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Monte Carlo simulation application to reliability assessment of an exemplary system operating at variable conditions

Keywords

system reliability, system operation process, Monte Carlo Simulation approach

Abstract

A semi-Markov process is applied to construct the multistate model of the system operation process and its main characteristics are determined. Analytical linking of the system operation process model with the system multistate reliability model is proposed to get a general reliability model of the complex system operating at varying in time operation conditions and to find its reliability characteristics. The Monte Carlo simulation algorithm based on the integrated general model of a complex multistate system reliability, linking its reliability model and its operation process model and considering variable at different operation states its reliability structure and its components reliability parameters is applied to the reliability evaluation of an exemplary system. Next the results of this simulation method application are illustrated and compared with the results obtained by the analytical method.

1. Introduction

Many technical systems belong to the class of complex critical infrastructure systems as a result of the large number of interacting components and subsystems they are built of and their complicated operating processes having significant influence on their safety. Considering the complexity of failure processes of real technical systems it seems reasonable to expand the two-state approach to multi-state approach in the system reliability analysis. The assumption that the system is composed of multistate components with reliability states degrading in time gives the possibility for more precise analysis of its reliability [3], [5]. This approach may be successfully applied analytically to a very wide class of real ageing technical systems and particularly for instance to reliability analysis and prediction of an exemplary system [4]. The reliability analysis of a system subjected to varying in time its operation process very often leads to complicated calculations and therefore it is difficult to implement analytical modelling, prediction and optimization. Especially it happens in the case when we assume the system multistate reliability model and the multistate model of its operation process. On the other hand, the complexity of the systems' operation processes and their influence on changing in time the systems' reliability parameters are very often met in real practice. Thus, the practical importance and need of an approach linking the system reliability model and the system operation process model into an integrated general model in reliability assessment of real technical systems is evident. Sometimes, the analytical determination of reliability characteristics of a complex multistate system leads to complicated formulae and therefore it is difficult to implement reliability modeling and prediction using this way. The Monte Carlo simulation method is a tool that often allows to simplify solving this problem [1], [6], [8].

In the paper, the general analytical approach to complex systems reliability analysis is shortly presented and next the Monte Carlo method based on this model is practically applied to examine the reliability of an exemplary system considered in [4] and its main reliability characteristics are found.

2. System operation process modelling

We assume that a system during its operation at the fixed moment t, $t \in \langle 0,+\infty \rangle$, may be at one of ν, $\nu \in N$, different operations states z_b, $b = 1,2,...,\nu$. Consequently, we mark by $Z(t)$, $t \in \langle 0,+\infty \rangle$, the system operation process, that is a function of a continuous variable t, taking discrete values at the set $\{z_1, z_2,...,z_\nu\}$ of the system operation states. We assume a semi-Markov model [2]-[3] of the system operation process $Z(t)$ and we mark by θ_{bl} its random conditional sojourn times at the operation states z_b, when its next operation state is z_l, $b, l = 1,2,...,\nu$, $b \neq l$.

Consequently, the operation process may be described by the following parameters:

- the vector of the initial probabilities of the system operation process $Z(t)$ staying at the particular operations states at the moment $t = 0$

$$[p_b(0)]_{1\times\nu} = [p_1(0), p_2(0),..., p_\nu(0)], \qquad (1)$$

where

$$p_b(0) = P(Z(0) = z_b),\ b = 1,2,...,\nu; \qquad (2)$$

- the matrix of the probabilities of the system operation process $Z(t)$ transitions between the operation states z_b and z_l, $b, l = 1,2,...,\nu$, $b \neq l$

$$[p_{bl}]_{\nu\times\nu} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1\nu} \\ p_{21} & p_{22} & \cdots & p_{2\nu} \\ \vdots & \vdots & \ddots & \vdots \\ p_{\nu1} & p_{\nu2} & \cdots & p_{\nu\nu} \end{bmatrix} \qquad (3)$$

where $p_{bb} = 0$ for $b = 1,2,...,\nu$;

- the matrix of the conditional distribution functions of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states

$$[H_{bl}(t)]_{\nu\times\nu} = \begin{bmatrix} H_{11}(t) & H_{12}(t) & \cdots & H_{1\nu}(t) \\ H_{21}(t) & H_{22}(t) & \cdots & H_{2\nu}(t) \\ \vdots & \vdots & \ddots & \vdots \\ H_{\nu1}(t) & H_{\nu2}(t) & \cdots & H_{\nu\nu}(t) \end{bmatrix} \qquad (4)$$

where

$$H_{bl}(t) = P(\theta_{bl} < t),\ H_{bb}(t) = 0, \qquad (5)$$

for $b, l = 1,2,...,\nu$, $b \neq l$.

3. Reliability of multistate system

In the multistate reliability analysis to define a system composed of n, $n \in N$ ageing components we assume that:

– E_i, $i = 1,2,...,n$, are components of a system,
– all components and a system under consideration have the set of reliability states $\{0,1,...,z\}$, $z \geq 1$,
– the reliability states are ordered, the state 0 is the worst and the state z is the best,
– the component and the system reliability states degrade with time t,
– $T_i(u)$, $i = 1,2,...,n$, $n \in N$, are independent random variables representing the lifetimes of components E_i in the reliability state subset $\{u, u + 1,...,z\}$, while they were in the reliability state z at the moment $t = 0$,
– $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset $\{u, u + 1,...,z\}$, while it was in the reliability state z at the moment $t = 0$,
– $s_i(t)$ is a component E_i reliability state at the moment t, $t \in \langle 0,\infty)$, given that it was in the reliability state z at the moment $t = 0$,
– $s(t)$ is the system reliability state at the moment t, $t \in \langle 0,\infty)$, given that it was in the reliability state z at the moment $t = 0$.

The above assumptions mean that the reliability states of the ageing system and components may be changed in time only from better to worse.

Definition 1. A vector

$$R_i(t,\cdot) = [R_i(t,0), R_i(t,1), \ldots, R_i(t,z)], \qquad (6)$$

for $t \in \langle 0,\infty)$, $i = 1,2,...,n$, where

$$R_i(t,u) = P(s_i(t) \geq u \mid s_i(0) = z) = P(T_i(u) > t) \qquad (7)$$

for $t \in \langle 0,\infty), u = 0,1,\ldots,z,$ is the probability that the component E_i is in the reliability state subset $\{u, u+1,...,z\}$ at the moment t, $t \in \langle 0,\infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multistate reliability function of a component E_i.

Definition 2. A vector

$$\boldsymbol{R}(t,\cdot) = [\boldsymbol{R}(t,0), \boldsymbol{S}(t,1), \ldots, \boldsymbol{R}(t,z)],\ t \in \langle 0,\infty), \qquad (8)$$

where

$$\boldsymbol{R}(t,u) = P(s(t) \ge u \mid s(0) = z) = P(T(u) > t) \quad (9)$$

for $t \in \langle 0,\infty)$, $u = 0,1,...,z$, is the probability that the system is in the reliability state subset $\{u,u + 1,...,z\}$ at the moment t, $t \in \langle 0,\infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multistate reliability function of a system.

Now, after introducing the notion of the multistate reliability analysis, we may define basic multistate reliability structures.

Definition 3. A multistate system is called series if its lifetime $T(u)$ in the reliability state subset $\{u,u + 1,...,z\}$ is given by

$$T(u) = \min_{1 \le i \le n}\{T_i(u)\},\ u = 1,2,...,z. \quad (10)$$

Definition 4. A multistate system is called series-parallel if its lifetime $T(u)$ in the reliability state subset $\{u,u + 1,...,z\}$ is given by

$$T(u) = \max_{1 \le i \le k}\{\min_{1 \le j \le l_i}\{T_{ij}(u)\}\},\ u = 1,2,...,z, \quad (11)$$

where k is the number of series subsystems linked in parallel and l_i is the number of components in the i^{th} series subsystem.

Definition 5. A multistate system is called series-"m out of k" system if its lifetime $T(u)$ in the reliability state subset $\{u,u+1,...,z\}$ is given by

$$T(u) = T_{(k-m+1)}(u),\ m = 1,2,...,k,\ u = 1,2,...,z, \quad (12)$$

where $T_{(k-m+1)}(u)$ is the $(k - m + 1)^{th}$ order statistic in the set of random variables

$$T_i(u) = \min_{1 \le j \le l_i}\{T_{ij}(u)\},\ i = 1,2,...,k,\ u = 1,2,...,z,$$

where l_i is the number of components in the i^{th} series subsystem.

4. Reliability of multistate system at variable operation conditions

We assume that every operation state of the system operation process $Z(t)$, $t \in \langle 0,\infty)$, described in Section 2, have an influence on the system reliability [3], [5]. Therefore, the system component's reliability at the particular operation state z_b, $b = 1,2,...,\nu$, can be described using the conditional reliability function

$$[\boldsymbol{R}_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t \mid Z(t) = z_b), \quad (13)$$

for $t \in \langle 0,\infty)$, $b = 1,2,...,\nu$, that is the conditional probability that the system component's conditional lifetime $T_i^{(b)}(u)$ is greater than t, while the system operation process $Z(t)$ is at the operation state z_b, $b = 1,2,...,\nu$ [3].

Further, we denote the unconditional reliability function of the system by

$$\boldsymbol{R}(t,u) = P(T(u) > t),\ t \in \langle 0,\infty), \quad (14)$$

where $T(u)$ is the system unconditional lifetime in the reliability state subset $\{u,u + 1,...,z\}$.

5. Monte Carlo approach to the system operation process and reliability evaluation

We denote by $z_b(q)$, $b = 1,2,...,\nu$, the realization of the system operation process initial operation state at the moment $t = 0$ generated from the distribution $[p_b(0)]_{1\times\nu}$. This realization is generated according to the formula

$$z_b(q) = \begin{cases} z_1, & 0 \le q < p_1(0), \\ z_2, & p_1(0) \le q < p_1(0) + p_2(0), \\ \vdots & \vdots \\ z_\nu, & \sum_{i=1}^{\nu-1} p_i(0) \le q \le 1, \end{cases} \quad (15)$$

where q is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$.

We denote by $z_{bl}(g)$, $l = 1,2,...,\nu$, $b \ne l$, the sequence of the realizations of the system operation process consecutive operation states generated from the distribution defined by $[p_{bl}]_{\nu\times\nu}$. Those realizations are generated according to the formula

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < p_{12}, \\ z_3, & p_{12} \le g < p_{12} + p_{13}, \\ \vdots & \vdots \\ z_\nu, & \sum_{i=1}^{\nu-1} p_{bi} \le g \le 1, \end{cases} \quad (16)$$

$$z_{bl}(g) = \begin{cases} z_1, & 0 \le g < p_{b1}, \\ \vdots & \vdots \\ z_{b-1}, & \sum_{i=1}^{b-2} p_{bi} \le g < \sum_{i=1}^{b-1} p_{bi}, \\ z_{b+1}, & \sum_{i=1}^{b-1} p_{bi} \le g < \sum_{i=1}^{b+1} p_{bi}, \\ \vdots & \vdots \\ z_v, & \sum_{i=1}^{v-1} p_{bi} \le g \le 1, \end{cases} \quad (17)$$

for $b = 2, 3, ..., v$,

$$z_{vl}(g) = \begin{cases} z_1, & 0 \le g < p_{v1}, \\ z_2, & p_{v1} \le g < p_{v1} + p_{v2} \\ \vdots & \vdots \\ z_{v-1}, & \sum_{i=1}^{v-2} p_{vi} \le g \le 1, \end{cases} \quad (18)$$

where g is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$.

We denote by $\theta_{bl}^{(i)}$, $b, l = 1,2,...,v,$, $i = 1,2,...,n_{bl}$, $b \ne l$, the realizations of the conditional sojourn time θ_{bl} of the system operation process generated from the distribution H_{bl}, where n_{bl} is the number of those sojourn time realizations during the experiment time $\tilde{\theta}$. The realizations are generated according to the formulae

$$\theta_{bl} = H_{bl}^{-1}(h), \; b, l = 1,2,...,v, b \ne l, \quad (19)$$

where $H_{bl}^{-1}(h)$ is the inverse function of the distribution function $H_{bl}(t)$ and h is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$, which in the case of exponential distribution

$$H_{bl}(t) = 1 - \exp[-\alpha_{bl} t], \; t \in \langle 0, \infty), \quad (20)$$

takes the following form

$$\theta_{bl} = -\frac{1}{\alpha_{bl}} \ln(1-h), \; b, l = 1,2,...,v, b \ne l. \quad (21)$$

5.1. Monte Carlo evaluation of the exemplary system operation process

The simulation is performed according to data given in [4]. The first step is to select the initial operation state $z_b(q)$, $b = 1, 2, 3, 4,$ at the moment $t = 0$, using formula (15), which is given by

$$z_b(q) = \begin{cases} z_1, & 0 \le q < 0.214 \\ z_2, & 0.214 \le q < 0.252 \\ z_3, & 0.252 \le q < 0.545 \\ z_4, & 0.545 \le q \le 1, \end{cases}$$

where q is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$. The next operation state z_l, $l = 1,2,3,4,$ is generated according to (16)-(18), from $z_{bl}(g)$, $b = 1,2,3,4,$ defined as

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < 0.22 \\ z_3, & 0.22 \le g < 0.54 \\ z_4, & 0.54 \le g \le 1, \end{cases}$$

$$z_{2l}(g) = \begin{cases} z_1, & 0 \le g < 0.2 \\ z_3, & 0.2 \le g < 0.5 \\ z_4, & 0.5 \le g \le 1, \end{cases}$$

$$z_{3l}(g) = \begin{cases} z_1, & 0 \le g < 0.12 \\ z_2, & 0.12 \le g < 0.28 \\ z_4, & 0.28 \le g \le 1, \end{cases}$$

$$z_{4l}(g) = \begin{cases} z_1, & 0 \le g < 0.48 \\ z_2, & 0.48 \le g < 0.7 \\ z_3, & 0.7 \le g \le 1. \end{cases}$$

For instance, if $z_b(g) = z_1$, then the next operation state would be z_2, z_3 or z_4 generated from $z_{1l}(g)$. Applying (19), the realizations of the empirical conditional sojourn times are generated according to the formulae

$$\theta_{12}(h) = -192\ln[1-h], \quad \theta_{13}(h) = -480\ln[1-h],$$

$$\theta_{14}(h) = -200\ln[1-h], \quad \theta_{21}(h) = -96\ln[1-h],$$

$$\theta_{23}(h) = -81\ln[1-h], \quad \theta_{24}(h) = -55\ln[1-h],$$

$$\theta_{31}(h) = -870\ln[1-h], \quad \theta_{32}(h) = -480\ln[1-h],$$

$$\theta_{34}(h) = -300\ln[1-h], \quad \theta_{41}(h) = -325\ln[1-h],$$

$$\theta_{42}(h) = -510\ln[1-h], \; \theta_{43}(h) = -438\ln[1-h], \quad (22)$$

where h is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$.

5.2. Monte Carlo approach to the exemplary system reliability modelling

The Monte Carlo simulation method uses a computational procedure and can provide the fairly accurate results in a relatively small amount of time [6], [8]. Obviously, the accuracy of the proposed Monte Carlo simulation method depends on the number of iterations. The is performed with N = 1 000 000 runs.

We can apply the Monte Carlo simulation method, according to the scheme presented in *Figure 1*.

At the beginning, we fix the following parameters:

- the number $N \in \boldsymbol{N} \setminus \{0\}$ of iterations (runs of the simulation) equal to the number of the system lifetime realizations;
- the function generating initial operation state $z_b(q), b = 1, 2, 3, 4,$ at the moment $t = 0,$ defined by (15);
- the functions generating next operation state $z_{bl}(g),\ b = 1,2,3,4,$ defined by (16)-(18),
- the matrix $[\alpha_{bl}], \alpha_{bl} \in \langle 0,\infty), b,l = 1,2,3,4, b \neq l$, of the intensities of the system operation process transitions between the operation states existing in [4];
- the system reliability parameters $\left[\lambda_{ij}^{(\upsilon)}(u)\right]^{(b)}$, $b,l = 1,2,3,4, b \neq l$, $i = 1,2,...,i^{(\upsilon)}$, $j = 1,2,..., j^{(\upsilon)}$, $u = 1,2,3$, $\upsilon = 1,2$, according to the *Table 1* in [4].

We declare the conditional sojourn times formulae (21) and the system component's lifetimes exponential sampling formula

$$[t_{ij}^{(\upsilon)}(u)]_n^{(b)} = -\frac{\ln[1-\left[f_{ij}^{(\upsilon)}(u)\right]^{(b)}]}{\left[\lambda_{ij}^{(\upsilon)}(u)\right]^{(b)}}, \tag{23}$$

where $\left[\lambda_{ij}^{(\upsilon)}(u)\right]^{(b)}$, are given according to the *Table 1* in [4] and $\left[f_{ij}^{(\upsilon)}(u)\right]^{(b)}$, $b,l = 1,2,3,4, b \neq l$, $i = 1,2,...,i^{(\upsilon)}$, $j = 1,2,..., j^{(\upsilon)}$, $u = 1,2,3$, $\upsilon = 1,2$, is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$.

In the next step we introduce:

- $n \in \{1,2,...,N\}$, as the subsequent iteration of the simulation and set $n = 1$;
- system component lifetimes exponential sampling formula $[t_{ij}^{(\upsilon)}(u)]_n^{(b)}$, according to (22) and set $[t_{ij}^{(\upsilon)}(u)]_n^{(b)} = 0$;
- the sum $\hat{\theta}$ of the realizations $\theta_{bl}^{(\kappa)}$, $\kappa = 1,2....,$ of the empirical conditional sojourn times and set $\hat{\theta} = 0$;
- $u \in \{1,2,...,z\}$, as the subsequent iteration in the loop and set $z = 3$;

As the algorithm progresses, we draw a random number q from the uniform distribution on the interval $\langle 0,1 \rangle$. Based on this random value, the realization

$$z_b(q),\ b = 1,2,3,4,$$

of the system operation process initial operation state at the moment $t = 0$ is generated according to the formula (15).

Next, we draw a random number g uniformly distributed on the unit interval. Concerning this random value, the realization

$$z_l(g),\ l = 1,2,3,4, l \neq b,$$

of the system operation process consecutive operation state is generated according to the formula (16).

Further, we generate a random number h from the uniform distribution on the interval $\langle 0,1 \rangle$, which we put into the formula (21) obtaining the realisation $\theta_{bl}^{(\kappa)},\ b,l = 1,2,3,4,\ b \neq l,\ \kappa = 1.$ Subsequently, for a particular initial operation state z_b, $b,l = 1,2,3,4,$ $b \neq l$, we draw a random number $\left[f_{ij}^{(\upsilon)}\right]^{(b)}$, $i = 1$, $j = 1$, $\upsilon = 1$, from the uniform distribution on the interval $\langle 0,1 \rangle$. Based on this random value, the realization $[t_{ij}^{(\upsilon)}(u)]_n^{(b)}$, $i = 1$, $j = 1$, $n = 1$, $u = z$, $\upsilon = 1$, of the considered system component lifetime realization is generated according the formula (22). We generate another random numbers $\left[f_{ij}^{(\upsilon)}\right]^{(b)}$, $i = 1,2,...,i^{(\upsilon)}$, $j = 1,2,..., j^{(\upsilon)}$, $\upsilon = 1,2$, from the uniform distribution on the interval $\langle 0,1 \rangle$ obtaining the realizations $[t_{ij}^{(\upsilon)}(u)]_n^{(b)}$, $i = 1,2,...,i^{(\upsilon)}$, $j = 1,2,..., j^{(\upsilon)}$, $n = 1$, $u = 1,2,3$, $\upsilon = 1,2$.

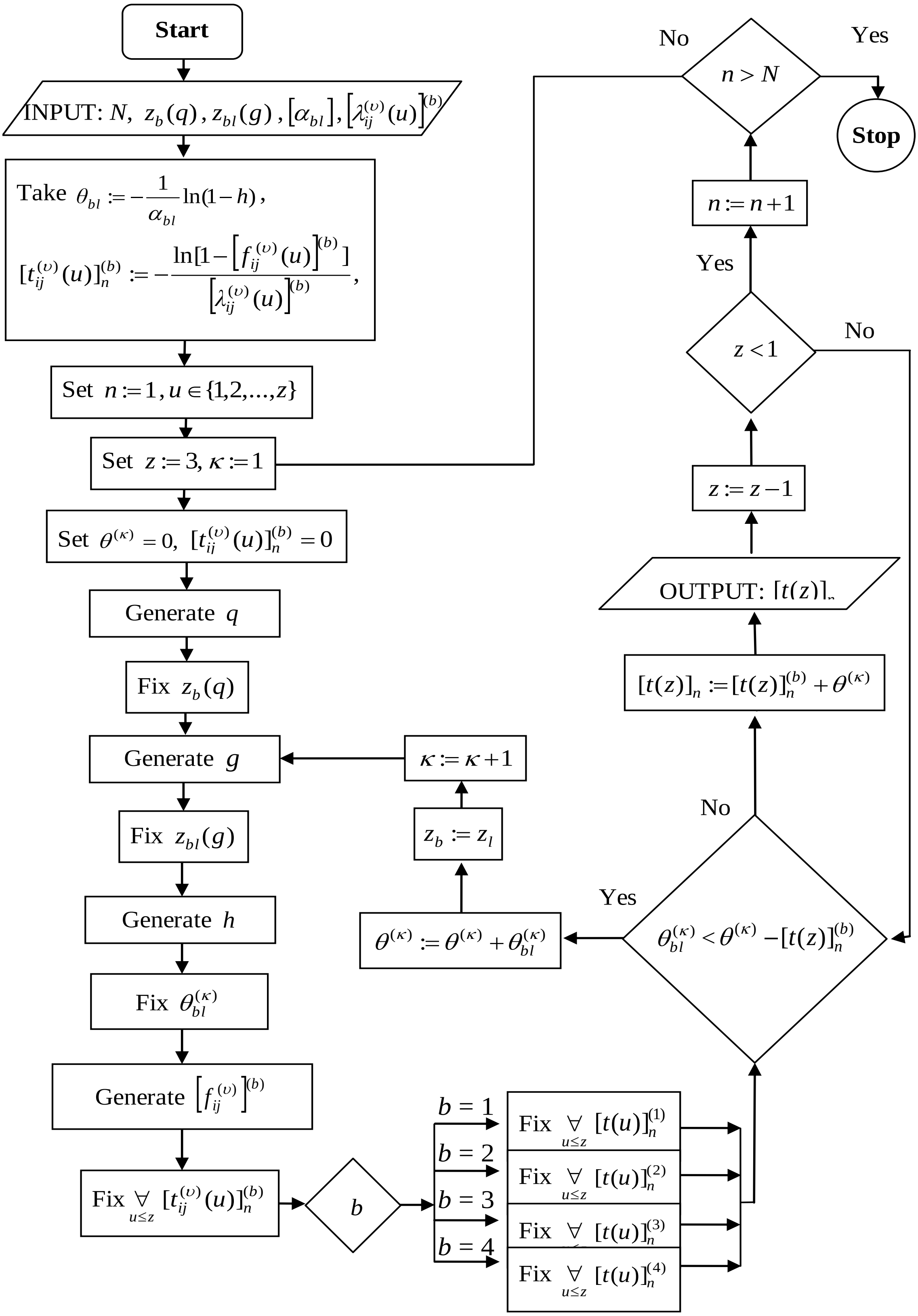

Figure 1. Monte Carlo algorithm for the exemplary system reliability evaluation

The realizations $[t^{(\upsilon)}(u)]_n^{(b)}$, $n = 1$, $u = z$, $\upsilon = 1,2$, of the system lifetime $[T^{(\upsilon)}(u)]_n^{(b)}$ in the reliability state subsets $\{u, u+1, ..., z\}$, $z = 3$, depend on the realizations $[t_{ij}^{(\upsilon)}(u)]_n^{(b)}$ given by (22) of the system component lifetimes $[T_{ij}^{(\upsilon)}(u)]_n^{(b)}$, $i = 1,2,...,i^{(\upsilon)}$, $j = 1,2,..., j^{(\upsilon)}$, $n = 1,2,...,N$, $u = z$, $\upsilon = 1,2$, and are calculated from the expression

$$[t(u)]_n = t([t_{ij}^{(\upsilon)}(u)]_n^{(b)}; b, l = 1,2,3,4,\ b \neq l,$$
$$i = 1,2,...,i^{(\upsilon)}, j = 1,2,..., j^{(\upsilon)}),$$
$$n = 1,2,...,N,\ u = z,$$

taking suitable explicit form dependent on the system structure according to (10)-(12):

$$[t(u)]_n^{(1)} = \max_{1 \le i \le 2}\left\{\min_{1 \le j \le 3}\left\{[t_{ij}^{(1)}(u)]_n^{(1)}\right\}\right\},$$

$$[t(u)]_n^{(2)} = \max_{1 \le i \le 4}\left\{\min_{1 \le j \le 3}\left\{[t_{ij}^{(2)}(u)]_n^{(2)}\right\}\right\},$$

$$[t(u)]_n^{(3)} = \min\left\{\max_{1 \le i \le 2}\left\{\min_{1 \le j \le 3}\left\{[t_{ij}^{(1)}(u)]_n^{(3)}\right\}\right\},\ \max_{1 \le i \le 4}\left\{\min_{1 \le j \le 3}\left\{[t_{ij}^{(2)}(u)]_n^{(3)}\right\}\right\}\right\},$$

$$[t(u)]_n^{(4)} = \min\left\{\max_{1 \le i \le 2}\left\{\min_{1 \le j \le 3}\left\{[t_{ij}^{(1)}(u)]_n^{(4)}\right\}\right\},\ [t^{(2)}{}_{(3)}(u)]_n^{(4)}\right\},$$

where $[t^{(2)}{}_{(3)}(u)]_n^{(4)}$ is the realization of the 3rd order statistic in the set of random variables $T_i(u)$ realizations

$$t_i(u) = \min_{1 \le j \le 2}\{t_{ij}^{(2)}(u)\},\ i = 1,2,3,4,$$

for $n = 1,2,...,N$, $u = z$.

If the realization of the empirical conditional sojourn time $\theta_{bl}^{(\kappa)}$, $b, l = 1,2,3,4$, $b \neq l$, $\kappa = 1$, is not greater than the realization of the difference between system conditional lifetime $[t(z)]_n^{(b)}$ and $\theta^{(\kappa)}$, we add to $\theta^{(\kappa)}$ the value $\theta_{bl}^{(\kappa)}$. The realization $[t(z)]_n$ is recorded, z_l is set as the initial operation state and we increase the value of κ. Otherwise, if the realization of the empirical conditional sojourn time $\theta_{bl}^{(\kappa)}$, $b, l = 1,2,3,4$, $b \neq l$, $\kappa = 1$, is greater than the realization of the difference between system conditional lifetime $[t(z)]_n^{(b)}$ and $\theta^{(\kappa)}$, we add to the system unconditional lifetime $[t(z)]_n$ the value $[t(z)]_n^{(b)}$, $b, l = 1,2,3,4$, $b \neq l$, $n = 1,2,...,N$. and record the realization $[t(z)]_n$. If the value of z is positive, we repeat the comparison for z – 1. each time, after completing all the steps in the loop, we record the rest realizations $[t(u)]_n$, for $u = 1,2,\ldots,z - 1$. Thus, if the value of z is negative, we can proceed replacing n with $n + 1$ and shift into the next iteration in the loop if $n < N$. In the other case, we stop the procedure.

The procedure of Monte Carlo simulation is performed with N = 1 000 000 runs.

The approximate mean value of the system lifetime is calculated as an arithmetic mean of all system lifetime realizations for N iterations, i.e.

$$\mu_s(u) = \frac{1}{N}\sum_{n=1}^{N} t_n(u), \tag{24}$$

where $u = 1,2,3$.

The approximate system lifetimes standard deviation is calculated as a square root of the average squared deviation from the mean value (24), i.e.

$$\sigma_s(u) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(t_k^{(\upsilon)}(u) - \mu_s(u)\right)^2}, \tag{25}$$

where $u = 1,2,3$, $\upsilon = 1,2,\ldots,5$.

The histograms of the exemplary system lifetimes in the particular reliability state subsets are illustrated in *Figure 2*. It can be seen that their shapes are similar to the shapes of the Weibull density functions.

The expected values of the system lifetimes in the reliability states subsets {1,2,3}, {2,3}, {3}, according to (24) respectively are

$$\mu_s(1) \cong 391.296,\ \mu_s(2) \cong 369.757,$$

$$\mu_s(3) \cong 351.403. \tag{26}$$

The expected values and standard deviations of the system lifetimes in the reliability states subsets {1,2,3}, {2,3}, {3}, according to (25) respectively are given as follows

$$\sigma_s(1) = 263.137, \ \sigma_s(2) = 250.955,$$

$$\sigma_s(3) = 240.592. \qquad (27)$$

The system lifetimes in the particular reliability states 1, 2, 3, respectively are [3]:

$$\bar{\mu}_s(1) \cong 21.539, \ \bar{\mu}_s(2) \cong 18.354,$$

$$\bar{\mu}_s(3) \cong 351.403. \qquad (28)$$

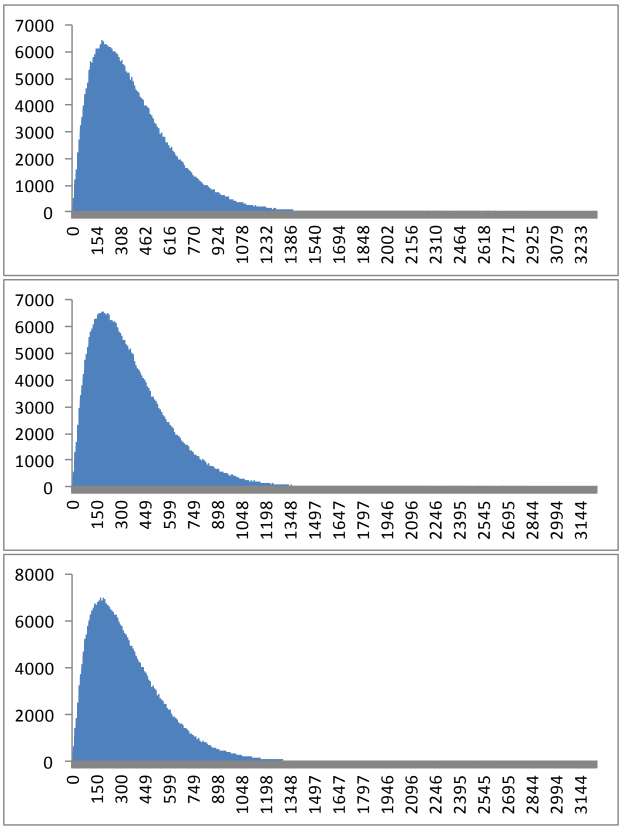

Figure 2. . The graph of the histograms of the exemplary system lifetimes in reliability state subsets {1,2,3}, {2,3}, {3}

6. Conclusions

The Monte Carlo simulation method was applied to the approximate evaluation of the maritime ferry technical system reliability main characteristics. The predicted approximate values of those characteristics given by (26)-(28) differ not much from their exact values determined by (50)-(52) in [4] for this system by applying the analytical method. This fact justifies a good accuracy of the Monte Carlo simulation method and suggests that this method can be applied instead of the analytical approach to reliability evaluation of this system applied in [4]. The sensibility of Monte Carlo simulation method application to other more complex systems reliability modeling and prediction is also reasonable.

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