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DISPERSION OF SH WAVES IN A VISCOELASTIC LAYER IMPERFECTLY BONDED WITH A COUPLE STRESS SUBSTRATE

Vikas Sharma
Department of Mathematics, Lovely Professional University, Phagwara, Punjab, India
e-mail: vikas.sharma@lpu.co.in; vikassharma10a@yahoo.co.in

Satish Kumar
School of Mathematics, Thapar University, Patiala, Punjab, India
e-mail: satishk.sharma@thapar.edu

The paper deals with propagation of SH waves in a viscoelastic layer over a couple stress substrate with imperfect bonding at the interface. A dispersion equation of SH waves in a viscoelastic layer overlying the couple stress substrate with an imperfect interface between them has been obtained. Dispersion equations for propagation of SH waves with perfectly bonded interface and slippage interface between two media are also obtained as particular cases. Effects of the degree of imperfectness of the interface are studied on the phase velocity of SH waves. The dispersion curves are plotted and the effects of material properties of both couple stress substrate and viscoelastic layer are studied. The effects of internal microstructures of the couple stress substrate in terms of characteristic length of the material are presented. The effects of heterogeneity, friction parameter and thickness of the viscoelastic layer are also studied on the propagation of SH waves.

Keywords: SH waves, couple stress theory, imperfect bonding, characteristic length, viscoelasticity

1. Introduction

The dynamical behaviour of near surface materials is very complicated and could not be explained on the basis of classical continuum mechanics. To explain the relationship between stress and strain at a particular subsurface point, the near surface of earth is modelled as a viscoelastic material (Butler, 2005). Wide variations of rocks erupted from volcanoes and scattering of high frequency seismic waves support the existence of small scale heterogeneity in the earth lithosphere. Hence, heterogeneity and viscoelasticity are to be considered for real characterisation of internal microstructure of solid earth.

Shear horizontal (SH) waves are linearly polarized in the direction normal to the direction of propagation and parallel to the surface. These waves propagate in a layer in contact with an elastic half space. The study of these guided waves has received much attention in the field of seismology for estimating damage capabilities of seismic waves. These waves are also helpful for studying surface mechanical properties of underlying solids in non destructive testing techniques and in electronics industry (Simonetti and Cawley, 2004; Qingzeng *et al.*, 2014). SH waves in a layered structure for a perfectly bonded interface between two media are studied by many researchers, but this condition is rarely achieved in reality. Due to certain reasons like thermal mismatch or some faults in the manufacturing process, cracks or defects may appear at the interface which leads to an imperfect interface. Components of the displacement field are not continuous at the common boundary of two media in the case of an imperfect interface. The difference in displacement fields is assumed to linearly depend upon the traction vector. These imperfections at the common boundary may affect the propagation of SH waves.

Bhattacharya (1970) pointed out some possible exact solutions to the SH-wave equation for inhomogeneous media. Schoenberg (1980) studied elastic wave behaviour across a linear slip interface by assuming that the displacement discontinuity is linearly related to stress traction, which itself is continuous across the interface. He studied the effects of interfacial compliances on reflection and transmission coefficients of plane harmonic waves. Liu *et al.* (2007) studied Love waves in layered graded composite structures with a rigid, slip and imperfectly bonded interface which was described using an interface shear spring model. They showed that for the imperfectly bonded interface, the phase velocity of Love waves changed in the range of velocities for the rigid and slip interface conditions. Nie *et al.* (2009) studied shear horizontal guided waves in a coupled plate consisting of a piezoelectric layer and a piezomagnetic layer. They assumed that both layers are transversely isotropic and are perfectly bonded at the interface. They concluded that phase velocity of SH waves approached the smaller bulk shear wave velocity of the two materials in the system with the increase in the wave number. Borcherdt (2009) studied the propagation of SH waves in viscoelastic media. Kumar and Chawla (2011) studied wave propagation at an imperfect boundary between the transversely isotropic thermodiffusive elastic layer and half space in the context of the Green-Lindsay theory. They presented the effects of various parameters involved in the problem on both phase velocity and attenuation of SH-waves. Singh *et al.* (2011) studied propagation of waves at an imperfectly bonded interface between two monoclinic thermoelastic half-spaces. Otero *et al.* (2011) studied dispersion relations for SH waves on a magnetoelectroelastic heterostructure with imperfect interfaces. They observed that with decreasing values of imperfect bonding parameter, propagation velocity also decreased. Cui *et al.* (2013) studied SH waves in a piezoelectric structure with an imperfectly bonded viscoelastic layer. Sahu *et al.* (2014) studied SH waves in a viscoelastic heterogeneous layer over a half space with self-weight. They studied the effects of gravity, heterogeneity and internal friction on propagation of SH waves in the viscoelastic layer over the half space. They observed that heterogeneity of the medium affected the velocity profile of SH wave significantly. Vardoulakis and Georgiadis (1997) studied SH surface waves in a homogeneous gradient-elastic half space with surface energy. They showed the existence of SH waves in a homogeneous gradient-elastic half space. Recently, Sharma and Kumar (2016) studied propagation of SH waves in layered media consisting of a viscoelastic layer perfectly bonded with a couple stress substrate.

In the classical elasticity, it is assumed that the matter is continuously distributed without any defects, and internal microstructure of the material is also ignored. Experimental results have shown that the materials having inner atomic structure or microstructures behave differently at the micro level as compared to macroscale. Due to these shortcomings of the classical elasticity, size dependent continuum mechanics has been developed, which accounts for the internal microstructure of the material and predicts the dependence of macroscopic response on microstructural parameters of the material. Voigt (1887) was first to generate the idea of couple stresses in the material. Cosserat and Cosserat (1909) gave a mathematical model involving couple stresses in the material but they did not give any specific constitutive relations. Later on, many researchers like Toupin (1962), Mindlin and Tiersten (1962), Koiter (1964), Eringen (1968) and Nowacki (1974) worked on this idea and presented many theories. In these theories, the concept of couple stress was introduced by defining the deformation of the material through displacement and an independent rotation vector, which were associated with stresses and couple stresses through constitutive relations. Due to rotation, the couple stress theory was able to explain the dispersive nature of waves, which was not captured by the classical theory of elasticity.

Many researchers have used couple stress theory to study problems of wave propagation in elastic media under different conditions. Sengupta and Ghosh (1974a,b) studied the effects of couple stresses in elastic media, they deduced the equations of surface waves in elastic media under the influence of couple stresses and observed that the couple stresses affect the velocity of Rayleigh and Love wave propagation. Das *et al.* (1991) studied thermo-viscoelastic Rayleigh waves under the influence of couple stress and gravity. Debnath and Roy (1988) studied propagation of edge waves in a thinly layered laminated medium with stress couples under initial stresses. They showed that for a specific compression, the presence of couple stresses increase the velocity of wave propagation with an increase in the wave number, whereas the trend was totally reversed when there was no couple stress. Georgiadis and Velgaki (2003) studied the dispersive nature of Rayleigh waves propagating along the surface of a half-space at high frequencies using the couple stress theory.

There were some difficulties with the original couple stress theory like indeterminacy of the spherical part of the couple stress tensor or involvement of separate material length scale parameters. Hadjesfandiari and Dargush (2011) proposed a consistent couple stress theory by considering true continuum kinematical displacement and rotation. In that proposed theory, it was shown that the couple-stress tensor was skew-symmetric and the skew-symmetric part of the gradient of the rotation tensor was the consistent curvature tensor. It is also shown that for an isotropic material two Lamé parameters (λ and μ) and one length scale parameter ($\eta = \mu l^2$) completely characterise the behaviour. Here, a length scale parameter l called the characteristic length is relevant for studies conducted at the micro or nano level for the materials which exhibit internal microstructures like composites or cellular solids. It is assumed that the characteristic length is comparable to the average cell size of the material.

Keeping in mind various factors affecting the dispersion of SH waves like nature of the interface between two media, properties of the half space and coated layer, we intend to study SH waves in a viscoelastic layer lying over a couple stress substrate with an imperfect interface between them. To study the effects of microstructures of the substrate on the propagation of SH waves, a model comprising of granular macromorphic rock (Dionysos Marble) exhibiting the properties of a couple stress solid underlying heterogeneous viscoelastic layer is employed. The couple stress theory proposed by Hadjesfandiari and Dargush (2011) is applied for observing the effects of microstructures of the material of the substrate in terms of the characteristic length and other parameters of the viscoelastic layer on the propagation of SH waves.

2. Formulation and solution of the problem

Consider a layer of a viscoelastic medium of thickness H lying over a couple stress substrate with microstructures. The interface between two media is assumed to be imperfect. The origin of the coordinate system $O(x, y, z)$ lies on the interfacial surface joining the substrate and layer of the viscoelastic medium. Here, the z axis is pointing vertically downwards into the half space, the interface between the layer and half space is given by $z = 0$ and the free surface of the layer is $z = -H$. For SH waves, displacement components and body forces are independent of the y co-ordinate, so if (u, v, w) are the displacement co-ordinates of a point, then $u = w = 0$ and v is a function of the parameters x, z and t.

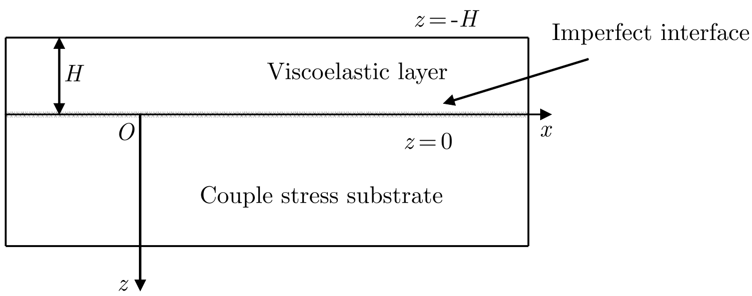

Fig. 1. Geometry of the problem

2.1. Couple stress half space

The basic governing equation of motion and constitutive relations of couple stress theory for an isotropic material in the absence of body forces (Hadjesfandiari and Dargush, 2011) are given by

$$(\lambda+\mu+\eta\nabla^2)\nabla(\nabla\cdot\mathbf{u})+(\mu-\eta\nabla^2)\nabla^2\mathbf{u}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2} \tag{2.1}$$

where λ and μ are Lamé constants, $\eta=\mu l^2$ is the couple-stress coefficient, l is the characteristic length, ρ is density of the material of the half space, and $\mathbf{u}$ is the displacement vector.

Let us assume that $\mathbf{u}=[0,v,0]$ and $\partial/\partial y\equiv 0$. Under these conditions, the equation of motion becomes

$$\Big(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\Big)-l^2\Big(\frac{\partial^4 v}{\partial x^4}+\frac{\partial^4 v}{\partial z^4}+2\frac{\partial^4 v}{\partial x^2\partial z^2}\Big)=\frac{1}{C_2^2}\frac{\partial^2 v}{\partial t^2} \tag{2.2}$$

where $C_2^2=\mu/\rho$.

We assume the solution to Eq. (2.2) to be $v=f(z)\exp[-\mathrm{i}(\omega t-kx)]$, where k is the wave number, $\omega=kc$ is the angular frequency and c is the phase velocity. Using this solution in Eq. (2.2), we get

$$\frac{d^4 f}{dz^4}-S\frac{d^2 f}{dz^2}+Pf=0 \tag{2.3}$$

where

$$S=2k^2+\frac{1}{l^2} \qquad P=k^4+\frac{k^2}{l^2}-\frac{\omega^2}{l^2C_2^2}$$

Since in the couple stress elastic half space the amplitude of waves decreases with an increase in depth, so the solution to the above differential equation becomes

$$f(z)=A_1\mathrm{e}^{-a_1 z}+B_1\mathrm{e}^{-b_1 z} \tag{2.4}$$

where

$$a_1=\sqrt{\frac{S+\sqrt{S^2-4P}}{2}} \qquad b_1=\sqrt{\frac{S-\sqrt{S^2-4P}}{2}}$$

and

$$v=(A_1\mathrm{e}^{-a_1 z}+B_1\mathrm{e}^{-b_1 z})\mathrm{e}^{-\mathrm{i}(\omega t-kx)} \tag{2.5}$$

The constitutive relations in the elastic half space are given by (Hadjesfandiari and Dargush, 2011)

$$\begin{aligned}&\sigma_{ji}=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})-\eta\nabla^2(u_{i,j}-u_{j,i})\\&\mu_{ji}=4\eta(\omega_{i,j}-\omega_{j,i}) \qquad \omega_i=\frac{1}{2}\epsilon_{ijk}u_{k,j}\end{aligned} \tag{2.6}$$

Here, u_i are displacement components, σ_{ji} is the non-symmetric force-stress tensor, μ_{ji} is the skew symmetric couple-stress tensor, δ_{ij} is Kronecker's delta, ϵ_{ijk} is the permutation tensor and $i,j,k=1,2,3$

$$\sigma_{yz}=\mu\frac{\partial v}{\partial z}+\mu l^2\Big(\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 v}{\partial z^3}\Big) \qquad \mu_{xz}=2\mu l^2\Big(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\Big) \tag{2.7}$$

Using Eq. (2.5) in eq. (2.7), we get

$$\begin{aligned}\sigma_{yz} &= \Big[\mu(-a_1A_1\mathrm{e}^{-a_1z} - b_1B_1\mathrm{e}^{-b_1z}) + \mu l^2(a_1A_1k^2\mathrm{e}^{-a_1z} \\ &\quad + b_1B_1k^2\mathrm{e}^{-b_1z} - a_1^3A_1\mathrm{e}^{-a_1z} - b_1^3B_1\mathrm{e}^{-b_1z})\Big]\mathrm{e}^{-\mathrm{i}(\omega t-kx)} \\ \mu_{xz} &= 2\mu l^2\Big[a_1^2A_1\mathrm{e}^{-a_1z} + b_1^2B_1\mathrm{e}^{-b_1z} - (A_1\mathrm{e}^{-a_1z} + B_1\mathrm{e}^{-b_1z})k^2\Big]\mathrm{e}^{-\mathrm{i}(\omega t-kx)}\end{aligned} \tag{2.8}$$

2.2. Heterogeneous viscoelastic layer

For the heterogeneity of the layer, we assume that properties of the medium change only in the z-direction. For SH waves propagating in the x-direction and causing displacement in the y-direction only, we shall assume that $\mathbf{u}_1 = [0, v_1, 0]$ and $\partial/\partial y \equiv 0$.

The equation of motion in the absence of body forces and under the above mentioned assumptions (Ravinder, 1968) is given by

$$\frac{\partial P_{xy}}{\partial x} + \frac{\partial P_{yz}}{\partial z} = \rho_1\frac{\partial^2 v_1}{\partial t^2} \tag{2.9}$$

where

$$P_{xy} = \Big(\mu_1 + \eta_1\frac{\partial}{\partial t}\Big)\frac{\partial v_1}{\partial x} \qquad\qquad P_{yz} = \Big(\mu_1 + \eta_1\frac{\partial}{\partial t}\Big)\frac{\partial v_1}{\partial z}$$

In the upper viscoelastic layer μ_1, η_1 and ρ_1 are assumed to be function of depth only and are given by

$$\mu_1 = \mu_0(1-\sin\alpha z) \qquad\qquad \eta_1 = \eta_0(1-\sin\alpha z) \qquad\qquad \rho_1 = \rho_0(1-\sin\alpha z) \tag{2.10}$$

where μ_0, η_0, ρ_0 are the constant values of μ_1, η_1 and ρ_1 at the interface of the layer and half space, and α is an arbitrary constant having dimensions of the inverse of length.

For the heterogeneous viscoelastic layer, Eq. (2.9) becomes

$$\Big(\mu_1 + \eta_1\frac{\partial}{\partial t}\Big)\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial}{\partial z}\Big[\Big(\mu_1 + \eta_1\frac{\partial}{\partial t}\Big)\frac{\partial v_1}{\partial z}\Big] = \rho_1\frac{\partial^2 v_1}{\partial t^2} \tag{2.11}$$

Now, assuming the solution $v_1 = v_L(z)\exp[-\mathrm{i}(\omega t - kx)]$, the equation of motion becomes

$$\frac{d^2v_L}{dz^2} + \frac{1}{\overline{\mu}_1}(\overline{\mu}_1)'\frac{dv_L}{dz} + \Big(\frac{\rho_1\omega^2}{\overline{\mu}_1} - k^2\Big)v_L = 0 \tag{2.12}$$

where $\overline{\mu}_1 = \mu_1 - \mathrm{i}\omega\eta_1$ and $(\overline{\mu}_1)' = d\overline{\mu}_1/dz$. Taking $v_L(z) = Y_1(z)/\sqrt{\overline{\mu}_1}$, Eq. (2.12) reduces to

$$\frac{d^2Y_1}{dz^2} + \left[\frac{1}{4(\overline{\mu}_1)^2}\Big(\frac{d\overline{\mu}_1}{dz}\Big)^2 - \frac{1}{2\overline{\mu}_1}\frac{d^2\overline{\mu}_1}{dz^2} + \frac{\rho_1\omega^2}{\overline{\mu}_1} - k^2\right]Y_1 = 0 \tag{2.13}$$

Solving this equation further, gives

$$\frac{d^2Y_1}{dz^2} + \Big[\frac{\alpha^2}{4} + \frac{\rho_0\omega^2}{\overline{\mu}_0} - k^2\Big]Y_1 = 0 \tag{2.14}$$

where $\overline{\mu}_0 = \mu_0 - \mathrm{i}\omega\eta_0$.

The solution to the above differential equation is

$$Y_1 = A\cos(mz) + B\sin(mz)$$

where A and B are arbitrary constants and

$$m^2 = \frac{\alpha^2}{4} + \frac{\rho_0\omega^2}{\overline{\mu}_0} - k^2$$

Hence $v_1 = v_L(z)\exp[-\mathrm{i}(\omega t - kx)] = (Y_1(z)/\sqrt{\overline{\mu}_1})\exp[-\mathrm{i}(\omega t - kx)]$, that

$$v_1 = \frac{1}{\sqrt{\overline{\mu}_0}}\frac{1}{\sqrt{1-\sin\alpha z}}[A\cos(mz) + B\sin(mz)]\mathrm{e}^{-\mathrm{i}(\omega t-kx)} \tag{2.15}$$

3. Boundary conditions

Boundary conditions to be satisfied at the free surface of the viscoelastic layer and at the interfacial surface between the viscoelastic layer and the couple stress half space are:

(i) The top surface of the viscoelastic layer should be stress free, so $P_{yz} = (\mu_1 + \eta_1\partial/\partial t)(\partial v_1/\partial z) = 0$ at $z = -H$.

(ii) The difference in displacement fields is assumed to depend linearly upon the traction vector, that is $P_{yz} = G(v - v_1)$ at $z = 0$, where G measures the degree of imperfectness at the interface.

(iii) The magnitude of shear stresses of both the couple stress substrate and the viscoelastic layer should be equal at the interface, that is $P_{yz} = \sigma_{yz}$ at $z = 0$.

(iv) The couple stress tensor μ_{xz} should vanish at the interface, that is $\mu_{xz} = 0$ at $z = 0$.

4. Derivation of secular equation

4.1. SH waves in the viscoelastic layer over the couple stress half space with an imperfect interface

Using the above mentioned boundary conditions, we get following four equations

$$\begin{aligned}
&\sqrt{\overline{\mu}_0}[2m(1+\sin(\alpha H))\sin(mH) + \alpha\cos(\alpha H)\cos(mH)]A \\
&\qquad + \sqrt{\overline{\mu}_0}[2m(1+\sin(\alpha H))\cos(mH) - \alpha\cos(\alpha H)\sin(mH)]B = 0 \\
&-\Big(\frac{\sqrt{\overline{\mu}_0}\alpha}{2} + \frac{G}{\sqrt{\overline{\mu}_0}}\Big)A - \sqrt{\overline{\mu}_0}mB + GA_1 + GB_1 = 0 \\
&\frac{\alpha\sqrt{\overline{\mu}_0}A}{2} + \sqrt{\overline{\mu}_0}mB + \mu a_1[1 + (a_1^2 - k^2)l^2]A_1 + \mu b_1[1 + (b_1^2 - k^2)l^2]B_1 = 0 \\
&(a_1^2 - k^2)A_1 + (b_1^2 - k^2)B_1 = 0
\end{aligned} \tag{4.1}$$

Equations (4.1) will have a non-trivial solution if the determinant of coefficients of the unknowns A, B, A_1, B_1 vanishes. Applying this condition to the above system of equations, we obtain the following secular equation for the SH waves in the heterogeneous viscoelastic layer imperfectly bonded to a couple stress half space with microstructures as

$$(-2T_1\overline{\mu}_0 m + T_2\overline{\mu}_0\alpha)[Q + G(b_1^2 - a_1^2)] + 2T_2QG = 0 \tag{4.2}$$

where

$$\begin{aligned}
T_1 &= 2m[1+\sin(\alpha H)]\sin(mH) + \alpha\cos(\alpha H)\cos(mH) \\
T_2 &= 2m[1+\sin(\alpha H)]\cos(mH) - \alpha\cos(\alpha H)\sin(mH) \\
Q &= \mu(k^2 - a_1^2)(k^2 - b_1^2)(a_1 - b_1)l^2 + \mu b_1(k^2 - a_1^2) - \mu a_1(k^2 - b_1^2)
\end{aligned}$$

Now, separating the real and imaginary parts of eq. (4.2), we get a dispersion equation of SH waves as

$$\begin{aligned}
&[-2R_1\mu_0 m_1 - 2R_1\omega\eta_0 m_2 - 2I_1\omega\eta_0 m_1 + 2I_1\mu_0 m_2 \\
&\qquad + \alpha(R_2\mu_0 + \omega\eta_0 I_2)][Q + G(b_1^2 - a_1^2)] + 2R_2QG = 0
\end{aligned} \tag{4.3}$$

and the imaginary part gives us the damping equation of SH waves as

$$\begin{aligned}
&[2R_1\omega\eta_0 m_1 - 2R_1\mu_0 m_2 - 2I_1\mu_0 m_1 - 2I_1\omega\eta_0 m_2 \\
&\qquad + \alpha(I_2\mu_0 - R_2\omega\eta_0)][Q + G(b_1^2 - a_1^2)] + 2I_2QG = 0
\end{aligned} \tag{4.4}$$

4.2. SH waves in the viscoelastic layer over the couple stress half space with a perfectly bonded interface

If in Eq. (4.2) $G \to \infty$, we get a secular equation for SH waves in the viscoelastic layer over the couple stress half space with a perfectly bonded interface. It is the same as that obtained by Sharma and Kumar (2016)

$$(-2T_1\overline{\mu}_0 m + T_2\overline{\mu}_0\alpha)(b_1^2 - a_1^2) + 2T_2Q = 0 \tag{4.5}$$

4.3. SH waves in the viscoelastic layer over the couple stress half space with a slip interface

If in Eq. (4.2), $G \to 0$, the secular equation for SH waves in the viscoelastic layer over the couple stress half space for a slippage interface is given by

$$(-2T_1\overline{\mu}_0 m + T_2\overline{\mu}_0\alpha)Q = 0 \tag{4.6}$$

where

$$\begin{aligned}
&T_1 = R_1 + \mathrm{i}I_1 \qquad T_2 = R_2 + \mathrm{i}I_2\\
&m = m_1 + \mathrm{i}m_2 \qquad m_1 = \sqrt{r}\cos\frac{\theta}{2} \qquad m_2 = \sqrt{r}\sin\frac{\theta}{2}\\
&R_1 = 2[1+\sin(\alpha H)](m_1S_1 - m_2S_2) + \alpha\cos(\alpha H)E_1\\
&I_1 = 2[1+\sin(\alpha H)](m_1S_2 + m_2S_1) - \alpha\cos(\alpha H)E_2\\
&R_2 = 2[1+\sin(\alpha H)](m_1E_1 + m_2E_2) - \alpha\cos(\alpha H)S_1\\
&I_2 = 2[1+\sin(\alpha H)](m_2E_1 - m_1E_2) - \alpha\cos(\alpha H)S_2\\
&S_1 = \sin(m_1H)\cosh(m_2H) \qquad S_2 = \cos(m_1H)\sinh(m_2H)\\
&E_1 = \cos(m_1H)\cosh(m_2H) \qquad E_2 = \sin(m_1H)\sinh(m_2H)\\
&F_1 = \frac{\rho_0\omega^3\eta_0}{\mu_0^2 + \omega^2\eta_0^2} \qquad F_2 = \frac{\alpha^2}{4} - k^2 + \frac{\rho_0\omega^2\mu_0}{\mu_0^2 + \omega^2\eta_0^2}\\
&r = \sqrt{F_1^2 + F_2^2} \qquad \tan\theta = \frac{F_1}{F_2}
\end{aligned}$$

5. Numerical results and discussion

(i) For the viscoelastic layer, various material parameters (Gubbins, 1990) are taken as $\rho_0 = 4705\,\mathrm{kg/m^3}$, $\mu_0 = 1.987 \cdot 10^{10}\,\mathrm{N/m^2}$, $\mu_0/\eta_0 = 10^6\,\mathrm{s^{-1}}$, $\beta_1 = \sqrt{\mu_0/\rho_0} = 2055\,\mathrm{m/s}$.

(ii) The material parameters for the couple stress half space which is made of Dionysos Marble (Vardoulakis and Georgiadis, 1997) are $\rho = 2717\,\mathrm{kg/m^3}$, $\mu = 30.5 \cdot 10^9\,\mathrm{N/m^2}$, $C_2 = \sqrt{\mu/\rho} = 3350\,\mathrm{m/s}$.

Dionysos Marble is a white fine-grained metamorphic marble with saccharoidal microstructure. To find the impact of characteristic length, different cases of characteristic length l comparable with the internal cell size of granular macromorphic rock $O(10^{-4})$ such as $l = 0.0001\,\mathrm{m}$, $l = 0.0004\,\mathrm{m}$, $l = 0.0008\,\mathrm{m}$ are considered. In all the figures, phase velocity is plotted using Eq. (4.3) and the real part of Eq. (4.5). Graphs of damping velocity are plotted using Eq. (4.4) and the imaginary part of Eq. (4.5).

5.1. Effects of degree of imperfectness at the interface

To study the role of degree of imperfectness of the interface on the propagation of SH waves in the viscoelastic layer over the couple stress substrate, curves are provided in Figs. 2a and 2b. Here, we have considered fixed values of other parameters as $\alpha H = 0.54$, characteristic length $l = 0.0004\,\mathrm{m}$ and friction parameter $\mu_1/\eta_1 = 10^6$. It can be observed in Fig. 2a that SH waves are dispersive, and the non dimensional phase velocity c/β_1 of SH waves decreases sharply with an increase in the non dimensional wave number kH before becoming asymptotically constant. It can also be observed from the profiles in Fig. 2a that an increase in the value of parameter G leads to an increase in the phase velocity of SH waves for any fixed value of the dimensionless wave number kH. Since the imperfectness is inversely proportional to G, so an increase in the imperfectness adversely affects the phase velocity, and the phase velocity is maximum when the interface is perfectly bonded $G \to \infty$. Figure 2b shows the variation in non dimensional damping velocity of SH waves with the non dimensional wave number for different values of the parameter G. It can be observed that the damping velocity increases with an increase in the parameter G. The damping velocity is also maximum when the interface is perfectly bonded.

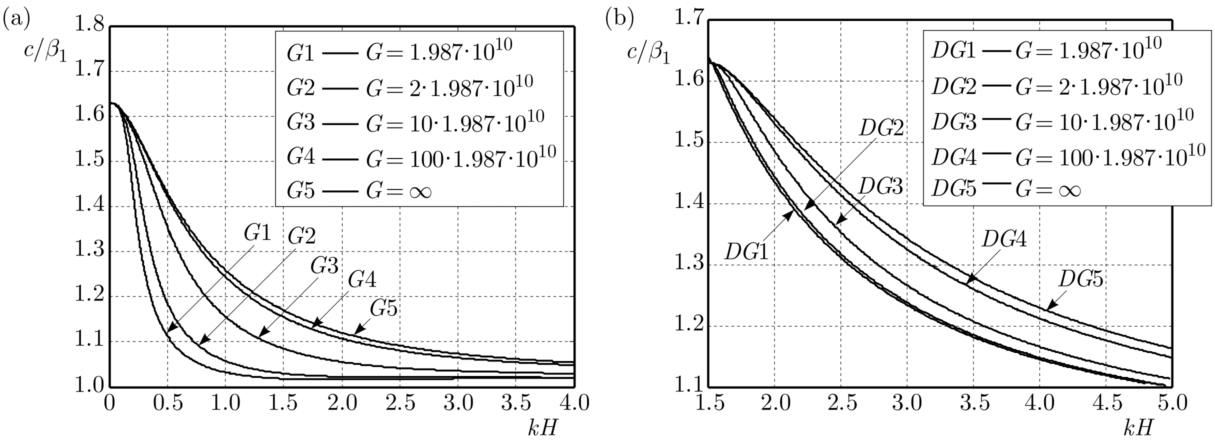

Fig. 2. (a) Phase and (b) damping velocity profiles of SH waves with the wave number for different values of G

5.2. Effects of the heterogeneity parameter

The role of the heterogeneity parameter on both the phase and damping velocities of SH waves is studied in Figs. 3a and 3b. Dispersion curves are provided for three different values of the heterogeneity parameter $\alpha H = 0.18$, 0.54 and 0.72. We have considered values of other parameters as $H = 0.09\,\mathrm{m}$, characteristic length $l = 0.0004\,\mathrm{m}$ and friction parameter $\mu_1/\eta_1 = 10^6$ and the value of $G = 2 \cdot 1.987 \cdot 10^{10}$. It can be observed that with the increasing value of the heterogeneity parameter αH, the phase velocity of SH waves decreases. Figure 3b shows variation of the damping velocity with the wave number for different values of the heterogeneity parameter. It can be seen that the damping velocity of SH waves increases with an increase in the heterogeneity parameter.

5.3. Effects of the friction parameter

Dispersion curves to demonstrate the role of the friction parameter on SH waves in the viscoelastic layer are provided in Figs. 4a and 4b for three different values of the friction parameter $\mu_1/\eta_1 = 7 \cdot 10^5\,\mathrm{s}^{-1}$, $10 \cdot 10^5\,\mathrm{s}^{-1}$, $80 \cdot 10^5\,\mathrm{s}^{-1}$. We have considered the values of other parameters as $\alpha H = 0.54$, characteristic length $l = 0.0004\,\mathrm{m}$ and $G = 2 \cdot 1.987 \cdot 10^{10}$. It can be observed from these figures that both the phase velocity and damping velocity of SH waves decrease with an increase in the value of the friction parameter μ_1/η_1.

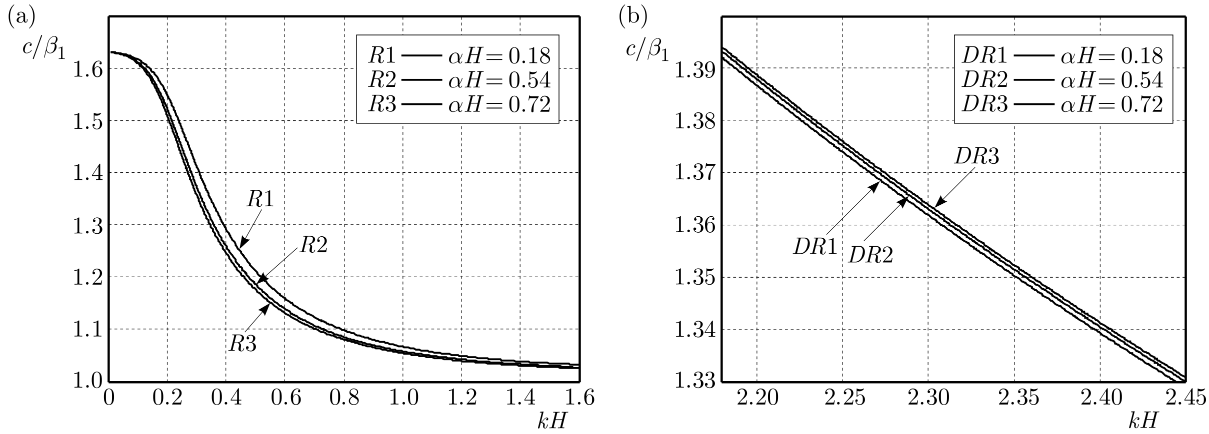

Fig. 3. (a) Phase and (b) damping velocity profiles of SH waves with the wave number for different values of αH

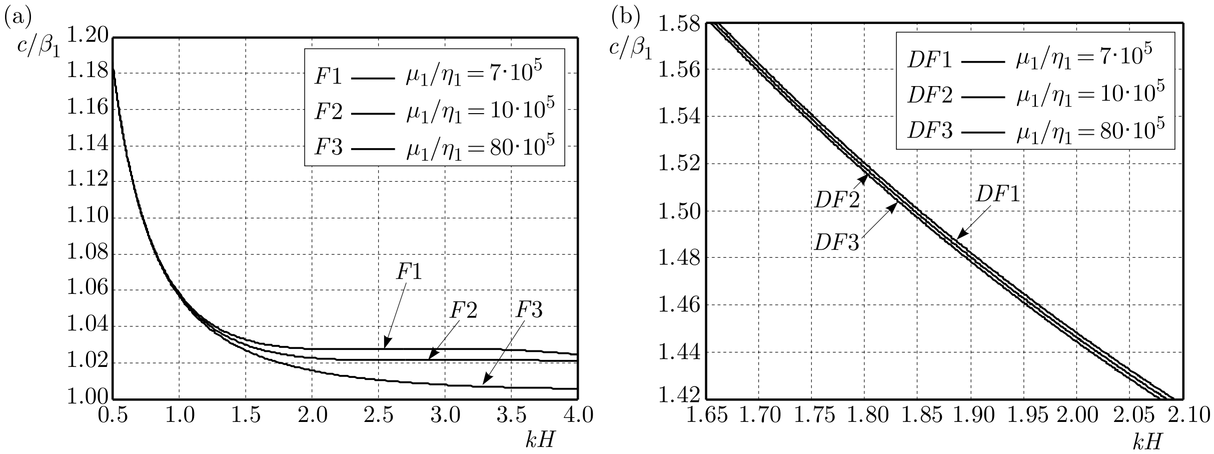

Fig. 4. (a) Phase and (b) damping velocity profiles of SH waves with the wave number for different values of μ_1/η_1

5.4. Effects of the thickness of the viscoelastic layer

Figure 5a shows variation in the phase velocity c/β_1 against the wave number kH for three different values of thickness, $H = 0.04\,\mathrm{m}$, $0.06\,\mathrm{m}$, $0.09\,\mathrm{m}$. The values of other fixed parameters are $\alpha H = 0.54$, $\mu_1/\eta_1 = 10^6$, $l = 0.0004$ and $G = 2 \cdot 1.987 \cdot 10^{10}$. It is observed that with the increasing value of thickness of the viscoelastic layer over the couple stress substrate, the phase velocity of SH waves also increases. Figure 5b shows variation of the damping velocity of SH waves with the wave number for different values of thickness of the viscoelastic layer. It can be seen that the damping velocity of SH waves decreases with an increase in thickness of the layer.

5.5. Effects of the internal microstructure of the substrate

For observing the effects of internal microstructure of the underlying substrate, variation in the phase velocity and damping velocity is shown against the wave number in Figs. 6a and 6b for three different values of the characteristic length $l = 0.0001\,\mathrm{m}$, $0.0004\,\mathrm{m}$, $0.0008\,\mathrm{m}$. The values of other fixed parameters are taken as $\alpha H = 0.54$, $\mu_1/\eta_1 = 10^6$ and $G = 2 \cdot 1.987 \cdot 10^{10}$. It can be observed that both the phase velocity and damping velocity of SH waves increase with an increase in the characteristic length l of the material.

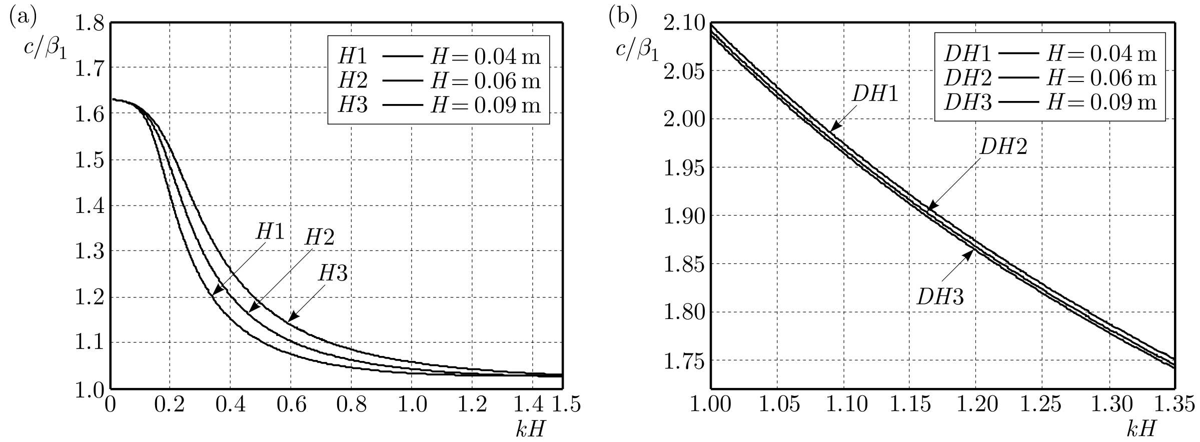

Fig. 5. (a) Phase and (b) damping velocity profiles of SH waves with the wave number for different values of H

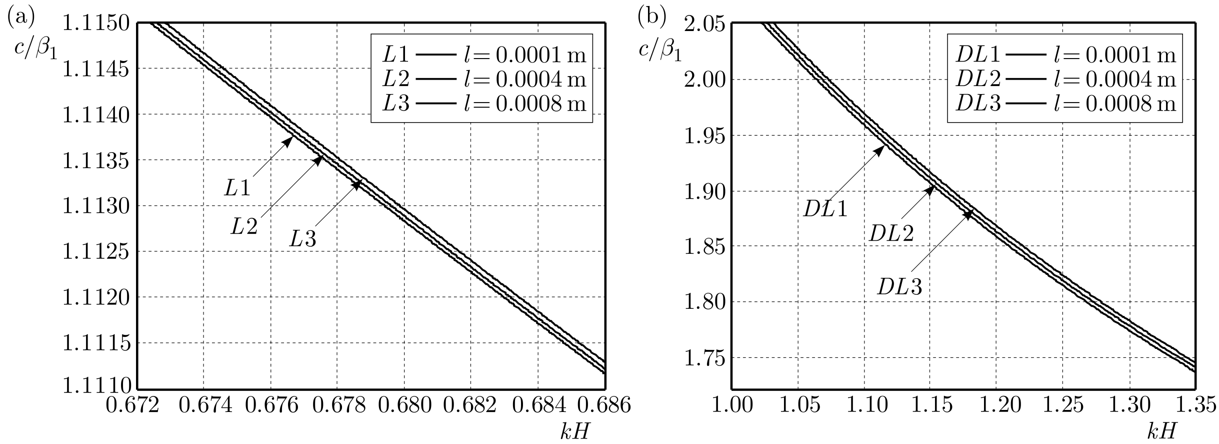

Fig. 6. (a) Phase and (b) damping velocity profiles of SH the waves with wave number for different values of l

6. Conclusion

Propagation of SH waves is studied in a viscoelastic layer bonded imperfectly with a couple stress substrate. Dispersion equations for propagation of SH waves with a perfectly bonded interface and a slippage interface between two media are also obtained as particular cases. The numerical results are presented graphically. Following major conclusions are drawn from the present study:

- SH waves show dispersion in the considered model. Initially, the phase velocity of SH waves decreases sharply with an increase in the wave number, then it becomes asymptotically constant for higher wave numbers.
- Imperfectness at the interface between two media has a significant effect on the phase and damping velocities of SH waves. It is observed that with the decreasing value of imperfectness at the interface, both the phase and damping velocities of SH waves increase. The phase and damping velocities are highest when the interface is perfectly bonded.
- The heterogeneity parameter of the viscoelastic layer has an adverse effect on the phase velocity but it favours the damping velocity. So, the phase velocity decreases and the damping velocity increases with an increase in the heterogeneity parameter.
- The friction parameter of the viscoelastic layer has an adverse effect on both the phase and damping velocities. Both of them decrease with the increasing value of this parameter.

- It is observed that with the increasing value of thickness of the viscoelastic layer over the couple stress substrate, the phase velocity of SH waves increases. The increasing value of thickness of the viscoelastic layer does not favour damping velocity.
- Characteristic length l which measures internal microstructure of the material of the underlying substrate favours both the phase and damping velocities of SH waves. These velocities increase with the increasing value of characteristic length of the underlying couple stress substrate.

The consideration of microstructural effects on the propagation of SH waves in this realistic model may provide possible applications to non-destructive testing techniques and other engineering fields. The results presented in this work may be applied to the designing of liquid viscosity sensors and biosensors. As the model is considered to replicate the internal structure of earth, so it may also find possible application to seismology or geomechanics engineering.

References

1. Bhattacharya S.N., 1970, Exact solution of SH-wave equation for inhomogeneous media, *Bulletin of Seismological Society of America*, **60**, 6, 1847-1859
2. Borcherdt R.D., 2009, *Viscoelastic Waves in Layered Media*, Cambridge University Press, New York
3. Butler D.K., 2005, *Near-Surface Geophysics*, Society of Exploration Geophysicists, Tulsa, Okla
4. Cosserat E., Cosserat F., 1909, *Théorie des corps deformables*, A. Hermann et Fils, Paris
5. Cui J., Du J., Wang J., 2013, Study on SH waves in piezoelectric structure with an imperfectly bonded viscoelastic layer, *Joint UFFC, EFTF and PFM Symposium*, 1017-1020
6. Das T.K., Sengupta P.R., Debnath L., 1991, Thermo-visco-elastic Rayleigh waves under the influence of couple stress and gravity, *International Journal of Mathematics and Mathematical Sciences*, **14**, 3, 553-560
7. Debnath L., Roy P.P., 1988, Propagation of edge waves in a thinly layered laminated medium with stress couples under initial stresses, *Journal of Applied Mathematics and Simulation*, **1**, 271-286
8. Eringen A.C., 1968, Theory of micropolar elasticity, [In:] *Fracture*, H. Liebowitz (Edit.), Academic Press, New York, 2, 662-729
9. Georgiadis H.G., Velgaki E.G., 2003, High-frequency Rayleigh waves in materials with microstructure and couple-stress effects, *International Journal of Solids and Structures*, **40**, 2501-2520
10. Gubbins D., 1990, *Seismology and Plate Tectonics*, Cambridge University Press, Cambridge
11. Hadjesfandiari A.R., Dargush G.F., 2011, Couple stress theory for solids, *International Journal of Solids and Structures*, **48**, 2496-2510
12. Koiter W.T., 1964, Couple stresses in the theory of elasticity I and II, *Proceedings of the Koninklijke, Nederlandse Akademie van Wetenschappen B*, **67**, 17-44
13. Kumar R., Chawla V., 2011, Wave propagation at the imperfect boundary between transversely isotropic thermodiffusive elastic layer and half-space, *Journal of Engineering Physics and Thermophysics*, **84**, 5, 1192-2000
14. Liu H., Yang J.L., Liu K., 2007, Love waves in layered graded composite structures with imperfectly bonded interface, *Chinese Journal of Aeronautics*, **20**, 210-214
15. Mindlin R.D., Tiersten H.F., 1962, Effects of couple-stresses in linear elasticity, *Archive for Rational Mechanics and Analysis*, **11**, 415-488

16. Nie G., An Z., Liu J., 2009, SH-guided waves in layered piezoelectric/piezomagnetic plates, *Progress in Natural Science*, **19**, 811-816

17. Nowacki W., 1974, *Micropolar Elasticity*, International Center for Mechanical Sciences, Courses and Lectures No: 151, Udine, Springer-Verlag, Wien-New York

18. Otero J.A., Calas H., Rodrguez-Ramos R., Bravo-castillero J., Aguiar A.R., Monsivais G., 2011, Dispersion relations for SH waves on a magnetoelectroelastic heterostructure with imperfect interfaces, *Journal of Mechanics of Materials and Structures*, **6**, 7/8, 969-994

19. Qingzeng M.A., Jingpin J.,Ping H.U., Xi Z., Bin W.U., Cunfu H.E., 2014, Excitation and detection of shear horizontal waves with electromagnetic acoustic transducers for nondestructive testing of plates, *Chinese Journal of Mechanical Engineering*, **27**, 2, 428-436

20. Ravindra R., 1968, Usual assumptions in the treatment of wave propagation in heterogeneous elastic media, *Pure and Applied Geophysics*, **70**, 1, 12-17

21. Sahu S.A., Saroj P.K., Dewangan N., 2014, SH-waves in viscoelastic heterogeneous layer over half-space with self-weight, *Archive of Applied Mechanics*, **84**, 235-245

22. Schoenberg M., 1980, Elastic wave behavior across linear slip interfaces, *Journal of the Acoustical Society of America*, **68**, 5, 1516-1521

23. Sengupta P.R., Ghosh B., 1974a, Effects of couple stresses on the propagation of waves in an elastic layer, *Pure and Applied Geophysics*, **112**, 331-338

24. Sengupta P.R., Ghosh B., 1974b, Effects of couple stresses on the surface waves in elastic media, *Gerlands Beitrage zur Geophysik, Leipzig*, **83**, 309-318

25. Sharma V., Kumar S., 2016, Influence of microstructure, heterogeneity and internal friction on SH waves propagation in a viscoelastic layer overlying a couple stress substrate, *Structural Engineering and Mechanics*, **57**, 4, 703-716

26. Simonetti F., Cawley P., 2004, On the nature of shear horizontal wave propagation in elastic plates coated with viscoelastic materials, *Proceedings of Royal Society – A*, **460**, 2197-2221

27. Singh J., Singh B., Ailawalia P., 2011, Propagation of waves at an imperfectly bonded interface between two monoclinic thermoelastic half-spaces, *Journal of Theoretical and Applied Mechanics*, **41**, 3, 77-92

28. Toupin R.A., 1962, Elastic materials with couple-stresses, *Archive for Rational Mechanics and Analysis*, **11**, 385-414

29. Vardoulakis I., Georgiadis H.G., 1997, SH surface waves in a homogeneous gradient-elastic half-space with surface energy, *Journal of Elasticity*, **47**, 147-165

30. Voigt W., 1887, Theoretische Studien fiber die Elastizitatsverhiltnisse der Kristalle, *Abh. Geschichte Wissenschaften*, **34**

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