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# ELEMENTS OF MATHEMATICAL LITERACY IN PRIMARY TEACHER TRAINING

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**Abstract.** Mathematical literacy is an ability to be developed not just in pupils but also in teachers. It is a constituent part of their undergraduate teacher training. The courses listed under the programmes of study offered by the Faculty of Education, University of Prešov provide a platform for developing mathematical literacy of students – prospective teachers in primary school. The courses syllabi include OECD PISA tasks. They are utilized as means of exploration of theoretical starting points in certain domains of arithmetic and algebra. The article offers the task analysis carried out from the aspect of the present elements of mathematical literacy. We also present some examples of the way how particular primary mathematics topics from the syllabus are interpreted in terms of mathematical literacy.

## 1. Introduction

Developing students' mathematical literacy is a part of undergraduate training of primary school teachers. Specific subject training in the given area is a precondition for teacher's competence to create and formulate tasks and activities which develop pupil's mathematical literacy. OECD PISA international assessment scheme defines mathematical literacy as *"individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen"* ([2], p. 7).

The team at the Department of Mathematical Education, Faculty of Education, University of Prešov has been researching since 2010 the project aimed at raising standards of subject specific preparedness of primary teachers to

develop pupils' mathematical literacy reflecting on the context of curricular reform as well as the OECD PISA and IEA TIMSS schemes. In view of the above context, more tasks on developing mathematical literacy and competence have been included into the modules and courses of teacher training.

Surveys carried out at the Faculty of Education, University of Prešov aimed at determining students' standards of mathematical literacy indicate that *"among the most problematic areas of mathematical literacy of students – prospective teachers in pre-elementary and elementary stage are: interpretation of data from graphs and tables"* ([3], p. 84). The authors of the survey state that some of the students have difficulty to solve the tasks in which it is necessary to apply lower secondary stage mathematics such as reading and writing decimals, roman numerals, percentage calculus, divisibility of natural numbers, elementary geometric and arithmetic terminology, and processing data from tables and graphs. The above results were obtained from the analysis of student's solutions of the test which included the released items from OECD PISA.

## 2. Arithmetic and algebra with didactics

Arithmetic and Algebra with Didactics is a course included into undergraduate teacher training programmes of study. The aims of the course are set as follows:

– to acquire the essential mathematical knowledge from arithmetic and algebra, followed by didactic transformation of them in conformity with the objectives of teaching mathematics in primary school as stipulated in the State Programme of Education ISCED 1;

– to introduce possible didactic interpretations of the theoretical knowledge in teaching mathematics in primary school;

- to identify relationships between the notions from the theoretical domains of arithmetic and algebra, and concepts developed in the minds of pupils of junior school age.

The course integrates the theoretical knowledge of arithmetic and algebra with didactic interpretation of it in teaching primary mathematics. The course contains the following elements of mathematical literacy:

Competences at the level of reproduction: mastery of mathematical terminology, reproduction of what is learnt, applying learnt algorithms and identifying analogy.

Competences at the level of connections: solving problems with known elements, linking several known methods, and the use of more complex procedures.

Competences at the level of reflection: the use of mathematical symbols and language.

## 3. Functions and dependence relations

Tasks containing elements of mathematical literacy, in the form of model situations and assignments, are used when presenting theoretical essentials of such notions from arithmetic and algebra which are used in primary mathematics on the level of propaedeutics (introduction). A function is one of the basic notions in algebra. This domain is included in the state mathematics curriculum in the *Sequences, Relations, Functions, Tables and Graphs* thematic area. The given thematic area provides for both finding quantitative relations and presentation of types of their systematic changes. Dependence relations are represented in the form of tables, graphs and diagrams [5].

Thinking in functions is important for resolving real life problems. Its rudiments start to be developed much earlier than the notion of a *function* is exposed in teaching mathematics. Children acquaint with dependence relations as early as in pre-school and primary school. The process of constructing concept of function is long-term and starts with solving tasks of propaedeutic character. In our view, it is important for primary mathematics teachers to be aware of those processes. By introducing the tasks taken from the real-life situations with the elements of mathematical literacy to undergraduate mathematical primary teacher training we strive to develop subject-specific and pedagogical competences of students, thus enhance their preparedness for teaching practice.

The notion of a function is introduced in school mathematics only in lower secondary stage, but the first conceptualisations of elementary functions as linear function and direct proportionality are already developed through some primary school activities.

In view of the above, the activities focusing on didactic processing and interpretation of tasks on functions are included in the *Arithmetic and Algebra with Didactics* course. Students are presented with theoretical points of departure such as: the definition of the notion of a function, the domain of definition of a function, the range of a function and the characteristics of the notions of dependent and independent variable. Elementary functions such as constant function, linear function and direct proportion are defined as well as the ways in which they can be assigned: by algorithm, where algorithm of a function is given by the formula for calculating the values of dependent variable, by table of function values, and by Cartesian graph.

The theoretical part of the given thematic area is made accessible through presenting the above notions in concrete examples. The analysis of task solutions is followed by another stage in which theoretical notes – definitions and characteristics of the notions relevant in the given domain – are explained. The above approach utilizes the elements of constructivist pedagogy in which

knowledge is constructed in human mind on the basis of his/her experience with solving task and experimenting [1].

OECD PISA 2003 released items are utilized by us in the thematic areas of *Solving Applied Tasks and Tasks Developing Specific Mathematical Thinking*. In the following part, we will introduce two tasks developed for the purpose of testing mathematical literacy, used in teaching a function. Before that, however, OECD PISA testing tasks will be characterised.

## 4. Characteristics of tasks from OECD PISA testing

Each PISA task contains three components: situations or context, mathematical content and competences. Situations or contexts refer to real life. Mathematical content is subdivided into four parts: quantity, space and shape, change and relationships, and uncertainty. Relationships are given by a variety of representations, including symbolic, graphic, tabular and geometrical. Uncertainty refers to probabilistic and statistical phenomena and relations. Competences are abilities which could be drawn upon in solving concrete task or problem. They are divided into three levels:

- the level of reproduction: routine linkage, reproduction of practised procedures;
- the level of connection: utilising practised procedures when solving new problems with different domains of mathematics, includes tasks of divergent character;
- the level of reflection: tasks aimed at argumentation, abstraction, creation and utilisation of new algorithms and the use of mathematical apparatus in unknown situations.

## 5. Tasks on mathematical literacy used in teacher training

The task named Growing Up (M150Q01, M150Q02, M150Q03) ([2], p. 12) is from the Change and Relationships group, recommended for the thematic area of Numerical Operations, Functions and Reading from Graphs. It is aimed at competences at reproduction and connection levels. The task contains the elements of assigning function and dependence relation by graph or table.

**Growing Up** ([2], p. 12)
*Youth grows taller. In 1998 the average height of both young males and young females in the Netherlands is represented in the graph (Figure 1).*
**Question 1:** *Since 1980 the average height of 20-year-old females has increased by 2.3 cm, to 170.6 cm. What was the average height of a 20-year-old female in 1980?*
**Question 2:** *According to this graph, on average, during which period in their life are females taller than males of the same age?*

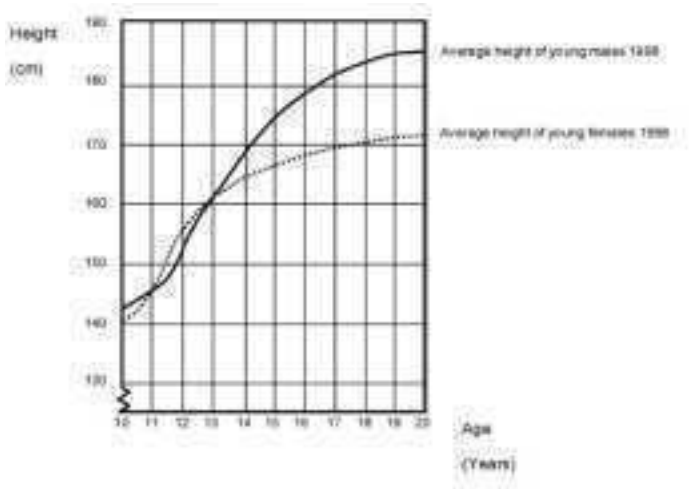


Figure 1

**Question 3:** *Explain how the graph shows that on average the growth rate for girls slows down after 12 years of age.*
Characteristics of the task:
Question 1: Solving the task does not require reading the data from the graph. It is indirectly formulated verbal task which relies on subtraction of decimal numbers. This part of the task is solved by students without problems.
Question 2: This part of the assignment requires reading and interpreting the data from the graph. It is important to identify a graph which represents the dependence of the female average height on age and the dependence of male average height on age. The data from both graphs are compared, which leads to the correct answer.
On the background of the graph, the notions of the domain of definition of a function (the values of age), the range of a function (the values of average height), dependent and independent variable (the average height is dependent on the age), a graph of a function, data on the $x$-axis and $y$-axis are explained to students. The function in the task is given by a graph.
Question 3: This part of the item requires reading and interpretation of the data from the graph as well as the ability to argue and give the rationale for his/her idea. Since the properties of a function (monotonicity: increasing and decreasing) are not defined yet, the students are expected to provide answers using imprecise mathematic terminology. The response should refer to the change of the gradient of the graph for the dependence of female height on age.
The reasoning may be based on comparing the given values in the graph, in the period after 12 years of age in the female group. Transcription of the data from the graph can also be used:
10 years – 140 cm
12 years – 155 cm. In the period preceding the 12th year of age (two years), the increase is by 15 cm. The yearly growth is 7.5 cm.
20 years – 172 cm. The period after the 12th year (by the 20th year, i.e. in 8 years) the growth is only at about 17 cm. The yearly growth is only 2 cm.

Age intervals can be arbitrary, but the result or the reasoning for the given fact are important. From the given data it is obvious that the acceleration of female growth has decreasing tendency after the 12th year of age.

Alternatively, it is possible to include transcription of the data from the graph into a table and interpret the rationale on the basis of the information from Table 1.

Table 1

| Age Interval | Height in the Given Period | Growth |
|---|---|---|
| 10. – 11. | 140 – 145 cm | 5 cm |
| 11. – 12. | 145 – 155 cm | 10 cm |

In a two-year period there is 15 cm growth, i.e. 7.5 cm a year in average.

Table 2

| Age Interval | Height in the Given Period | Growth |
|---|---|---|
| 12. – 13. | 155 – 160 cm | 5 cm |
| 13. – 14. | 160 – 165 cm | 5 cm |
| 14. – 15. | 165 – 167 cm | 2 cm |
| 15. – 16. | 167 – 169 cm | 2 cm |
| 16. – 17. | 169 – 170 cm | 1 cm |
| 17. – 18. | 170 – 170.5 cm | 0.5 cm |
| 18. – 19. | 170.5 – 171 cm | 0.5 cm |
| 19. – 20. | 171 – 171.5 cm | 0.5 cm |

The growth in eight-year period is 16.5 cm, i.e. 2 cm a year in average. From the above data it follows that acceleration of female growth after the 12th year of age is decreasing.

Building upon the presented contextual situation, it is possible to formulate another task aimed at the third level of competences – reflection, for example, to compare the acceleration of female and male growth in the given periods and give reasons to one's answers. The task is of divergent character due to the possibility of different solutions. A student can use the data represented in graphs or transform them into a table, and the answer will follow from the data presented in the table.

The task Exchange Rate (M413Q01, M413Q02, M413Q03) includes a Quantity domain. It develops competences on reproduction and reflection levels and could be classified under Direct Proportionality thematic area. Therefore, it is included in the Arithmetic and Algebra with Didactics syllabus in the part which deals with the theoretical fundaments of an elementary function - direct proportion.

**Exchange rate** ([2], p. 14)

*Mei-Ling from Singapore was preparing to go to South Africa for 3 months as an exchange student. She needed to change some Singapore dollars (SGD) into South African rands (ZAR).*

**Question 1:** *Mei-Ling found out that the exchange rate between Singapore dollars and South African rands was: 1 SGD = 4.2 ZAR*

*Mei-Ling changed 3000 Singapore dollars into South African rands at this exchange rate. How much money in South African rands did Mei-Ling get?*

**Question 2:** *On returning to Singapore after 3 months, Mei-Ling had 3 900 ZAR left. She changed this back to Singapore dollars, noting that the exchange rate had changed to:*

*1 SGD = 4.0 ZAR*

*How much money in Singapore dollars did Mei-Ling get?*

**Question 3:** *During these 3 months the exchange rate had changed from 4.2 to 4.0 ZAR per SGD. Was it in Mei-Ling's favour that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rands back to Singapore dollars? Give an explanation to support your answer.*

Characteristics of the task:

The task is aimed at propaedeutics of direct proportion and at assigning a function by algorithm. We have devised a task for students in which they were expected to create an algorithm for a function on the basis of the given information.

Question 1 is aimed at the interpretation of a simple mathematical model, and the solution requires applying multiplication of decimal numbers.

1 SGD = 4.2 ZAR
3 000 SGD = $x$ ZAR
3 000 × 4.2=12 600 (ZAR)

Direct proportionality is used, i.e. the amount of SGD will increase as many times more as will the amount of ZAR: increase in the value of one variable is in direct proportion to the increasing value of another variable.

Question 2 requires the interpretation of a simple quantitative model (direct proportionality) and applying it together with basic decimal numbers numeric operation. It is important to identify the operation to be applied.

3 900 ZAR = $x$ SGD
4.0 ZAR = 1 SGD
3 900:4=975 (SGD)

It is a direct proportionality relation when the rule of three is proper: the ratio 3 900:4 equals to the ratio $x$:1.

The students create algorithm of the function from the task: $y = 4.2x$; $y = 4x$. On the background of the algorithm structure it is easy to identify direct proportionality.

Question 3: this part of the task is about explanation and reasoning of one's own thinking procedure. For example, it suffices to verify how many Singapore Dollars one gets at the exchange rate of 4.2 ZAR = 1 SGD

3 900 ZAR = $x$ SGD
3 900:4.2 = 928.57 SGD

From the given, it is obvious that at this exchange rate one get less SGD for 3 900 ZAR than at the exchange rate which was current to the date (she got 975 SGD). Lower exchange rate is more profitable. It is not necessary to use numeric operations and state exact sum which she would get at the given exchange rates. Reasoning, though, must be precise and clear. There is also an option to draw the graphs of both functions with the table of domain and range values, and to present own reasoning based on the given data from the graphs and the table.

## 5. Conclusion

We have outlined one possibility to develop mathematical literacy of prospective primary school teachers. Teachers should realize that in teaching mathematics they can take advantage of real situations which correspond with the inner world of junior school age children [4]. In the future, more tasks which context reflects real life situation will be included into the curriculum of the programmes taught at the Faculty of Education, University of Prešov. The tasks of the given character are instrumental in presenting not only pedagogical approaches to the particular domains of mathematics but also theoretical points of departure of the given problem.

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