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# Convex optimization of thrust allocation in a dynamic positioning simulation system

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**Abstract**

Vessels conducting dynamic positioning (DP) operations are usually equipped with thruster configurations that enable the generation of resultant force and moment in any direction. These configurations are deliberately redundant in order to reduce the consequences of thruster failures and increase the safety. On such vessels a thrust allocation system must be used to distribute the control actions determined by the DP controller among the thrusters. The optimal allocation of thrusters' settings in DP systems is a problem that can be solved by several convex optimization methods depending on criteria and constraints used. The paper presents linear programming (LP) and quadratic programming (QP) methods adopted in the DP control model which is being developed at the Maritime University of Szczecin for ship simulation purposes.

## Introduction

A convex optimization problem is one of the form (Boyd & Vandenberghe, 2009):

$$\begin{aligned} &\text{minimize} \quad f_0(x) \\ &\text{subject to} \quad f_i(x) \le b_i,\ i = 1,\ldots,m \end{aligned} \tag{1}$$

where the functions $f_0,\ldots,f_m : \mathbb{R}^n \rightarrow \mathbb{R}$ are convex, i.e., satisfy:

$$f_i(\alpha x + \beta y) \le \alpha f_i(x) + \beta f_i(y)$$

for all $x, y \in \mathbb{R}^n$ and all $\alpha, \beta \in \mathbb{R}$ with $\alpha + \beta = 1$, $\alpha \ge 0$, $\beta \ge 0$. In general, there is no analytical formula for the solution of convex optimization problems, but there are very effective methods for solving them, such as: the least-squares in quadratic programming (QP), linear programming (LP) or interior-point methods that are used in second-order cone programming (SOCP) or geometric programming (GP) (Boyd & Vandenberghe, 2009).

Dynamic Positioning (DP) system can be defined as a system that automatically controls a vessel, by the influence of external excitations, to maintain her position and headed exclusively by means of active thrust. The DP system divides forces among a ship's thrusters to achieve a resultant force and momentum equal to the one set by the control system. Optimization of thrust allocation is based on minimization of energy usage (need for power or fuel if feasible), additionally taking into account limitations like forbidden zones for thrusters' settings (individual and relative to each other – for instance in the case of opposing thruster pairs).

The optimal allocation of forces generated by thrusters in DP systems is a problem that can be solved by several convex optimization methods depending on the criteria and constraints used. A survey of these methods, including direct allocation by QP, has been presented by Johansen and Fossen (Johansen & Fossen, 2013). This paper presents linear programming (LP) and quadratic programming (QP) methods adopted in a DP control model developed at the Maritime University of Szczecin for ship simulation purposes.

## Generation of Forces with Thrusters

For a DP control, similar to ship simulation, a ship's hull can be treated as a rigid body with the center of mass at the origin $p = 0 \in \mathbb{R}^2$. Measurements of the position of the vessel are then compared with the required position. The difference is fed into an Extended Kalman Filter (EKF) and PID-controller to convert this to the resultant force and momentum required to correct the position. The allocation unit controls the thrusters, which must generate the component forces of the resultant one. Therefore the model of thrust allocation used for a vessel with $i^{th}$-number of azimuth thrusters can be built according to the geometrical relations presented in Figure 1.

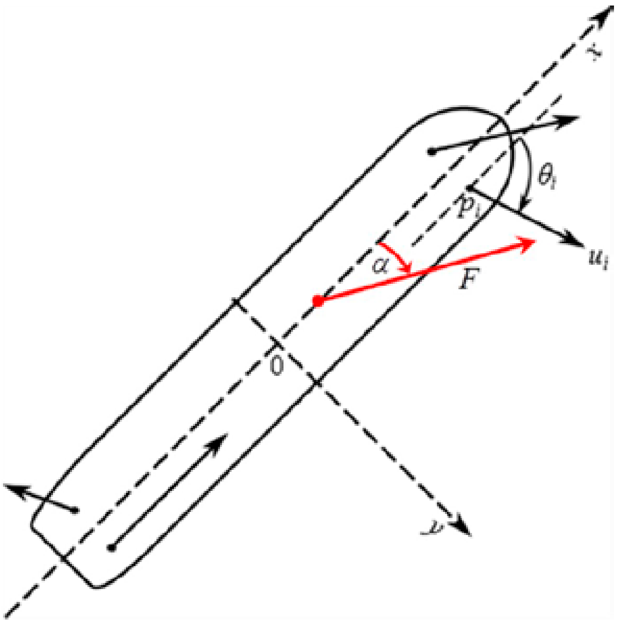



**Figure 1. Thrust forces acting on a vessel with $i^{th}$ number of azimuth thrusters**

The assumptions of the model are:

- The vessel's position is stabilized at low speed (less than 2 knots or 1 m/s), and the center of mass (force reference origin) is the fixed rotation center.
- There are n component forces with magnitude $u_i$ [kN] or [tf], acting at $p_i = (p_{ix}, p_{iy})$ [m, m], in direction $\theta_i$ [deg], $i = 1,2,\dots,n$.
- The resultant force [kN] or [tf]:

$$F = \sqrt{F_x^2 + F_y^2} \quad (2)$$

- The resultant longitudinal force (horizontal in ship-body frame) [kN] or [tf]:

$$F_x = \sum_{i=1}^{n} u_i \cos\theta_i \quad (3)$$

- The resultant transverse force (vertical in ship-body frame) [kN] or [tf]:

$$F_y = \sum_{i=1}^{n} u_i \sin\theta_i \quad (4)$$

- The resultant torque (moment of the resultant force) [kNm] or [tfm]:

$$M_z = \sum_{i=1}^{n} (p_{iy} u_i \cos\theta_i - p_{ix} u_i \sin\theta_i) \quad (5)$$

- The force limits [kN] or [tf]:

$$0 \le u_i \le u_{\max} \quad (6)$$

- The thruster angle limits [rad] or [deg]:

$$\pi < \theta_i \le 2\pi \quad (7)$$

- The energy or fuel usage is strictly dependent on $u_i$ and in the case of the same type of thrusters, the total energy is assumed to be linearly correlated to:

$$\sum_{i=1}^{n} u_i = u_1 + u_2 + \ldots + u_n \quad (8)$$

The problem to solve is as follows: find the appropriate values for $u_i$ and $\theta_i$ that yield the desired resultant force and moment while minimizing the fuel or energy usage. Note that the problem is considered to be 3-DOF (degrees of freedom) or solved in 2-dimensional space. In fact, any movement in the $z$-direction (up/down) or around the $x$- and $y$-axis is ignored because common actuators in offshore vessels do not have the ability to produce thrust in these directions. This clearly reduces the complexity of the problem.

## LP problem solution

The standard form of constrained LP can be given in matrix notation as:

$$\underset{x}{\text{minimize}} \quad c^T x$$
$$\text{subject to} \quad Ax = b,\ x \ge 0 \quad (9)$$

where: $A \in P^{m \times n}$; $x, c \in P^n$; $b \in P^m$.

For the thruster allocation problem with variables $u_i$ and $\theta_i$ the formulation of the objective function and constraints is:

$$\text{minimize} \quad \mathbf{1}^T u$$
$$\text{subject to} \quad fu^T = F^T$$
$$0 \le u_i \le u_{\max},\ 0 \le \theta_i < 2\pi,\ i = 1,\dots,n \quad (10)$$

where:

$\mathbf{1} = [1,1,1,\dots,u_n]$ of size $n$;

$u = [u_1, u_2,\dots,u_n]$;

$$f =$$

$$= \begin{bmatrix} \cos\theta_1 & \dots & \cos\theta_n \\ \sin\theta_1 & \dots & \sin\theta_n \\ p_{1y}\cos\theta_1 - p_{1x}\sin\theta_1 & \dots & p_{ny}\cos\theta_n - p_{nx}\sin\theta_n \end{bmatrix} \tag{11}$$

$F = [F_x, F_y, M_z]$;

$F_x$, $F_y$, $M$ – designated longitudinal, transverse forces, and moment.

## QP problem solution

The standard form of constrained QP can be given in matrix notation as:

$$\begin{aligned} &\underset{x}{\text{minimize}} \quad x^T P x + q^T x + r \\ &\text{subject to} \;\; Gx \preceq h \\ &\qquad\qquad\quad Ax = b \end{aligned} \tag{12}$$

where:

$P \in \Sigma_+^n$; $G \in \mathrm{P}^{m\times n}$; $A \in \mathrm{P}^{o\times n}$; $x,q,r \in \mathrm{P}^n$; $h \in \mathrm{P}^m$;

$b \in \mathrm{P}^o$;

$\Sigma_+^n$ is the set of symmetric, positive semidefinite, $n \times n$ matrices.

For the thruster allocation problem with variables $u_i$ and $\theta_i$ transformed to $f_{xi}$ and $f_{yi}$ (longitudinal and transverse components of forces $u_i$), the formulation of the objective function and constraints is:

$$\begin{aligned} &\text{minimize} \quad \mathbf{1}^T\left(f_x^2 + f_y^2\right) \\ &\text{subject to} \;\; F_x = \mathbf{1}^T f_x \\ &\qquad\qquad\quad F_y = \mathbf{1}^T f_y \\ &\qquad\qquad\quad M_z = \mathbf{1}^T\left(p_x \bullet f_y - p_y \bullet f_x\right) \\ &\qquad\qquad\quad \max\left(f_x^2 + f_y^2\right) \le f_{\max}^2 \end{aligned} \tag{13}$$

where:

$$\begin{aligned} f_x &= [f_{x1}, f_{x2}, \dots, f_{xn}] \\ f_y &= [f_{y1}, f_{y2}, \dots, f_{yn}] \end{aligned} \tag{14}$$

$$\begin{aligned} f_{xi} &= u_i \cos\theta_i \\ f_{yi} &= u_i \sin\theta_i \\ u_i^2 &= f_{xi}^2 + f_{yi}^2 \end{aligned} \tag{15}$$

and $F_x$, $F_y$, $M_z$ are the designated longitudinal, transverse forces, and moment constraints analogous to the LP problem. If the final constraints worked out by EKF and PID are in the form of (see Figure 1):

$F$ – resultant force;

$\alpha$ – orientation of the resultant force;

$M_z$ – resultant momentum;

$f_{\max}$ – maximum individual thruster force

then:

$$\begin{aligned} F_x &= F\cos\alpha \\ F_y &= F\sin\alpha \end{aligned} \tag{16}$$

The formula (13) can be further extended by additional constraints on the thrusters' work sectors (limits of $\theta_i$ or ratio $f_{yi}/f_{xi}$).

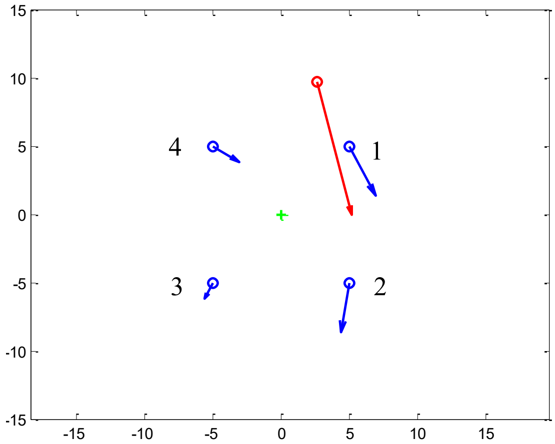
## Implementation in DP Simulation System

The algorithms, which solve (10) and (13) by applying Newton's method to a sequence of equality constrained problems, have been developed in Matlab with either Optimization or the CVX Toolbox (Boyd & Vandenberghe, 2009) and afterwards translated to C#. The tests proved that though generally, the solution efficiency of LP is better than the efficiency of QP (which formally is a further generalization of LP), while in the case of the thruster allocation algorithm QP was more robust and faster. The main reasons of this have been nonlinearities in constraints (11) where trigonometric functions are directly applied. The problem elaborated as (13) avoids non-convexity as all constraints are strictly convex, and it can be classified into a linearly constrained QP. Because $f_x^2$ and $f_y^2$ are convex functions, their point wise maximum $f_{\max}^2$ is also convex and can be re-expressed as two separate inequalities using a standard LP formulation. The route of QP is also followed by most of the authors dealing with thrust allocation in ship borne DP systems (Lindfors, 1993; Gierusz & Tomera, 2006; Ruth, 2008; Wit, 2009; Daalen et al., 2011; Rindarøy, 2013).

The examples of thrust allocation to four azimuth thrusters, calculated by the model adopted in the DP simulation system established by the Maritime University of Szczecin, with the resultant force in various four quadrants of ship-body fixed co-ordinate system (360° clockwise), are presented in the Figures 2, 3, 4, 5.

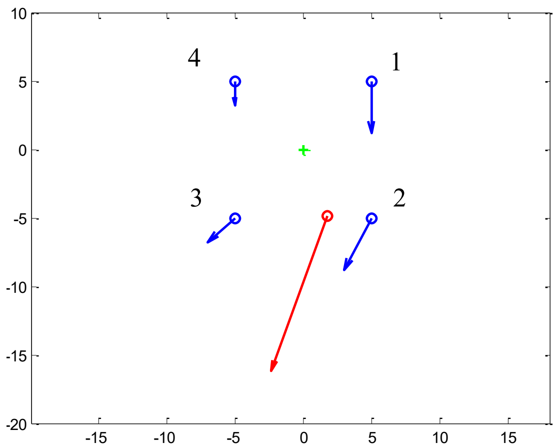
The co-ordinates $p_{xi}$ [m] and $p_{yi}$ [m] of azimuth thrusters in the figures 2, 3, 4 and 5 have been set for visualization purposes as: $p_x$ = [5;5;–5;–5], $p_y$ = [5;–5;–5;5] and $f_{\max}$ = 5 tf.

It must be stressed that the developed model focuses on the allocation part of the full closed loop control system which is used to keep the vessel in a stationary position. The allocation unit receives the required total force and momentum from the PID controller and generates the appropriate control signals to the available thrusters of the vessel.



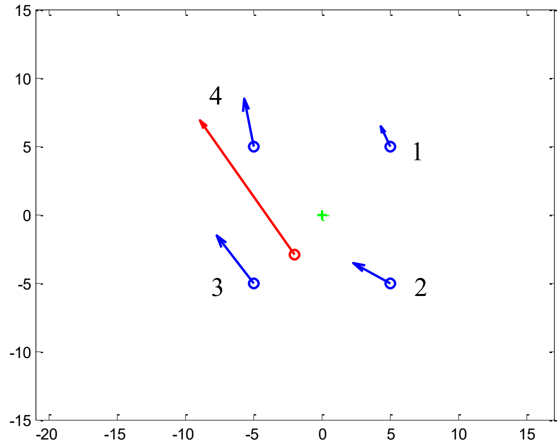

| $F$ [tf] | $\alpha$ [deg] | $M_z$ [tfm] |
|---|---|---|
| 10.00 | 165.0 | 50.00 |
| $i$ | $u_i$ | $\theta_i$ |
| 1 | 4.12 | 152.63 |
| 2 | 3.71 | 189.34 |
| 3 | 1.31 | 207.37 |
| 4 | 2.23 | 121.55 |

**Figure 2. Example of thrust forces allocation with the resultant force in the 1st quadrant of a ship-body fixed co-ordinate system**



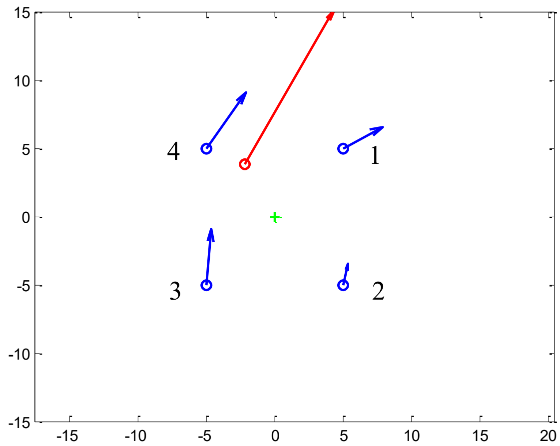

| $F$ [tf] | $\alpha$ [deg] | $M_z$ [tfm] |
|---|---|---|
| 12.00 | 200.0 | 40.00 |
| $i$ | $u_i$ | $\theta_i$ |
| 1 | 3.82 | 180.39 |
| 2 | 4.32 | 207.95 |
| 3 | 2.72 | 228.08 |
| 4 | 1.82 | 180.82 |

**Figure 3. Example of thrust forces allocation with the resultant force in the 2nd quadrant of a ship-body fixed co-ordinate system**




| $F$ [tf] | $\alpha$ [deg] | $M_z$ [tfm] |
|---|---|---|
| 12.00 | 325.0 | 40.00 |
| $i$ | $u_i$ | $\theta_i$ |
| 1 | 1.63 | 333.69 |
| 2 | 3.09 | 298.18 |
| 3 | 4.40 | 321.80 |
| 4 | 3.53 | 348.22 |

**Figure 4. Example of thrust forces allocation with the resultant force in the 3rd quadrant of a ship-body fixed co-ordinate system**




| $F$ [tf] | $\alpha$ [deg] | $M_z$ [tfm] |
|---|---|---|
| 13.00 | 30.0 | 50.00 |
| $i$ | $u_i$ | $\theta_i$ |
| 1 | 3.27 | 61.45 |
| 2 | 1.61 | 13.48 |
| 3 | 4.08 | 5.27 |
| 4 | 4.98 | 35.27 |

**Figure 5. Example of thrust forces allocation with the resultant force in the 4th quadrant of a ship-body fixed co-ordinate system**

Note that the problem is simplified by considering it to be 2-dimensional. In fact, any movement in the up/down direction is ignored due to its periodic behavior and no necessity, and no ability to control it. In addition, the presented algorithm is limited to the azimuth thrusters, which are able to direct their thrust in 360 degrees around the *z*-axis.

As a numerical method, the established model solves the optimization almost in real time: the computation can be treated as instantaneous compared to the typical timescales of the vessels dynamics, as it takes less than 0.5 s to calculate the results.

Nevertheless it should also be noted that optimizing only the allocation system might not be ideal. A model-predictive approach that combines the Extended Kalman Filter (EKF), PID, and allocation unit might lead to even better results. Another important aspect of this approach would be the concept of time horizon: the power can be minimized over a given period, for example the next minute. However, this would require a full model of the vessel, together with models of the wind, current, and the waves to be implemented.

## Conclusions

A thrust allocation system must be used to distribute the control actions determined by the DP controller among the thrusters. The allocation problem can be translated to a constrained optimization problem. The quadratic programming (QP) method has been developed for this purpose in the DP ship simulation model. The tests proved that the optimization algorithm translated into C# programming language worked efficiently using interior-point methods (Boyd & Vandenberghe, 2009) to solve the problem by applying Newton's method to a sequence of equality constrained problems. The further development including extra constraints like limits to the thrusters' work sector (forbidden zones) and non-azimuth thrusters, or model predictive approach will continue in the future.

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