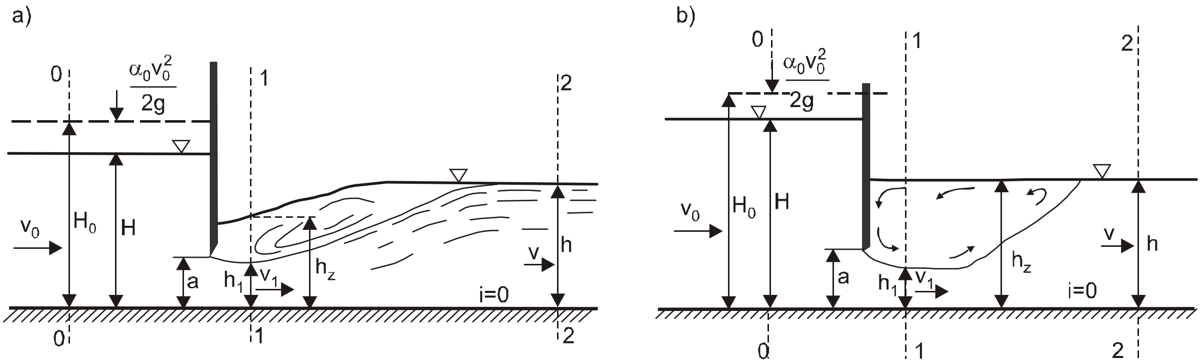

Fig. 2. Submerged flow through the sluice gate: a) $h_z < h$, b) $h_z \approx h$; the tailwater curve is always above the jump curve: h_z = the depth near the sluice gate (m); other symbols: a, g, H_0, H, h_1, h, i, v_0, v_1, v, α_0, as in Fig 1; source: own elaboration

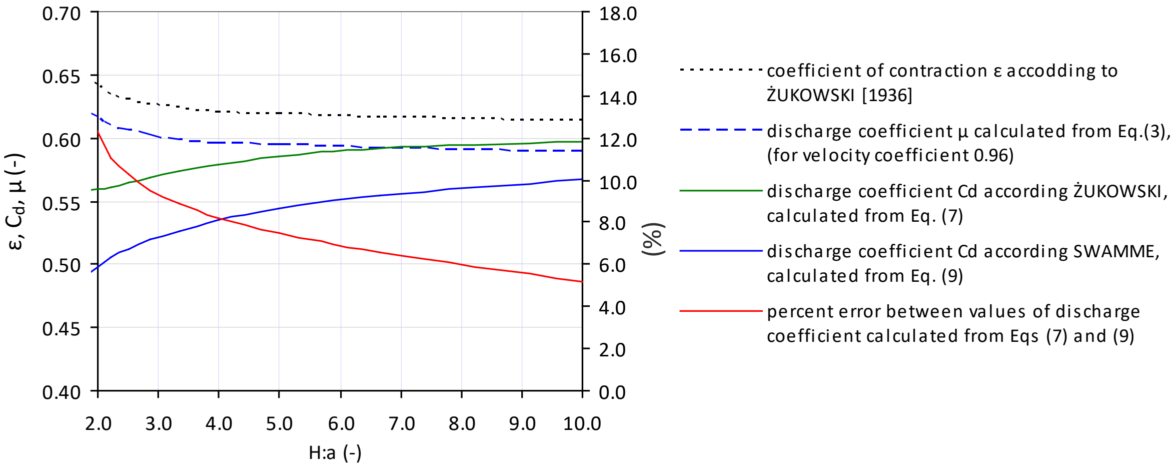

Fig. 3. Values of contraction coefficient ε according to ZHUKOVSKIY [1936] and discharge coefficients μ and C_d calculated on their basis and values of discharge coefficient C_d for free flow through the sluice gate according to SWAMEE [1992]; source: own study

In the case of a submerged flow through the sluice gate, the discharge coefficient depends on the upstream and downstream depths and the gate-opening height.

SWAMEE [1992] used Eq. (6) to calculate submerged flow through the sluice gate. However, this requires a distinction between the discharge coefficients for a free and submerged flow through the sluice gate. According to SWAMEE [1992], submerged flow (Fig. 2a, b) occurs when:

$$H < 0.81h\left(\frac{h}{a}\right)^{0.72} \qquad (10)$$

The equation for calculating the coefficient of discharge for a submerged flow through the sluice gate is given by SWAMEE [1992]:

$$C_d = \frac{0.611\left(\frac{H-a}{H+15a}\right)^{0.072}(H-h)^{0.7}}{\left\{0.32\left[0.81h\left(\frac{h}{a}\right)^{0.72}-H\right]^{0.7}+(H-h)^{0.7}\right\}} \qquad (11)$$

Values of the discharge coefficients during the transition of a free flow into a submerged flow through the sluice gate are shown in Figure 4.

To calculate a submerged flow through the sluice gate, two formulas are commonly used: Eq. (5) and Eq. (6). In the first case, discharge coefficient is defined as a function of H:a ratio and in the second case, discharge coefficient is calculated from Eq. (11).

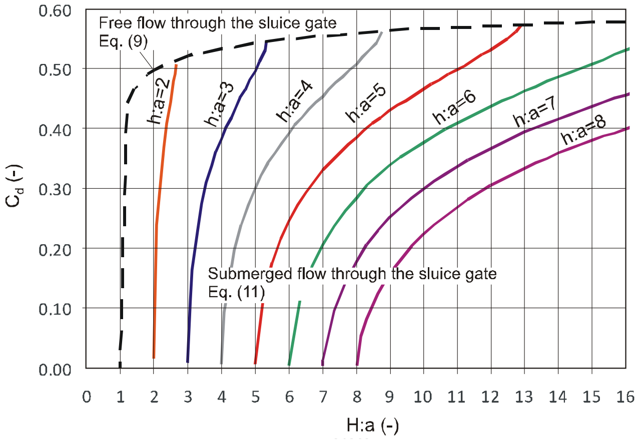

Fig. 4. Values of discharge coefficient C_d for different H:a and h:a ratios in the case of a free flow through the sluice gate; source: own elaboration on base Equations (9) and (11)

EXPERIMENTAL TESTS OF A FLOW THROUGH THE SLUICE GATE

The experimental tests were carried out at the hydraulic laboratory of the Faculty of Civil and Environmental Engineering of the Warsaw University of Life Sciences. The open-channel used in this experiment is a model of prototype check structure used in practice in 1:2 scale

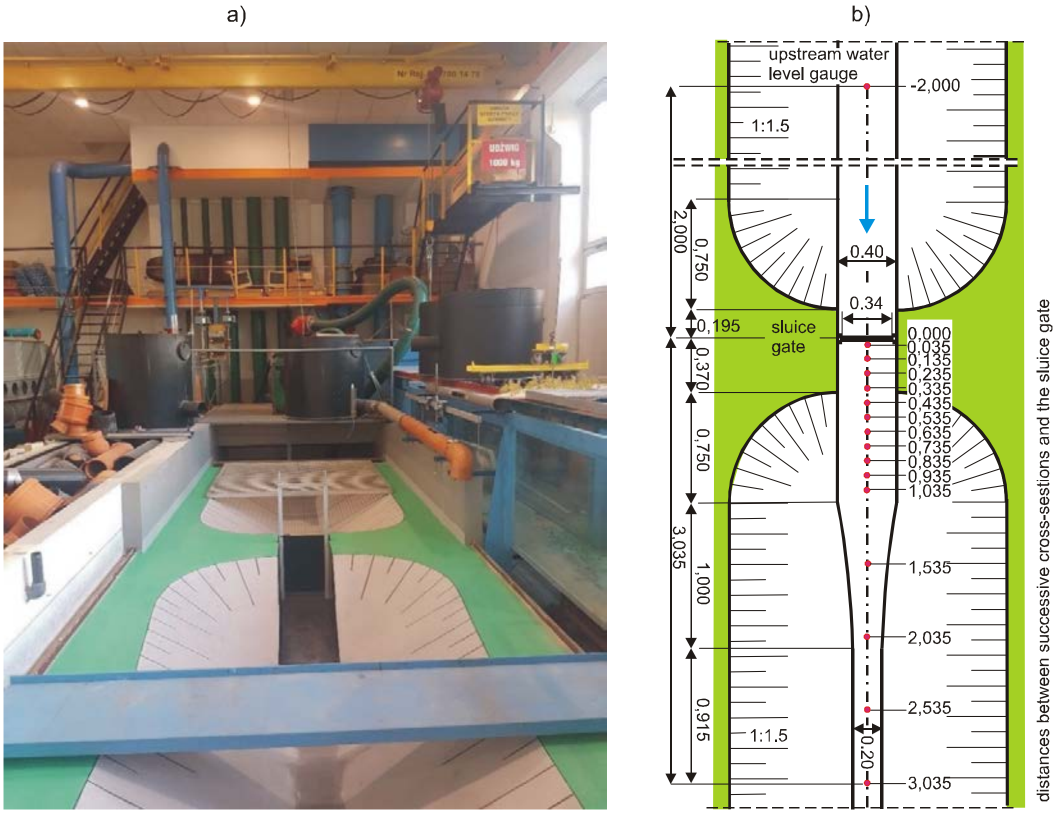

Fig. 5. Experimental setup: a) view from the downstream side, b) scheme of the top view of the sluice gate model with the location of measurement points (dimensions are given in meters); source: own elaboration

[CBSiPWM 1969]. The sluice gate was made from a rectangular PVC plate with dimensions of 0.43 × 0.50 m and a thickness of 0.01 m. The sluice gate has been placed in guides which were fixed directly to the check structure abutments. C-shaped guides reduced the width of the cross section of the outflow to $b = 0.34$ m. The inlet to the open-channel model has been equipped with a calming chamber. Volumetric flow rate was measured with an inductive flowmeter. The flow conditions were determined by setting the tailwater depth. For this purpose, a hinged flap was mounted on the downstream side of the gate. The flap had dimensions of 2.30 × 0.61 m and was placed at a distance of 3.5 m from the sluice gate. By raising the flap, an increase of tailwater depth was obtained. This allowed the experiment to be carried out under various flow conditions. The tailwater depth h was calculated as for the flow in the channel of trapezoidal cross-section, with bottom width of 0.20 m, slope inclination of 1:1 and values of roughness coefficient equal to 0.031 $\mathrm{m}^{-1/3}\cdot\mathrm{s}$, 0.034 $\mathrm{m}^{-1/3}\cdot\mathrm{s}$ and 0.037 $\mathrm{m}^{-1/3}\cdot\mathrm{s}$. Upstream water level was measured at a distance of 2.0 m from the sluice gate. Water level gauge which measured the tailwater depth was mounted on a cart on the rails. Depth measurements were made along the channel axis. The first depth measurement cross-section was located 0.035 m below the sluice gate. The distances between successive cross-sections equaled 0.10 m. After exceeding the distance of 1.035 m below the sluice gate, subsequent cross-sections were located at a distance of 0.50 m from each other. The last measurement cross-section was 3.035 m below the sluice gate (Fig. 5). Water depths were taken using a level gauge with an accuracy of 0.1 mm.

RESULTS AND DISCUSSION

The obtained data were used to create the water surface profiles. Examples of measured downstream water surface profiles for different values of roughness coefficient, flow rate and gate-opening height are shown in Figures 6 and 7.

As shown in Figures 6 and 7, values of measured downstream tailwater depth change with the flow rate and gate-opening height but tend to stabilize at a distance of approx. 1.0 m from the sluice gate (Fig. 5).

In order to indicate the location of the cross-section at which the downstream water depth should be measured to obtain the calculated value of the water flow rate equal to the measured one, an analysis was carried out based on the previously mentioned discharge equations. To calculate a submerged water flow rate through the sluice gate, Equations (5) and (6) are commonly used. For this reason, both equations were used in the analyses:

1. Based on Equation (5), for measured flow rate, the water depth h_z was calculated. The discharge coefficient was calculated as for a free flow through the sluice gate. For a specific H/a ratio, contraction coefficient ε was determined and for velocity coefficient $\varphi = 0.96$, discharge coefficient was calculated from Equation (3). Hence, the value of water depth h_z was calculated on the basis of measured flow rate from:

$$h_z = H_0 - \frac{Q^2}{\mu^2 a^2 b^2 2g} \tag{12}$$

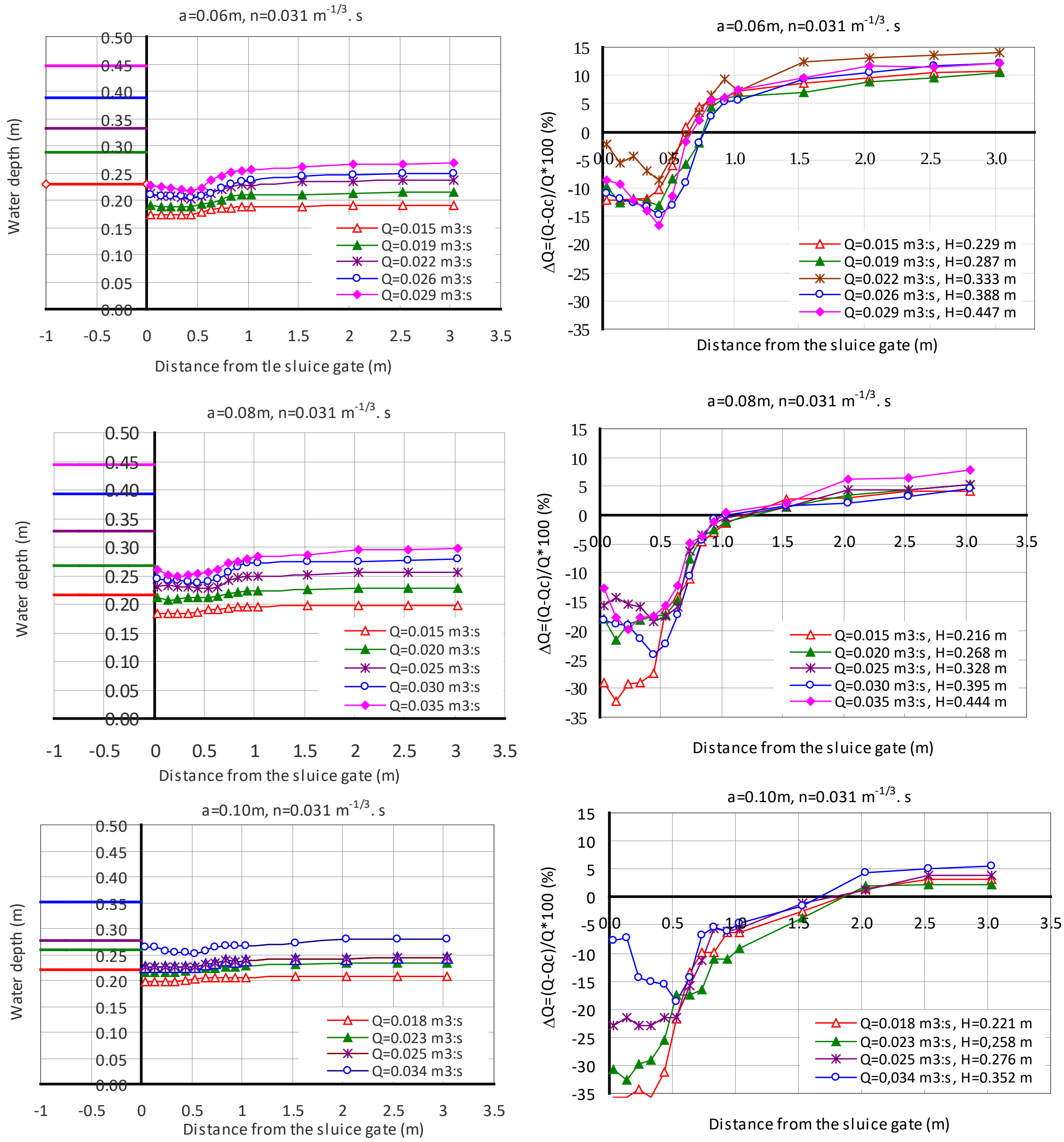

Fig. 6. Measured downstream water surface profiles and plots percent of errors between measured flow rates Q and calculated flow rates Q_c during a submerged water flow through the sluice gate for a constant value of roughness coefficient n and different values of gate-opening height a; source: own study

On the created water surface profile, a water depth calculated from Equation (12) was indicated so that its distance from the sluice gate L_z could be determined (Fig. 8).

Additionally, in each cross-section of the downstream water surface profile, the percent error between the measured flow rate Q and calculated flow rate Q_c was calculated:

$$\frac{Q-Q_c}{Q}100\% \tag{13}$$

2. From the Equation (6), for each measured downstream tailwater depth h_{di}, the water flow rate Q_c was calculated. This required calculating the value of discharge coefficient from the Equation (11). Based on the water surface profiles, the distance from the sluice gate L_d was determined at which the downstream water depth h_d allowed to obtain zero percent error between measured and calculated flow rate. This methodology is illustrated in Figure 9.

The values of downstream water depth h_z and h_d are shown in Figure 10 as a function of H:a ratio. As shown in Figure 11, values of h_z and h_d increase linearly with the increase of upstream depth H for a constant gate-opening height a. The values of h_z are slightly higher then h_d. The linear regression equations describing the changes of h_d and h_z at a given gate-opening height are shown in Table 1.

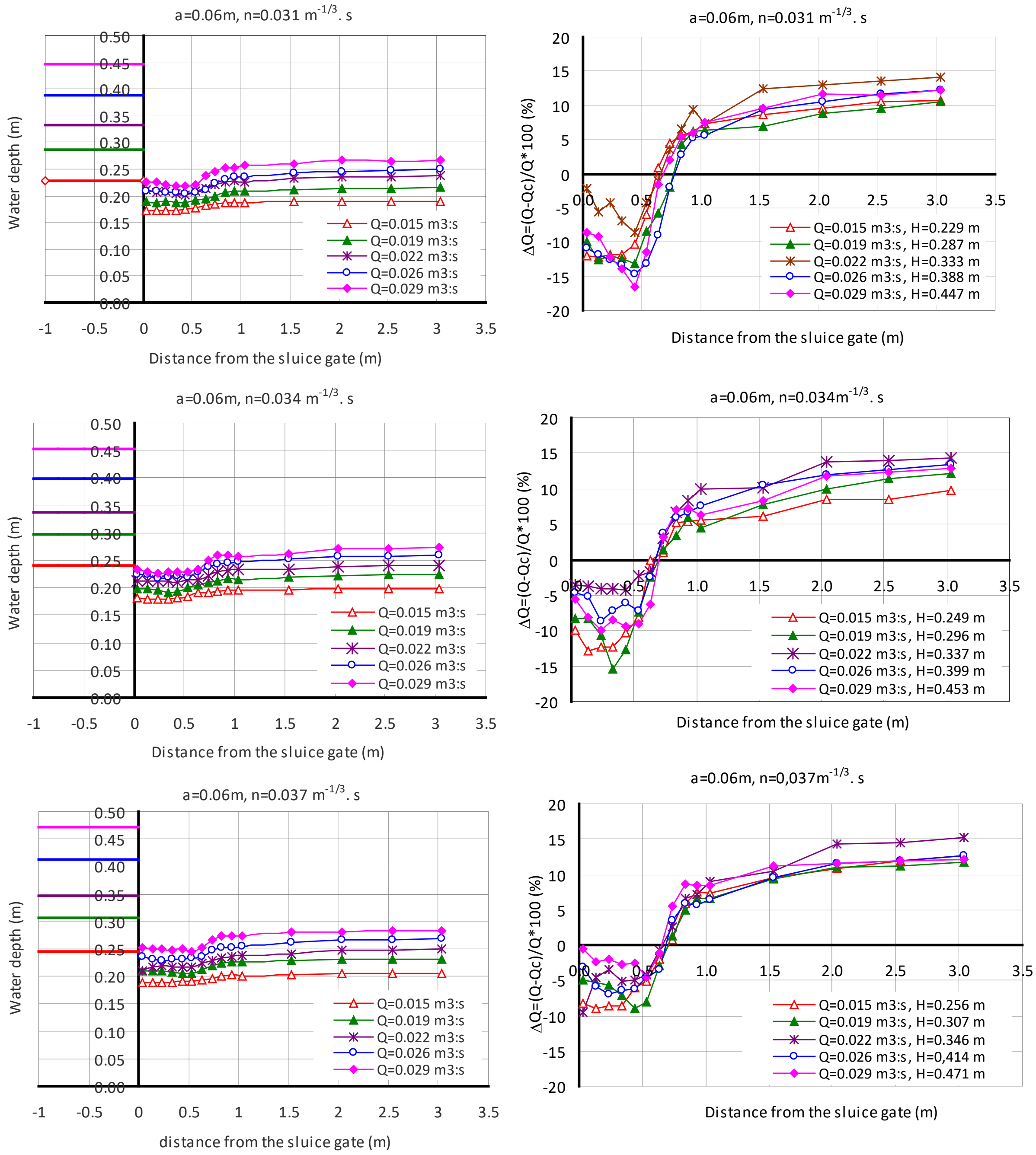

Fig. 7. Measured downstream water surface profiles and plots of percent errors between measured flow rates Q and calculated flow rates Q_c during a submerged water flow through the sluice gate for a constant gate-opening heights a and different values of roughness coefficient n; source: own study

Values of the distance from the sluice gate L_z increase with the increase of the upstream water depth H and with the increase of the gate-opening height (Fig. 11).

For a gate-opening height of $a = 0.10$ m, water depth h_z is lower than all the other measured values of downstream water depth, therefore at this gate-opening height it was not possible to determine the distance L_z. This indicates limited possibilities of using the Eq. (5) to calculate the flow rate based on measurements of downstream tailwater depth (Fig. 12).

Values of the distance from the sluice gate L_d, for which measured downstream water depth h_d allow to accurately calculate a flow rate from Eq. (6) are shown in Figure 12 as a function of H:a ratio. The values of distance L_d increase when the gate-opening height increases. This distance is independent of the upstream water depth, i.e. independent of H:a ratio, but also independent of the roughness coefficient, which is shown in Figure 7. If gate-opening height is $a = 0.06$ m, the downstream water depth h_d should be measured in a cross-section located at a distance of $L_d \approx 0.67$ m from the sluice gate, i.e. $L_d \approx 10a$. Increasing the gate-opening height to $a = 0.08$ m means that the downstream tailwater depth should be measured at an average distance of $L_d \approx 1.19$ m, i.e. $L_d \approx 15a$. Further increasing the gate-opening height to $a = 0.10$ m shifts the measurement cross-section to $L_d \approx 1.78$ m, i.e. $L_d \approx 18a$ (Fig. 13).

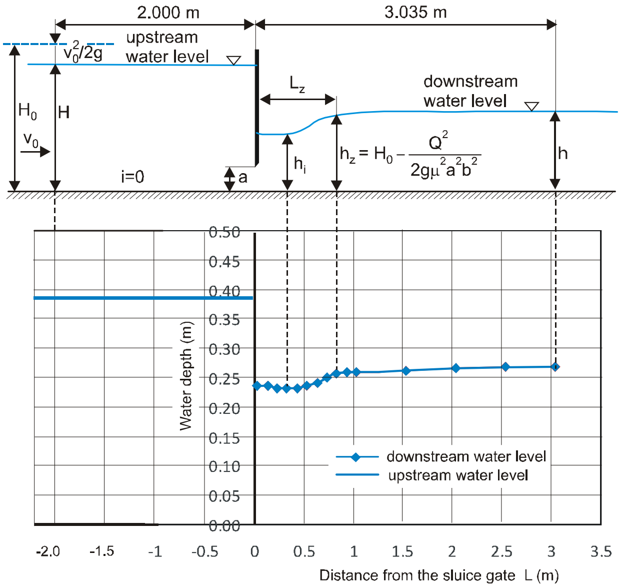

Fig. 8. Determining of the distance from the sluice L_z at which the downstream water depth h_z used for calculating the flow rate from Eq. (5) is equal to the measured flow rate; h_i = measured depth below the sluice gate (m); a, g, H_0, H, h, i, v_0 as in Figure 1; source: own study

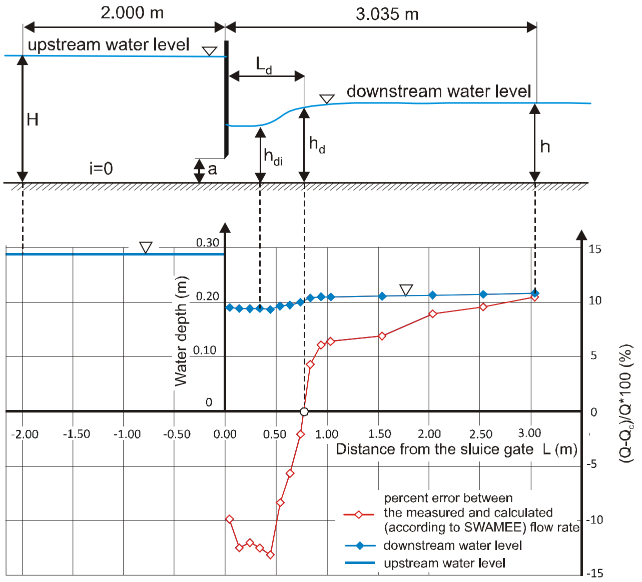

Fig. 9. Determining of the distance from the sluice gate L_d, at which the downstream water depth h_d used for calculating the flow rate from Eq. (6) is equal to the measured flow rate; h_{di} = measured depth below the sluice gate (m); a, H, h, i as in Figure 1; source: own study

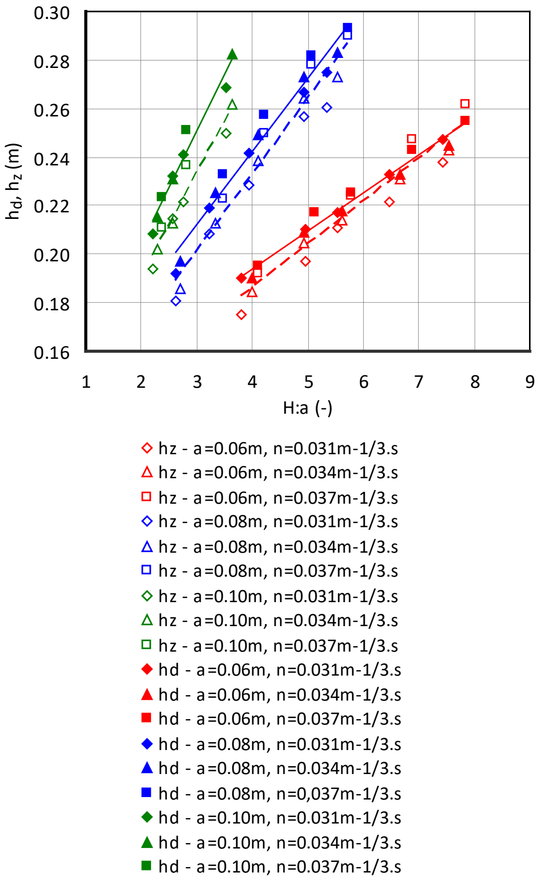

Fig. 10. Values of downstream water depths h_z and h_d, for which values of flow rate calculated from Eqs. (5) and (6) are equal to the values of measured flow rate; a, H as in Figure 1; source: own study

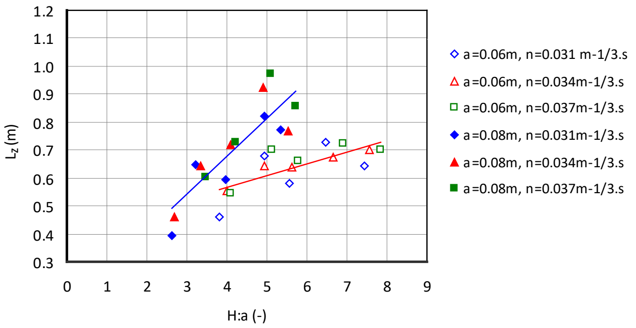

Fig. 11. Distance from the sluice gate L_z for which values of flow rate calculated from Eq. (6) using water depth h_z are equal to values of measured flow rate; other symbols: a, H as in Fig 1; source: own study

Table 1. The linear regression equations describing the changes of h_d and h_z at a given gate-opening height

Gate-opening height a (m)	Water depth h_z (m)	Water depth h_d (m)	Distance from the sluice gate L_d (m)
0.06	$h_z = 0.018(H{:}a) + 0.115 \pm 3\sigma$ $\sigma = 0.007$; $R^2 = 0.923$	$h_d = 0.016(H{:}a) + 0.132 \pm 3\sigma$ $\sigma = 0.003$; $R^2 = 0.983$	$L_d = 0.67 \pm 3\sigma$ $\sigma = 0.026$; i.e. $L_d \approx 10a$
0.08	$h_z = 0.032(H{:}a) + 0.105 \pm 3\sigma$ $\sigma = 0.008$; $R^2 = 0.951$	$h_d = 0.030(H{:}a) + 0.121 \pm 3\sigma$ $\sigma = 0.006$; $R^2 = 0.968$	$L_d = 1.19 \pm 3\sigma$ $\sigma = 0.071$; i.e. $L_d \approx 15a$
0.10	$h_z = 0.042(H{:}a) + 0.106 \pm 3\sigma$ $\sigma = 0.005$; $R^2 = 0.944$	$h_d = 0.047(H{:}a) + 0.111 \pm 3\sigma$ $\sigma = 0.005$; $R^2 = 0.963$	$L_d = 1.78 \pm 3\sigma$ $\sigma = 0.077$; i.e. $L_d \approx 18a$

Explanations: R^2 = regression coefficient, σ = standard deviation; h_z and h_d as in Figure 10.
Source: own study.

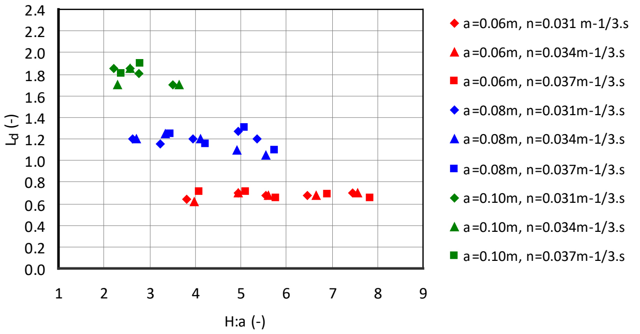

Fig. 12. Distance from the sluice gate L_d for which values of flow rate calculated from Eq. (6) using water depth h_d are equal to values of measured flow rate; *a*, *H* as in Figure 1; source: own study

The influence of gate-opening height on the location of the measurement cross-section is also illustrated in Figure 6.

The analyzes show that if the measurement cross-section of the downstream tailwater depth is located at a distance from the sluice gate L_i which is lower than the distance L_d ($L_i < L_d$), values of water depth h_i are lower than values of h_d ($h_i < h_d$). Values of flow rate calculated using downstream water depth h_i are 35% higher than the measured flow rate.

If the measurement cross-section of the downstream tailwater depth is located at a distance from the sluice gate L_i which is higher than the distance L_d ($L_i > L_d$), values of water depth h_i are slightly higher than the values of h_d ($h_i > h_d$). Values of flow rate calculated using downstream water depth h_i are approx. 5–15% lower than the measured flow rate.

Experimental tests were carried out at $H{:}a \in (2.2 \div 8)$. For this range of upstream water depth to gate-opening height ratio, if the downstream water depth is measured at a distance from the sluice gate of $L_d > 1.8$ m (i.e. $L_d \approx 18a$), the error between calculated and measured flow rate will not exceed 15%.

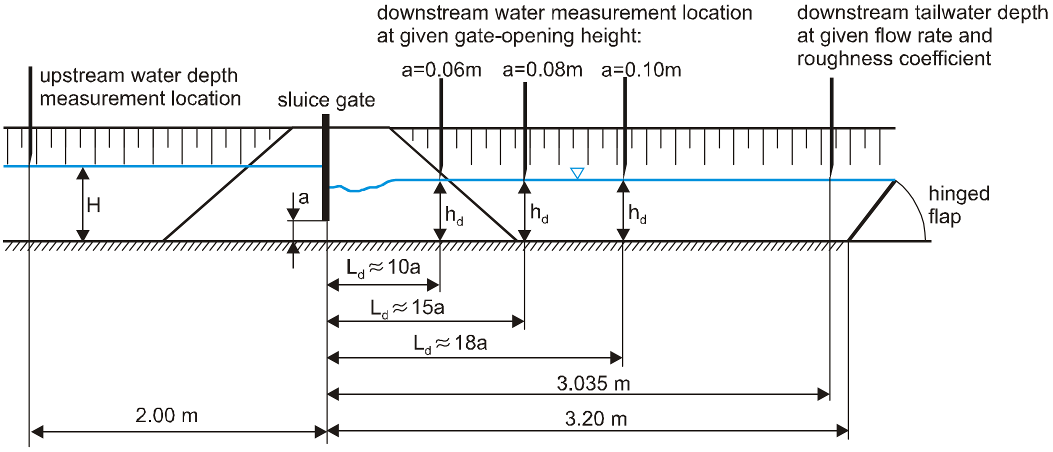

Fig. 13. Scheme of downstream water depth h_d measurement locations for various values of gate-opening height *a*; L_d = distance from the sluice gate for which values of flow rate calculated from Eq. (6) using water depth h_d are equal to values of measured flow rate; *a*, *H* as in Figure 1; source: own study

CONCLUSIONS

Based on experimental tests, it has been shown that Equations (6) and (11) can be used to calculate the volumetric flow rate through the sluice gate in the case of a submerged flow. In order to obtain accurate values of flow rate, the downstream tailwater depth should be measured at the appropriate distance from the sluice gate. If the downstream water depth is measured at a distance from the sluice gate of $L_d > 1.8$m (i.e. $L_d \approx 18a$), the error between calculated and measured flow rate will not exceed 15%.

Equation (5) does not allow to indicate the measurement cross-section of the downstream tailwater depth for which it could be possible to accurately calculate flow rate through the sluice gate using the discharge coefficient μ according to ZHUKOVSKIY (Fig. 3).

ACKNOWLEDGEMENTS

We thank anonymous reviewers for the valuable suggestions, which helped us improving the quality of our paper.

FUNDING

This research was funded by the National Centre for Research and Development, grant number [347837/11/ NCBR/2017], “Technical innovations and system of monitoring, forecasting and planning of irrigation and drainage for precise water management on the scale of drainage / irrigation system”.

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