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Orginal Article

Load and resistance factors for prestressed concrete girder bridges

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Abstract: There has been a considerable progress in the reliability-based code development procedures. The load and resistance factors in the AASHTO bridge design code were determined using the statistical parameters from the 1970's and early 1980's. Load and resistance factors were determined by first fixing the load factors and then calculating resistance factors. Load factors were selected so that the factored load corresponds to two standard deviations from the mean value and the resistance factors were calculated so that the reliability index is close to the target value. However, from the theoretical point of view, the load and resistance factors are to be determined as coordinates of the so-called "design point" that corresponds to less than two standard deviations from the mean. Therefore, the optimum load and resistance factors are about 10% lower than what is in the AASHTO LRFD Code. The objective of this paper is to revisit the original calibration and recalculate the load and resistance factors as coordinates of the "design point" for prestressed concrete girder bridges. The recommended new load and resistance factors provide a consistent reliability and a rational safety margin.

Keywords: design point, design formula, prestressed concrete girders, resistance factor, reliability index, bridge live load, safety margin

1. Introduction

The basis for the current AASHTO LRFD Code [1] was developed in the 1980's [8]. The major conceptual change from the Standard Specifications [2] was the introduction of four types of limit states and corresponding load and resistance factors.

The basic design formula for structural components in the Standard Specifications [2] is:

$$1.3D + 2.17\left(L + I\right) < \phi R \tag{1}$$

where:

D = dead load;

L = live load (HS-20);

I = dynamic load;

R = resistance (load carrying capacity);

ϕ = resistance factor (by default = 1).

On the other hand, the equivalent design formula in the AASHTO LRFD Code [3] is:

$$1.25D + 1.50D_w + 1.75\left(L + I\right) < \phi R \tag{2}$$

where:

D_w = dead load due to wearing surface;

L = live load (HL-93);

ϕ = 1 for steel girders and pre-tensioned concrete girders and 0.9 for reinforced concrete T Beams.

Comparison of Eq. 1 and Eq. 2 indicates that the differences are on the load side only. The role of load and resistance factors is to provide safety margins, i.e. load factors are to increase the design loads so that there is an acceptably low probability of being exceeded. Role of resistance factor is to decrease the design load carrying capacity, to result in an acceptably low probability of exceeding the critical level. However, if ϕ = 1, then resistance is not reduced and most of the safety reserve is on the load side of Eq. 1 and 2.

Therefore, there is a need to determine values of load and resistance factors that would represent rational and optimum safety margins. The derivation procedure involves the reliability analysis procedure and calculation of the so-called "design point" [10]. The product of load and load factor can be referred to as a factored load, and the product of resistance and resistance factor is a factored resistance. The coordinates of the design point are values of factored load and factored resistance corresponding to the minimum reliability index. The objective of this paper is to calculate the optimum load and resistance factors for selected representative bridge components and then propose a modified design formula to replace Eq. 2.

2. Limit state function and reliability index

For each limit state, a structural component can be in two states: safe when resistance, R, exceeds the load, Q, and unsafe (failure) when load exceeds resistance. The boundary between safe and unsafe states can be represented by the limit state function, in a simple form such as:

$$g = R - Q = 0 \tag{3}$$

Since R and Q can be considered as random variables, the probability of failure, PF, is equal to probability of g being negative,

$$P\_F = P \cdot \left(g < 0\right) \tag{4}$$

In general, R and Q can be functions of several variables such as dead load, live load, dynamic load, strength of material, dimensions, girder distribution factors, and so on. Therefore, the limit state function can be a complex function:

$$g\left(X_1,...,X_n\right)=0 \tag{5}$$

A direct calculation of probability of failure can be difficult, in particular when g is non-linear. Instead, reliability index, β, can be calculated and the relationship between, β, and the probability of failure, P_F, is as follows:

$$P_F g=\Phi(-\beta) \tag{6}$$

and

$$\beta=-\Phi^{-1}\left(P_F\right) \tag{7}$$

where:

Φ = cumulative distribution function of the standardized normal random variable;

Φ-1= the inverse of Φ [10].

There are several formulas and analytical procedures available to calculate β. If the limit state function is linear, and all the variables are normal (Gaussian), i.e.

$$g\left(X_1,...,X_n\right)=a_0+\sum_{i=1}^{1} a_i X_i \tag{8}$$

then

$$\beta=\frac{\mu_g}{\sigma_g} \tag{9}$$

where:

$$\mu_g=g\left(\mu_1,...,\mu_n\right) \tag{10}$$

μ_i = mean value of $X_{i;}$

$$\sigma_g=\sqrt{\sum\left(a_1\sigma_i\right)^2} \tag{11}$$

σ_i = standard deviation of Xi.

If the variables are non-normal, then Eq. 9 can be used as an approximation. Otherwise, a more accurate value of β can be calculated using an iterative procedure developed by Rackwitz and Fiessler [17]. However, in practical cases the results obtained using Eq. 9 can be considered as accurate.

If the limit state function is nonlinear, then accurate results can be obtained using Monte Carlo simulations [10].

3. Design point

The result of reliability analysis is reliability index, β. In addition, the reliability analysis can be used to determine the coordinates of the "design point", i.e. the corresponding value of factored load for each load component and value of factored resistance. For the limit state function in Eq. 5, the design point is a point in n-dimensional space, denoted by (X_1^*, …, X_n^*), that satisfies Eq. 5, and if failure is to occur, it is the most likely combination of X_1^*, …, X_n^* [10].

For example, if the limit state function is given by Eq. 3, and R and Q are normal random variables, then the coordinates of the design point are [10]:

$$R^* = \mu_R - \frac{\beta \sigma_R^2}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{12}$$

$$Q^* = \mu_Q + \frac{\beta \sigma_Q^2}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{13}$$

If R and Q are not both normally distributed then R^* and Q^* can be calculated by iterations using Rackwitz and Fiessler procedure [17]. However, a relatively wider range of design point coordinates corresponds to the same value of reliability index, so in practice, Eq. 12 and Eq. 13 can be used even for non-normal distributions.

4. Statistical Parameters of Load Components

The basic load combination for bridge components include dead load, D, dead load due to the wearing surface, D_W, live load, L, and dynamic load, I. Each random variable is described by its cumulative distribution function (CDF), including the mean and standard deviation. It is also convenient to use the bias factor which is the ratio of mean-to-nominal value, denoted by λ, and the coefficient of variation, V, equal to the ratio of the standard deviation and the mean. Both λ and V are non-dimensional.

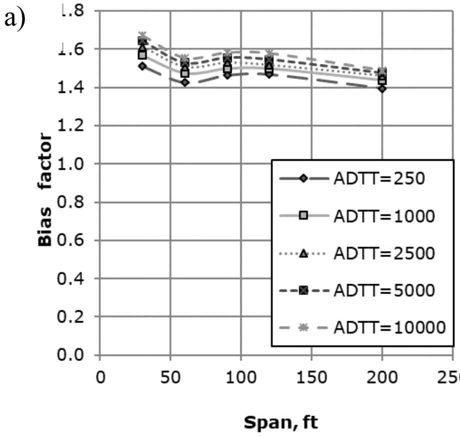
The total load is a sum of $D + D_w + (L + I)$. Dead load is time invariant so the only time-varying load components are L and I. In the original code calibration [12], the maximum expected 75-year live load was considered, therefore, the same time period is considered in this paper.

The statistical parameters of dead load that were used in the original calibration have not been challenged so far. Therefore, for factory-made components (structural steel and precast/prestressed concrete) $\lambda = 1.03$ and $V = 0.08$. For the cast-in-place concrete, $\lambda = 1.05$ and $V = 0.10$. For the wearing surface it is assumed that the mean thickness is 3.5in (90 mm) with $\lambda = 1.00$ and $V = 0.25$.

The live load parameters used in the original calibration were based on the Ontario truck survey data [18], with less than 10,000 vehicles, because no other reliable data was available at that time. In the meantime, a considerable weight-in-motion (WIM) database was collected by the FHWA. Therefore, the statistical parameters for live load are taken from the recent SHRP2 R19B report [15]. The processed data included 34 million vehicles from 37 locations in 18 states. For each location, the annual number of vehicles was 1 to 2 million.

Live load is the effect of trucks, therefore, the vehicles in the WIM databasebase were run over influence lines to determine the moments and shears. CDF's of the maximum simple span moments were calculated for 30 ft (9 m), 60 ft (18 m), 90 ft (27 m), 120 ft (36 m) and 200 ft (60 m). For an easier interpretation of the results, the moments were divided by the corresponding HL-93 moments [1]. For the considered locations, the maximum ratios were about 1.35-1.40 of HL-93.

The cumulative distribution functions were extrapolated to predict the mean maximum 75 year moment. The ratio of mean-to-nominal value, or bias factor for live load moment, is plotted vs. span length in Fig. 1 for the average daily truck traffic (ADTT) from 250 to 10,000.



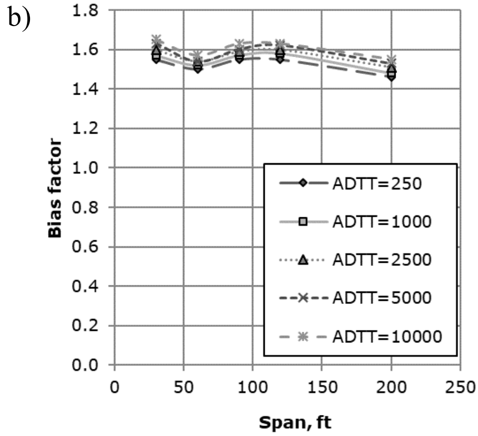


Fig. 1. Bias factor vs. span length for the maximum 75 year: a) moment, b) shear (1ft=3.05 m). *Source:* own study

Field tests showed that dynamic load practically does not depend on the truck weight [9]. Therefore, dynamic load factor decreases for heavier trucks. It is further reduced when a multiple presence of trucks is considered, in particular for side-by-side occurrence [12]. Therefore, in the reliability analysis, the mean value of the dynamic load factor is taken as 0.10.

The coefficient of variation for static and dynamic live load is taken as 0.14. The total load as a sum of several components can be considered as a normal random variable.

5. Statistical parameters of resistance

The load carrying capacity is considered as a product of three factors representing the uncertainties involved in material properties, dimensions/geometry and the analytical model. The statistical parameters, bias factor, λ, and coefficient of variation, *V*, that were used in the original calibration are listed in Table 1.

Table 1. Statistical parameters of resistance from NCHRP report 368. *Source:* [12]

Material	Moment		Shear	
	λ	*V*	λ	*V*
Steel – Non-composite	1.12	0.1	1.14	0.105
Steel – Composite	1.12	0.1	1.14	0.105
Reinforced Concrete	1.14	0.13	1.2	0.155
Prestressed Concrete	1.05	0.075	1.15	0.14

Since the original calibration, a considerable research was performed in conjunction with revision of the ACI 318 Code [13, 14, 16]. The data base included compressive strength of concrete, yield strength of reinforcing bars and tensile strength of prestressing strands. The results pointed out that the material properties are more predictable than 30 years ago. There is a reduction in coefficient of variation because of more efficient quality control procedures. It was observed that compressive strength of concrete has a bias factor of 1.3 for f_c' = 3000 psi (21 MPa) and 1.1 for f_c' = 12,000 psi (85 MPa), and corresponding coefficient of variation varies from 0.17 for f_c' = 3000 psi (21 MPa) to 0.10 for f_c' = 12,000 psi (85 MPa). For reinforcing steel, $\lambda = 1.13$ and $V = 0.03$, and for prestressing strands $\lambda = 1.04$ and $V = 0.015$. These material parameters can serve as a basis for revising the resistance models for bridge components. It is estimated that the mean load carrying capacity of bridge girders is higher by 5 to 10% compared to what was considered in the original calibration. However, since additional analysis is required to develop updated statistical parameters for resistance of bridge components, in this paper, the reliability analysis is carried out using the parameters from Table 1.

6. Representative Design Cases

The reliability indices are calculated for the design cases considered in the original calibration using Eq. 9 [12]. The results are shown in Fig. 2 for prestressed concrete girders, Fig. 3 for reinforced concrete T-beams and Fig. 4 for steel girders. For each material, the analysis is performed for spans: 30, 60, 90, 120 and 200 ft (9, 18, 27, 36 and 60 m), and girder spacing 4, 6, 8, 10 and 12 ft (1.2, 1.8, 2.4 and 3.6 m). For reinforced concrete T-beams the span length was limited to 60 ft (18 m). The analysis was performed for ADTT from 250 to 10,000.

The resulting reliability indices are about 3.5, with a small degree of variation. This is an indication that the code is consistent.

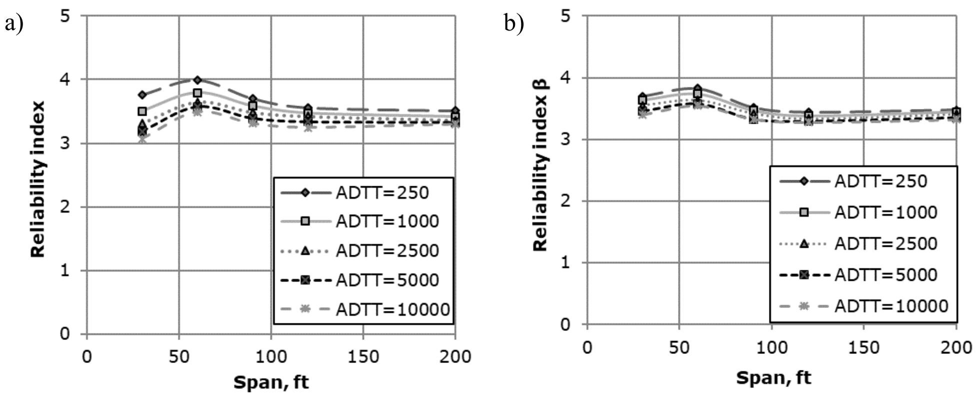


Fig. 2. Reliability index vs. span length for: a) moment, b) shear, for prestressed concrete girders (1 ft = 3.05 m). *Source:* own study

7. Optimum load and resistance factors

The reliability indices are calculated for the design cases considered in the original calibration. For these design cases, the parameters of the design point were also calculated using Eq. 12 and Eq. 13.

For each load component, X, the optimum load factor, γ_X, is (Eq. 14):

$$\gamma_X = \frac{\lambda_X X^*}{\mu_X} \tag{14}$$

where:

λ_X = bias factor of X;

X^* = coordinate of the design point;

μ_x = mean value of X;

and for resistance (Eq. 15):

$$\phi = \frac{\lambda_R R^*}{\mu_R} \tag{15}$$

Therefore, for D_1 (weight of factory made elements), the load factor, γ_{D1}, is:

$$\gamma_{D1} = \frac{\lambda_{D1} D_1^*}{\mu_{D1}} \tag{16}$$

for D_2 (weight of cast-in-place concrete), the load factor γ_{D2} is:

$$\gamma_{D2} = \frac{\lambda_{D2} D_2^*}{\mu_{D2}} \tag{17}$$

for D_3 (weight of the wearing surface), the load factor γ_{D3} is:

$$\gamma_{D3} = \frac{\lambda_{D3} D_3^*}{\mu_{D3}} \tag{18}$$

For live load, L, the load factor γ_L is:

$$\gamma_L = \frac{\lambda_L L^*}{\mu_L} \tag{19}$$

The dead load factors calculated using Eq. 16-18 are follows:

- for D_1, γ_{D1} =1.05-1.1;
- for D_2, γ_{D2}=1.10-1.17;
- for D_3, γ_{D3}=1.03-1.

As an example, the dead load factors γ_{D2} load factors are shown in Fig. 3 for prestressed concrete girders.

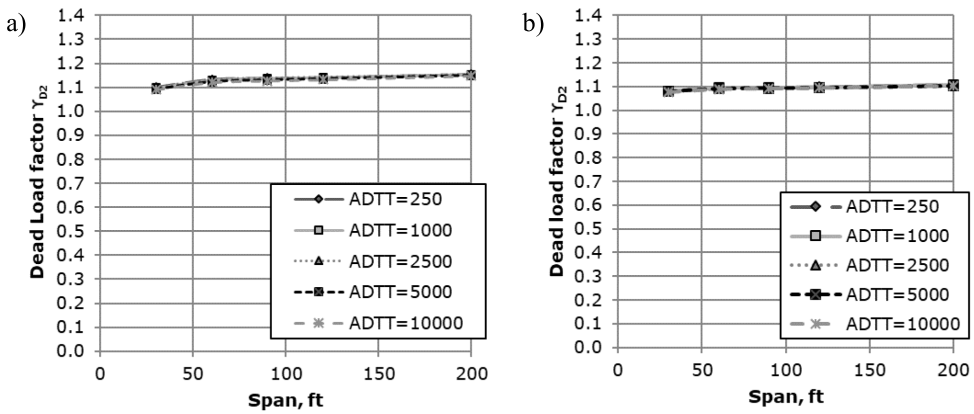


Fig. 3. Dead load factors vs. span length for: a) moment, b) shear, for prestressed concrete girders (1 ft = 3.05 m). *Source:* own study

The calculated live load factors are shown in Fig. 4 for prestressed concrete girders. For most cases, the optimum live load factor γ_L is between 1.4 and 1.55 for ADTT = 10,000 and the range is 1.3 to 1.5 for ADTT = 250. Therefore, 1.55 can be considered as a conservative value of live load, even for ADTT = 10,000.

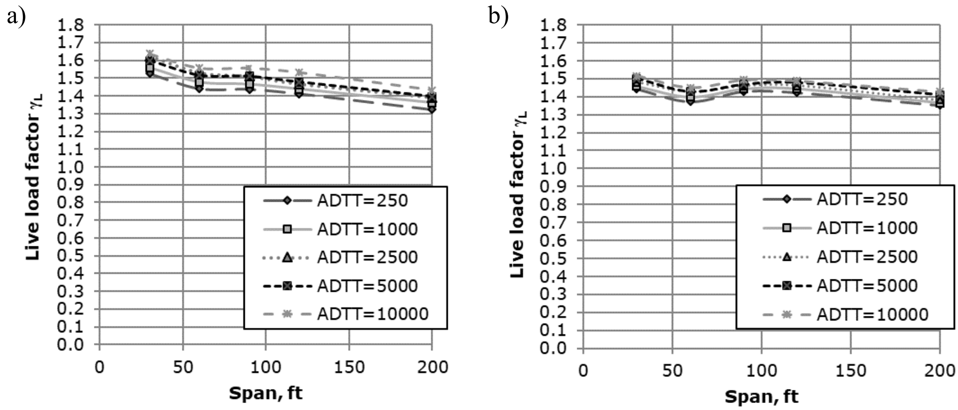


Fig. 4. Live load factor vs. span length for: a) moment, b) shear, for prestressed concrete girders (1 ft = 3.05 m). *Source:* own study

The resistance factors were calculated using Eq. 15. The results are presented in Fig. 5 for prestressed concrete girders, and they are summarized in Table 2.

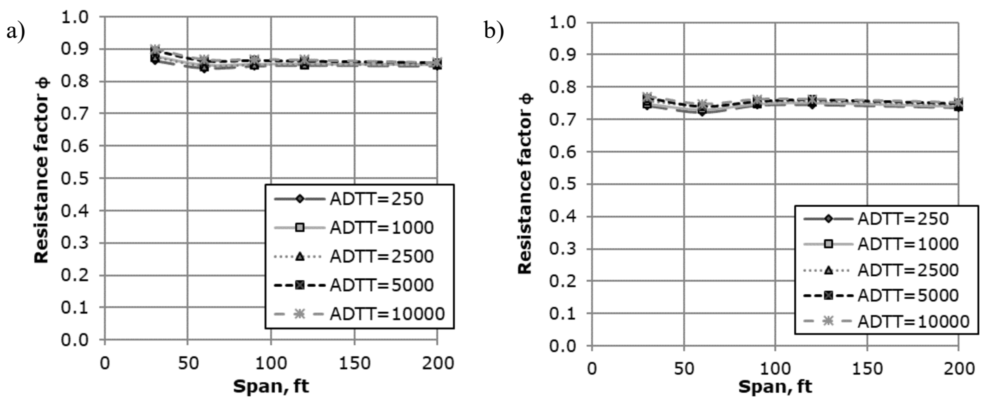


Fig. 5. Resistance factor vs. span length for: a) moment, b) shear, for prestressed concrete girders (1 ft = 3.05 m). *Source:* own study

Table 2. Current AASHTO resistance factors and calculated resistance factors

Material	Resistance Factor ϕ in Current AASHTO LRFD [3]		Calculated Resistance Factor ϕ	
	Limit State			
	Moment	Shear	Moment	Shear
Prestressed Concrete	1.00	0.9	0.85	0.75

8. Recommended load and resistance factors

The load and resistance factors corresponding to the coordinates of the design point are about 10-15% lower than what is in the current AASHTO Code [1]. The reliability indices calculated for design according to AASHTO [1] are consistent at about 3.5 level (Fig. 2-4). However, the bias factor for live load (Fig. 1) is higher for short spans compared to other span lengths which is an indication that the design live load for short spans has to be increased.

The calculated dead load factor for three components D_1, D_2 and D_3 is 1.05-1.17. For the dead load due to wearing surface, the statistical parameters are based on assumption about future overlays, and for simplicity of the code, one dead load factor of 1.20 is recommended for all dead load components.

The calculated values of live load factor γ_L are between 1.40 and 1.50. Higher value shows only for a short span, and it is due to the design load being too low. Therefore, live load factor can be 1.50 but a conservative 1.60 is recommended.

Calculated values of resistance factor corresponding to the design point are shown in Table 2. However, it is recommended to increase the listed values by 0.05, which is justified because of conservatism in the dead load factor and live load factor. The recommended "ϕ" factors are as shown in Table 3.

$$1.20\left(D+D_w\right)+1.6\left(L+I\right)<\phi R \tag{20}$$

Table 3. Current AASHTO resistance factors and recommended resistance factors. *Source:* [3]

Material	Resistance Factor ϕ in Current AASHTO LRFD		Recommended Resistance Factor ϕ	
	Limit State			
	Moment	Shear	Moment	Shear
Prestressed Concrete	1.00	0.9	0.9	0.8

The reliability indices are calculated for the recommended load and resistance factors and compared to the reliability indices corresponding to the current AASHTO and Eq. 2. The results are shown as scatter plot in Fig. 6 for moment and shear. The required moment carrying capacity corresponding to the recommended load and resistance factors is about 35% higher than for the current AASHTO [1] and for shear capacity it is about 5% higher than the current AASHTO [1].

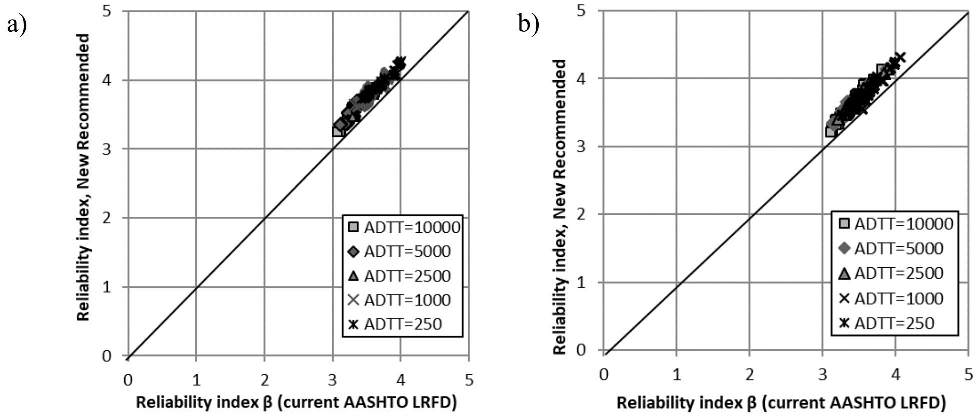


Fig. 6 Reliability indices for new recommended load and resistance factors vs. current AASHTO code: a) moment, b) shear. *Source:* own study.

Recommended load and resistance factors are 1.20 for dead load, 1.60 for live load and resistance factors of 0.90 for steel and P/C girders. Incidentally, these load and resistance factors would then be the same as in ASCE Standard 74 [6], ACI 318 [4], AISC LRFD [5] and National Design Specification for Wood [7].

9. Conclusions

Load factors in the AASHTO LRFD Code [1] were selected so that factored load corresponds to two standard deviations from the mean value. In this study, the optimum load factors are determined as corresponding to the "design point" and they are about 10% lower than specified in the code. The corresponding resistance factors are calculated as corresponding to the target reliability index. The resulting factors are also about 10% lower than in AASHTO Code. The acceptability criterion is, as in the original calibration, closeness to the target reliability index. The selection of load and resistance factors is checked on a set of representative bridges, the same as used in NCHRP Report 3689 [12]. In general, recommended load and resistance factors are about 10% lower than in the current AASHTO [1]. The reliability indices calculated for design cases using the current and recommended new load and resistance factors show a very good agreement.

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