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The study of pulse laser propagation through breast tissues by means of the Green function method

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Propagation of a short pulse through human breast tissues is studied by numerically solving the diffuse equation. Different numerical methods, such as Monte Carlo and finite difference time domain, have been used to study the short pulse laser propagation through biological tissues. In this paper, we use boundary integral method to study the laser pulse–tissue interaction. The diffuse equation is used to examine the propagation of laser into biological tissues, and boundary integral method is used to alter this equation to the integral form and the result is solved by using boundary element method. To verify the precision of a boundary element method code, we compared the obtained results with those obtained by finite difference time domain method. In addition, the effects of different optical parameters of breast tissues, *i.e.*, reduced scattering and absorption coefficients, on time evolution of a diffusely reflected pulse are studied.

Keywords: breast tissue, short pulse, boundary element method, time broadening.

1. Introduction

Recent rapid advances in the short pulse laser with duration of picoseconds have opened up many emerging areas and a lot of attention is now being paid to the transient diffuse equation in biological tissues. A blooming application in this domain is optical tomography which is able to detect tumors in biological tissues by utilizing the short-pulsed near infrared laser [1]. This imaging can be applied to study the properties of biological tissues [1–5]. The optical imaging has several advantages over the established imaging methods such as ultrasound, X-ray computed tomography, and magnetic resonance imaging. Moreover, the short-pulse near infrared laser is a safer and cheaper alternative to traditional imaging techniques [5]. The temporal diffused signals convey the additional information which is often not available in large pulse width or continuous wave lasers. In optical imaging with short-pulse irradiation, in spite of continuous wave lasers, the time derivative of the radiation intensity in the diffuse equation can no longer be neglected. As compared with a continuous wave, in short-pulse optical tomography, more information can be achieved by studying the temporal distribution of reflectance [4–11]. That is because the multiple scattering of photons inside tissues can increase the temporal distribution of reflected signals [1, 12].

In 1989, PATTERSON and his colleagues analytically solved the diffuse equation for a special case. In this study, the illuminating source is a Dirac delta and the sample is a semi-infinite slab [12]. In 2001, the optical properties of the female breast tissue during femtosecond laser pulse propagation were studied. In this paper, the experimental data of the temporal spread of the ultrashort pulse during the transmission through the breast tissue have been analyzed using the Patterson analytical expression derived from the diffusion theory [13].

As presented in [5, 12, 13], the behaviour of a short-pulse laser inside biological tissues can be studied by the diffuse equation, but for a complicated shape of a phantom or expanded laser beam, the solution of this equation is difficult, so it should be solved by numerical methods like Monte Carlo (MC) and finite-difference time-domain (FDTD). However, MC and FDTD are time-consuming [11–20] and they are not suitable for special applications like image reconstruction, so a new fast method must be used.

Boundary integral method (BIM) can also be used to solve the diffuse equation. As this method requires a surface tessellation, the computation time is reduced and the accuracy of results increases [21]. The diffusion equation in a frequency domain has also been numerically solved by BIM [22–27].

As mentioned earlier, most studies of the propagation of a short laser pulses in the breast tissue are based on results presented in Patterson's papers. But, in current studies, the illuminating source is a Gaussian beam and Patterson's assumption cannot be applied. Hence, the analytical solution is not appropriate. In this paper, to simulate the diffusion of a photon, a numerical method based on the Green theorem is presented. First, the appropriate Green function to convert the diffusion equation to the integral form by using Green's second theorem is presented [25]. Next, the surface integral is discretized by using boundary element method (BEM) and the resulting integral is numerically solved [23]. Using this technique, we have calculated the intensity and temporal evolution of a diffusely reflected pulse from the tissue. Furthermore, the effects of absorption and the reduced scattering coefficient on time broadening of a diffusely reflected pulse are also studied. In [24], we have solved the temporal diffuse equation and the penetration of diffused photon intensity inside a biological sample has been calculated. But, in this work, the effects of reduced scattering on reflectance are studied. These results show that the effects of optical properties on reflectance can be studied by BIM.

2. Review of diffuse equation

Let us consider a pulsed collimated beam incident on a biological medium (female breast tissue) perpendicularly to its boundary plane. The propagation of this pulse in the medium is described by the diffuse equation [27]

$$\frac{\partial}{c\partial t}\Phi(\mathbf{r}, t) - D\nabla^2\Phi(\mathbf{r}, t) + a\,\Phi(\mathbf{r}, t) - \Sigma(\mathbf{r}, t) = 0 \tag{1}$$

where $\Phi(\mathbf{r}, t)$ and $\Sigma(\mathbf{r}, t)$ are, respectively, the fluency and the isotropic source term at the position $\mathbf{r}$ and at the moment of time t. The velocity of light is shown by c. The parameter $D = [3(a + \sigma')]^{-1}$ is the diffusion coefficient, where a and $\sigma' = \sigma(1 - g)$ are the absorption and the reduced scattering coefficients, respectively; σ and g are the scattering coefficient and the anisotropic factor, respectively. A boundary condition is given by

$$\Phi(\mathbf{r}_s, t) - 2C_R D\,\hat{\mathbf{n}}\cdot\nabla\Phi(\mathbf{r}_s, t) = 0 \tag{2}$$

That $C_R = (1 + R)/(1 - R)$ where R is the Fresnel reflection coefficient; $\hat{\mathbf{n}}$ is the normal vector and $\mathbf{r}_s$ is the position vector on the boundary of the sample Γ_s. Note, $\hat{\mathbf{n}}\cdot\nabla = \partial/\partial n$.

The boundary integral method is based on the Green function [22]. The Green function of Eq. (1) in the domain Ω can be considered as a solution of the following equation:

$$\frac{\partial}{c\partial t}G(\mathbf{r}, \mathbf{r}'; t, t') - D\nabla^2 G(\mathbf{r}, \mathbf{r}'; t, t') + a\,G(\mathbf{r}, \mathbf{r}'; t, t') = -\delta(\mathbf{r} - \mathbf{r}')\delta(t - t') \tag{3}$$

where δ is the Dirac delta function. The Green function $G(\mathbf{r}, \mathbf{r}'; t, t')$ can be interpreted as the intensity of light generated by a point flash light located at the position $\mathbf{r}'$ and applied at the time t'.

First, by applying the Laplace transform $L(f(t)) = \int_0^\infty \exp(-st)f(t)\mathrm{d}t$ on t in Eq. (3), the equation is transformed into

$$\frac{s}{c}G(\mathbf{r}, \mathbf{r}'; s, t') - D\nabla^2 G(\mathbf{r}, \mathbf{r}'; s, t') + a\,G(\mathbf{r}, \mathbf{r}'; s, t') = -\exp(\mathbf{r} - \mathbf{r}')\delta(-st') \tag{4}$$

then Eq. (4) can be rearranged as

$$\nabla^2 G(\mathbf{r}, \mathbf{r}'; s, t') - \kappa^2 G(\mathbf{r}, \mathbf{r}'; s, t') = \frac{-\delta(\mathbf{r} - \mathbf{r}')}{D}\exp(-st') \tag{5}$$

where $\kappa^2 = s/cD + a/D$. Then, applying the Fourier transform $\tilde{f}(t) = \int_{-\infty}^{\infty}\exp(-ikr)f(r)\mathrm{d}r$ on $\mathbf{r}$, we obtain

$$(k^2 + \kappa^2)\tilde{G}(k, \mathbf{r}'; s, t') = \frac{\exp(-i\mathbf{k} \cdot \mathbf{r}')}{D} \exp(-st') \tag{6}$$

where $\tilde{G}$ is the Fourier transform of G. The inverse Fourier and Laplace transforms on the Green function stated in Eq. (6) give the Green function as [25]

$$G(\mathbf{r}, \mathbf{r}'; t, t') = \frac{H(t-t')}{\sqrt{c\left[4\pi D(t-t')\right]^3}} \exp\left[-ca(t-t')\right] \exp\left[-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4Dc(t-t')}\right] \tag{7}$$

where $H(t - t')$ is the Heaviside function.

For an extended source, reflectance is obtained by using Green's second theorem:

$$\Phi(\mathbf{r}, t) = \int_{\Omega}\int_0^t S(\mathbf{r}', t') G(\mathbf{r}, t; \mathbf{r}', t') \mathrm{d}\mathbf{r}' \mathrm{d}t' + \\ - \int_{\Gamma_s}\int_0^t \left[\frac{c}{C_R D} \Phi(\mathbf{r}_s, t') G(\mathbf{r}, t; \mathbf{r}_s, t') - \Phi(\mathbf{r}_s, t') \frac{\partial}{\partial n} G(\mathbf{r}, t; \mathbf{r}_s, t')\right] \mathrm{d}\mathbf{r}_s \mathrm{d}t' \tag{8}$$

The intensity of a diffused short pulse can be calculated from Eq. (8) that is the integral form of the diffusion equation. BEM can be used to solve Eq. (8). In this method, the boundary of the sample is first discretized to elements. Then, the observation point $\mathbf{r}$ is located on the surface of the tissue and an equation containing fluency at that point is achieved. Locating the observation point on different nodes, a system of equations is obtained which gives the fluency at those points. Finally, one can solve that set of equations and calculate the fluency at any arbitrary point inside and outside the sample [23].

3. Results

In this paper, we use BEM to solve the diffuse equation. At first, the precision of results obtained by BEM was evaluated by comparing them with the results obtained by FDTD method. Figure 1 illustrates the value of maximum of reflectance for a different number of nodes in FDTD method and for a specific mesh resolution in BEM. The illuminating beam has Gaussian temporal evolution with duration $\tau = 10$ ps and the sample is a semi-infinite medium with the absorption and reduced scattering coefficients of $a = 0.02$ mm^{-1} and $\sigma' = 1.5$ mm^{-1}, respectively, and $n = 1.3$. These optical properties are similar to the female breast tissue [25]. The radial distance from the location of illumination ρ, is a key parameter in diffuse optical imaging; in this graph $\rho = 1.0$ mm. We observe that by increasing the number of nodes in the former technique the corresponding results approach those obtained by BEM while the computational time in FDTD is more than four times longer [24].

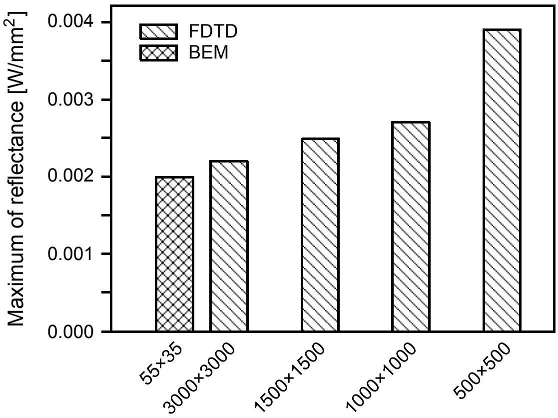

Fig. 1. Comparison between the calculated value of the maximum of reflectance using BEM and FDTD. The error between BEM and FDTD is less than 10%.

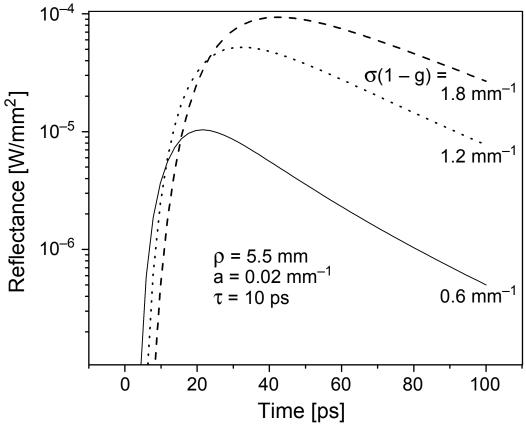

Fig. 2. Temporal distribution of reflectance for different values of reduced scattering coefficient. The optical properties are similar to the female breast tissue [25].

After verifying the precision of the method, the effects of different parameters on temporal evolution of a diffusely reflected pulse from the breast tissue are studied. Temporal broadening of the reflected pulse is due to scattering and absorption of diffused photons in the tissue. Therefore, it is important to study such effect in more detail. The broadening of reflectance is presented in Fig.2. The results presented in this figure show that the peak value of the reflectance increases for larger scattering coefficients. That is because in such a case more photons are scattered from the direction of illumination and, therefore, the scattering probability increases. The results are in agreement with those reported in [24–27]. One can see that the duration of a reflectance pulse is altered by increasing the value of reduced scattering. So, the dependence of pulse duration on scattering is studied (Fig. 3). One can see that the pulse is broadened by

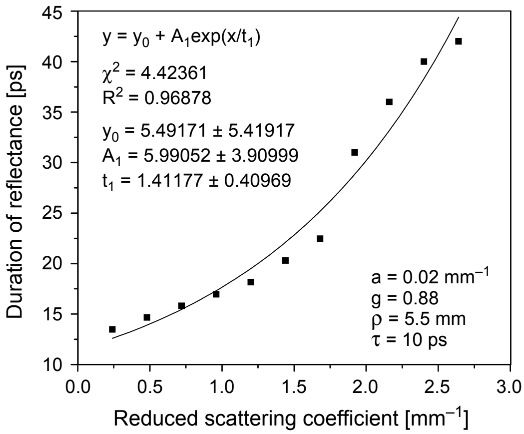

Fig. 3. Duration of reflectance pulse for different values of reduced scattering coefficient.

an increase of the reduced scattering coefficient. The results are best fitted to $\tau \propto \exp(\sigma'/1.14)$. It shows that the reflected pulse duration increases exponentially by increasing the reduced scattering coefficient. That is because for larger reduced scattering, the photons can be scattered and diffused to farther distance and some diffused photons undergo larger delay time before coming back to the surface.

To investigate the effect of the absorption coefficient on the reflectance, the temporal distribution of reflectance is calculated for two different values of the absorption coefficient (Fig. 4). One can see that the peak value of reflectance decreases by an increase of the value of the absorption coefficient, which is because, for a larger absorption coefficient, the majority of launched photons are absorbed in the tissue and therefore the intensity of the reflectance decreases.

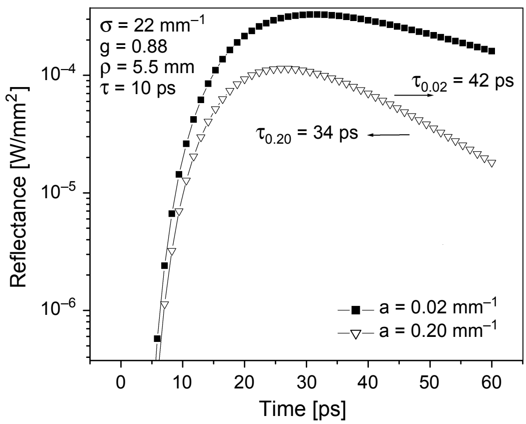

Fig. 4. Temporal distribution of the reflectance for two different values of absorption coefficient.

In addition, in Figure 4, the pulse duration of reflectance for two different values of the absorption coefficient is not the same. The reflectance for different values of the absorption coefficient is depicted in Fig. 5. It can be seen that the pulse duration of the reflectance decreases when the absorption coefficient increases; the results are best fitted to $\tau \propto \exp(-a/0.27)$. As we mentioned, for a larger absorption coefficient, the majority of photons are absorbed in the tissue and the remaining photons which are scattered in close areas can return to the surface of the tissue and therefore they undergo lower delay time before coming back to the surface.

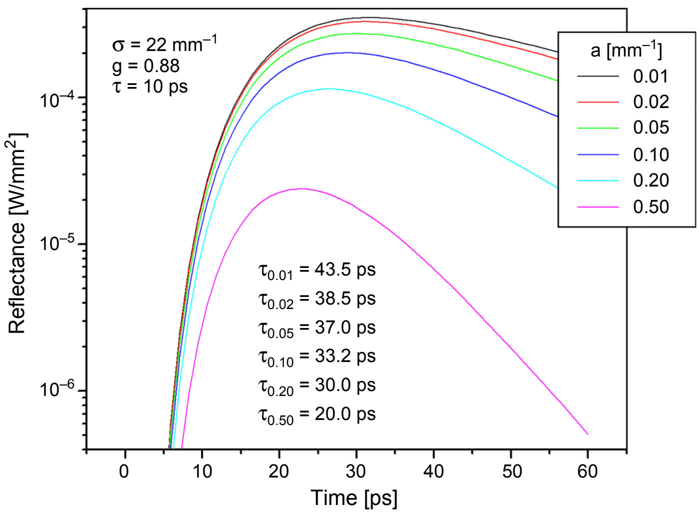

Fig. 5. Temporal distribution and duration of the reflectance for different values of absorption coefficient.

As we can see from the figures, the variation in pulse duration of reflectance for breast tissues is proportional to $\exp(-a/0.27 + \sigma'/1.14)$, so an increase in absorption and reduction of scattering coefficients, as is the case in cancer process, lead to the modification of the duration of the reflectance pulse. Therefore, while studying the duration of the reflectance pulse, we can collect some information about the cancer progress. That is because, during the progress in malignancy, the value of scattering and the absorption coefficients of lesions change, and these changes can modify the width of the reflected pulse. Therefore, the variation in broadening of the reflected pulse can be assumed as a criterion in optical mammography.

4. Conclusions

In this paper we used BIM to convert the differential diffusion equation to the integral form. The resulting integral equation is numerically solved by using BEM. Propagation of short laser pulses through the female breast tissue is studied by this method. It is shown that the optical parameters of the tissue, *i.e.*, absorption and scattering coefficients, significantly change the pulse duration of diffusely reflected pulse from the surface of the tissue. The results are compared with those obtained by FDTD method, and a good agreement between them is observed. The study of propagation of the pulse laser into the breast tissue by using BEM is the main aim of this paper. Moreover, the evaluation of this numerical method to simulate the diffusion of ultrashort pulse laser is another important aim. As mentioned in Introduction, the diffusion of optical imaging requires a fast and accurate numerical method. Results presented in current study show that the BEM can be applied as a fast method. The accuracy of results obtained by BEM can be compared with FDTD. The presented results prove that the accuracy of BEM is high and this numerical method can be used as an effective method in optical mammography.

References

[1] AN W., RUAN L.M., TAN H.P., QI H., LEW Y.M., *Finite element simulation for short pulse light radiative transfer in homogeneous and nonhomogeneous media*, Journal of Heat Transfer **129**(3), 2007, pp. 353–369.

[2] LIU F., YOO K.M., ALFANO R.R., *Ultrafast laser-pulse transmission and imaging through biological tissues*, Applied Optics **32**(4), 1993, pp. 554–558.

[3] BRAUN M., GILCH P., ZINTH W., *Ultrashort Laser Pulse in Biology and Medicine*, Springer, New York, 2007.

[4] OKUTUCU T., YENER Y., *Radiative transfer in participating media with collimated short-pulse Gaussian irradiation*, Journal of Physics D: Applied Physics **39**(9), 2006, pp 1976–1983.

[5] WANG L.V., WU H.I., *Biomedical Optics: Principles and Imaging*, Wiley-Interscience, New Jersey, 2007.

[6] PING SUN, XIANPING CAO, RUNQIU YANG, FENGHUA XIE, JIAQI DING, FUQIANG ZHANG, *Basic research on determining optical properties of tissues* in vivo *by measuring diffuse reflectance with a charge-coupled device*, Optica Applicata **41**(3), 2011, pp. 541–555.

[7] PAL G., BASU S., MITRA K., VO-DINH T., *Time-resolved optical tomography using short-pulse laser for tumor detection*, Applied Optics **45**(24), 2006, pp. 6270–6282.

[8] CHAI J.C., *Transient photon transport in turbid media with an irregularly shaped foreign objects*, Optical Review **10**(6), 2003, pp. 609–610.

[9] DAS C., TRIVEDI A., MITRA K., VO-DINH T., *Short pulse laser propagation through tissues for biomedical imaging*, Journal of Physics D: Applied Physics **36**(14), 2003, pp. 1714–1721.

[10] MITRA K., KUMAR S., *Development and comparison of models for ligh-pulse transport through scattering-absorbing media*, Applied Optics **38**(1), 1999, pp. 188–196.

[11] HALL D.J., HEBDEN J.C., DELPY D.T., *Imaging very-low-contrast objects in breastlike scattering media with a time-resolved method*, Applied Optics **36**(28), 1997, pp. 7270–7276.

[12] PATTERSON M.S., CHANCE B., WILSON B.C., *Time resolved reflectance and transmittance for the non-invasive measurement of tissue optical properties*, Applied Optics **28**(12), 1989, pp. 2331–2336.

[13] ZACHARAKIS G., ZOLINDAKI A., SAKKALIS V., FILIPPIDIS G., PAPAZOGLOU T.G., TSIFTSIS D.D., KOUMANTAKIS E., In vitro *optical characterization and discrimination of female breast tissue during near infrared femtosecond laser pulses propagation*, Journal of Biomedical Optics **6**(4), 2001, pp. 446–449.

[14] TRIVEDI A., BASU S., MITRA K., *Temporal analysis of reflected optical signal for short pulse laser interaction with nonhomogeneous tissue phantoms*, Journal of Quantitative Spectroscopy and Radiative Transfer **93**(1–3), 2005, pp. 337–348.

[15] GAN K.B., ZAHEDI E., MOHD ALI M.A., *Investigation of optical detection strategies for transabdominal fetal heart rate detection using three-layered tissue model and Monte Carlo simulation*, Optica Applicata **41**(4), 2011, pp. 885–896.
[16] HASEGAWA Y., YAMADA Y., TAMURA M., NOMURA Y., *Monte Carlo simulation of light transmission through living tissues*, Applied Optics **30**(31), 1991, pp. 4515–4520.
[17] LIEBERT A., TERAJEWICZ N., MANIEWSKI R., NILSSON G., DE MUL FRITS F.M., *Estimation of scattering volume in short distance reflectance measurements by Monte Carlo modelling*, Optica Applicata **32**(4), 2002, pp. 709–720.
[18] HIELSCHER A.H., ALCOUFFE R.E., BARBOUR R.L., *Comparison of finite-difference transport and diffusion calculations for photon migration in homogeneous and heterogeneous tissues*, Physics in Medicine and Biology **43**(5), 1998, pp. 1285–1302.
[19] PLUCINSKI J., *Accelerated Monte Carlo method for computation of photon migration by matrix description of photon direction*, Optica Applicata **35**(4), 2005, pp. 977–983.
[20] MROCZKA J., SZCZEPANOWSKI R., *Modeling of light transmittance measurement in a finite layer of whole blood – a collimated transmittance problem in Monte Carlo simulation and diffusion model*, Optica Applicata **35**(2), 2005, pp. 311–331.
[21] TANIFUJI T., HIJIKATA M., *Finite difference time domain (FDTD) analysis of optical pulse responses in biological tissues for spectroscopic diffused optical tomography*, IEEE Transactions on Medical Imaging **21**(2), 2002, pp. 181–184.
[22] TANIFUJI T., OHTOMO T., OHMOFRI D., ISHIKAWA T., *Time-resolved reflectance of an optical pulse in adult heads based on the finite difference time domain (FDTD) analysis*, Optical Review **12**(1), 2005, pp. 42–45.
[23] PARIS F., CANAS J., *Boundary Element Method: Fundamentals and Applications*, University Press, Oxford, 1997.
[24] ANSARI M.A., MASSUDI R., *Boundary integral method for simulating laser short-pulse penetration into biological tissues*, Journal of Biomedical Optics **15**(6), 2010, article 065009.
[25] GHOSH N., MOHANTY S.K., MAJUMDER S.K., GUPTA P.K., *Measurement of optical transport properties of normal and malignant human breast tissue*, Applied Optics **40**(1), 2001, pp. 176–184.
[26] SRINIVASAN S., POGUE B.W., CARPENTER C., YALAVARTHY P.K., PAULSEN K., *A boundary element approach for image-guided near-infrared absorption and scatter estimation*, Medical Physics **34**(11), 2007, pp. 4545–4557.
[27] ANSARI M.A., MASSUDI R., *Study of light propagation in Asian and Caucasian skins by means of the boundary element method*, Optics and Laser in Engineering **47**(9), 2009, pp. 965–970.

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