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OPTIMIZATION OF A DIRECTIONAL BOREHOLE TRAJECTORY AS THE CRITERION OF MINIMUM COST OF PERFORMANCE

OPTYMALIZACJA TRAJEKTORII OTWORU KIERUNKOWEGO DLA KRYTERIUM MINIMALIZACJI KOSZTU JEGO WYKONANIA

New drilling technical and technological solutions enhance the development of directional drilling, enabling better control of direction of drilling and real-time 3D information about the position of the bit in the borehole rendered by the control and measurement apparatuses. The 2D and 3D method of designing the trajectory of a borehole axis has been modified in this paper. In both cases the minimum cost of drilling of a borehole in an interval from the beginning of its deflecting (KOP) up to the ultimate destination point was assumed to be the optimization criterion.

Another assumption says that the admissible dogleg will never be exceeded over the entire interval of the borehole axis.

Commonly more and more boreholes are performed from one place or one drilling platform. Such drilling is aimed at, e.g. developing unconventional deposits of natural gas or improving the depletion factor by injecting water or CO_2. In such cases the boreholes are located densely, which may result in their colliding. Hence, attention was drawn to the fact that in such cases inaccuracies of apparatuses measuring the angle of deflection of the borehole, azimuth, length of the borehole and formation of error ellipsoids around a given point should be accounted for. The assumed method of positioning the borehole axis is also important. For a considerable length of the borehole axis the calculations are simple but time-consuming, therefore specialist computer programs are recommended. The trajectories of directional boreholes should be designed taking into account the position of the neighboring boreholes and inaccuracy of measuring equipment, as in this way the level of risk of potential collisions of boreholes axes can be determined.

Keywords: directional boreholes, trajectories, cost of drilling, colliding borehole axes

Wprowadzanie najnowszych rozwiązań technicznych i technologicznych do wiertnictwa sprzyja rozwojowi wierceń kierunkowych. Jest to możliwe dzięki szerokim możliwościom sterowania kierunkiem wiercenia i aparaturze kontrolno-pomiarowej mogącej w czasie rzeczywistym przekazywać informacje o położeniu świdra w przestrzeni trójwymiarowej. W artykule zmodyfikowano projektowanie w przestrzeni dwu i trójwymiarowej trajektorii osi otworu kierunkowego. Założono, że kryterium optymalizacji

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w obu przypadkach jest minimum kosztu wykonania otworu w interwale od początku jego kierunkowania (KOP) aż do końcowego celu.

Przyjęto także ograniczenie mówiące o tym, że na całym analizowanym interwale osi otworu nie zostanie przekroczona dopuszczalna krzywizna.

W praktyce wiertniczej obserwuje się coraz to więcej otworów wykonywanych z jednego placu lub z jednej platformy wiertniczej. Przykładowo są to wiercenia mające na celu udostępniania złóż gazu niekonwencjonalnego lub poprawienie współczynnika sczerpania złóż poprzez zatłaczanie do nich wody lub CO_2. W takich przypadkach zazwyczaj zwiększa się zagęszczenie otworów, co może w niekorzystnych przypadkach prowadzić do ich kolizji. Z tego powodu zwrócono uwagę, że w takich sytuacjach wskazane jest uwzględnienie niedokładności przyrządów mierzących kąt odchylenia osi otworu i jej azymut oraz długość otworu i tworzenie wokół danego punktu pomiarowego elipsoidy błędu. Tutaj istotna jest także przyjęta metodyka określania przestrzennego położenia osi otworu. Dla znacznej długości osi otworu obliczenia są proste lecz bardzo czasochłonne i dlatego warto skorzystać z programów komputerowych. Zatem zaleca się projektować trajektorie osi otworów kierunkowych z uwzględnieniem położenia sąsiednich otworów i niedokładności przyrządów pomiarowych, co umożliwia określenie poziomu ryzyka w zakresie potencjalnych kolizji osi otworów.

Słowa kluczowe: otwory kierunkowe, trajektorie, koszty wiercenia, kolizje osi otworów

Introduction

The development of modern directional drilling techniques also opens possibilities for directional drilling control, mainly the change of the angle of angle of deflection, azimuth and intensity of spatial deflection (Mitchell & Miska, 2011). They are more and more frequently used by, e.g. geologists who select a few points on one trajectory of a directional borehole axis. This usually results in a more complex trajectory and additional complications during realization of the borehole.

Recently an intense development of shale gas development and exploitation has been observed in the World. One of the most important components influencing the profitability of this type of production is the cost of performing numerous boreholes and their fracturing. By drilling a few or a dozen of boreholes in one site considerably reduces the cost of preparation of the area and considerably reduces the environmental impact of drilling operations. This solution, however, elongates the summaric length of drilling, and brings about a risk of collision between the neighboring boreholes. Therefore the design of borehole trajectories should be based on the criterion of optimized cost of drilling, and in view of technical limitations of drilling process, assumed depth of initial dogleg (KOP), target point and axes of neighboring boreholes.

Methods of designing the trajectory of directional borehole axis

A number of methods of designing the trajectory of a directional borehole axis exist in the World (Mitchell & Miska, 2011). They arbitrarily assume a definite type of trajectory, e.g. „J" or „S" type (Gonet, 1987). Other methods account for strength parameters of the string (Miska S. & Miska W., 2006), e.g. criterion of minimum dogleg curvature. However, more attention is paid to the cost of drilling as it significantly affects the economic aspect of the undertaking, and influences further decisions on, e.g. where or not to prospect and develop shale gas fields.

In this situation the author proposes a method of designing a directional borehole trajectory based on the criterion of the cost of drilling, casing and sealing the casing (Gonet, 1987). It also addresses the state of the directional drilling techniques and technologies, i.e. also indicates the most economic borehole profile.

Objective function F_c was written as:

$$F_c = \sum_{j=1}^{m} K_j \cdot l_j \Rightarrow \text{minimum} \tag{1}$$

where:

K_j — cost of drilling rocks, casing and sealing 1 m long casing in the j-th uniform price layer in the planned borehole,

l_j — length of borehole axis in the j-th uniform price layer,

m — number of uniform price layers.

Under the notion of „uniform price layer" we understand layers, for which the total cost of drilling, casing and sealing the casing per 1 meter is the same or almost the same.

Therefore, to fully prepare the data, the uniform price layers should be determined in a directional profile of the borehole interval placed on a vertical plane crossing the initial point of deflection (KOP) and target point in its azimuth.

The boundaries between specific layers are connected with:

- depth at which the borehole has been cased,
- depth of sealing the casing,
- change of borehole diameter,
- change of drilling method (rotary, with downhole motor, drill or bit),
- change of macroscopically homogeneous rocks (Miska, 1979; Wojtanowicz, 1975).

Each layer is ascribed a unit cost of drilling, casing and sealing. The dip of layers is also specified individually for each layer (Gonet, 1981). If the layers are parallel, then the directions of their dip are equal (Fig. 1) and the calculations are shorter than for non-parallel layers. In the latter case the optimum profile is determined by the successive approximation method taking into account the spatial location of layers.

For the sake of generalization, the first vertical section of the borehole has not been considered. Therefore, analyses will start for the profile of the directional borehole axis from the point where the first interval with increased angle of deflection is drilled with the deflecting tool (Matanović, 2007; Mitchell & Miska, 2011; Pinka et al., 2007) to the final point of the directional borehole.

Two orthogonal Cartesian coordinates systems with a common point in the origin of the system (beginning of deflection of the borehole from vertical) have been presented in fig. 1.

OX axis of the first coordinates system is vertical and downwardly oriented, whereas OY axis is horizontal. Ox axis of the other coordinates system is perpendicular to the boundary of the first and the second layer. This signifies that the other coordinates system was rotated by the directional dip of layers δ. The coordinates of the bottom of the directional borehole P_2 corresponding to the depth of deflected borehole H_o (vertically) and to deflection A_p (horizontally) in the other coordinates system should be calculated from the formulae:

$$H = H_o \cos\delta + A_p \sin\delta \tag{2}$$

$$A = H_o \sin\delta + A_p \cos\delta \tag{3}$$

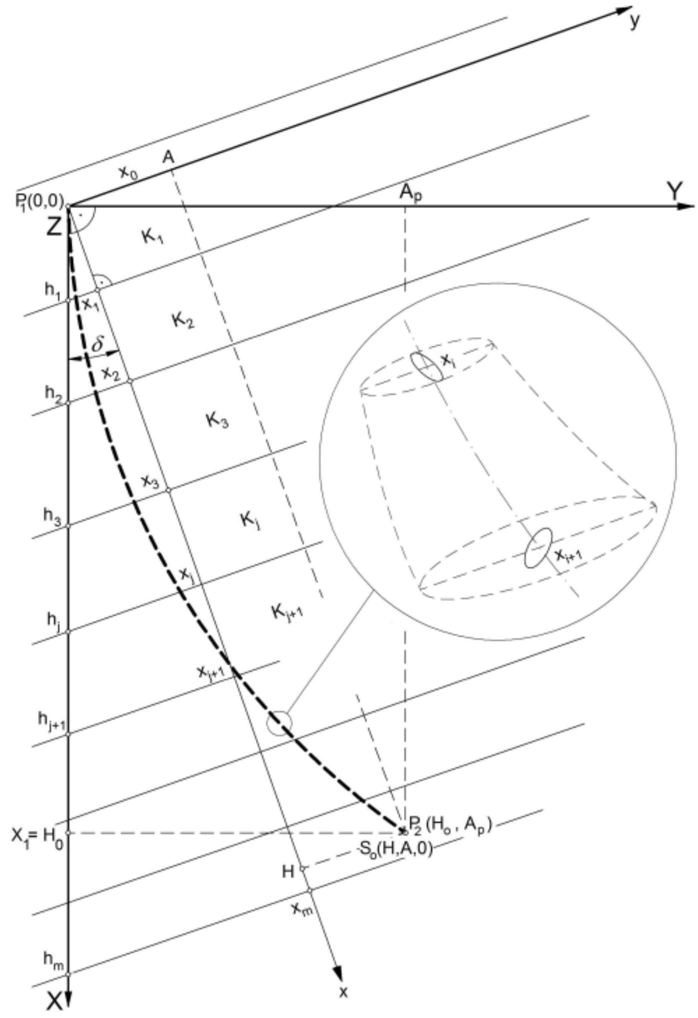

Fig. 1. Scheme of trajectory of a directional borehole

The coordinates of specific boundaries of layers measured in vertical h_j in the other coordinates system can be determined from the equation:

$$x_j = h_j \cos\delta \qquad (i = 1, 2, \ldots, m) \tag{4}$$

The trajectory of borehole axis described by function $y = y(x)$, which should link points $P_1(0, 0)$ and $S_0(H, A)$ and realize the objective function (1), is unknown. The analysis of various trajectories was followed by an assumption that it was a third order polynomial with unknowns a, b, c, d:

$$y = ax^3 + bx^2 + cx + d \tag{5}$$

Two factors were determined from the condition that curve (5) crosses the beginning of deflection P_1 and end of directional borehole S_0:

$$d = 0 \wedge a = \frac{A}{H^3} - \frac{b}{H} - \frac{c}{H^2} \tag{6}$$

After substituting them to equation (5) the final equation of borehole axis trajectory (5) takes the form:

$$y = \left(\frac{A}{H^3} - \frac{b}{H} - \frac{c}{H^2}\right) x^3 + bx^2 + cx \tag{7}$$

The length l_j of directional borehole trajectory between two neighboring boundaries x_j and x_{j-1} required for the objective function (1) is determined from the equation:

$$l_j = \int_{x_{j-1}}^{x_j} \sqrt{1 + y'^2}\, dx \tag{8}$$

At small values of the angle of deflection of borehole axis, the equation for the length of the trajectory between neighboring points can be simplified. Assuming that $|y'| < 1$ (angle of deflection of borehole axis) $\alpha \in \left(-\frac{\pi}{4} + \delta, \frac{\pi}{4} + \delta\right)$ and developing the integrand (8) into Maclaurin series and accounting for first two elements of the developed function, we get equation (8) in the form:

$$l_j = \int_{x_{j-1}}^{x_j} \left(1 + \frac{1}{2} y'^2\right) dx \tag{9}$$

After calculating the first derivative of function (7) and substituting it to equation (9) we have:

$$\begin{aligned} l_j = {} & \frac{0,9}{H^2}\left(\frac{A^2}{H^4} + b^2 + \frac{2bc}{H} + \frac{c^2 - 2Ab}{H^2} - \frac{2Ac}{H^3}\right)\Delta x_j^5 + \frac{1,5}{H}\left(\frac{Ab}{H^2} - b^2 - \frac{bc}{H}\right)\Delta x_j^4 + \\ & + \left(\frac{2b^2}{3} + \frac{Ac}{H^3} - \frac{bc}{H} - \frac{c^2}{H^2}\right)\Delta x_j^3 + bc\Delta x_j^2 + \left(1 + \frac{c^2}{2}\right)\Delta x_j^1 \end{aligned} \tag{10}$$

where:

$$\Delta x_j^r := x_j^r - x_{j-1}^r \qquad r = 1, 2, 3, 4, 5 \tag{11}$$

For angles of deflection of borehole axis not belonging to $\alpha \in \left(-\frac{\pi}{4} + \delta, \frac{\pi}{4} + \delta\right)$ it is recommended that the first derivative of function (7) were substituted to integral (8) and calculated. An alternative simplified solution lies in assuming coordinates of an indirect point at the trajectories and determining the trajectory on a shortened interval with its final angle of deflection. Then, the coordinates system should be rotated by the previously determined angle of deflection of borehole axis and the procedure of defining the trajectory repeated once or twice, depending on the target.

To begin drilling of the first dogleg interval is technically limited therefore a specific drilling method is selected for the planned stage of realization. It can be assumed that the initial angle of deflection of borehole axis α is known. After accounting for the directional dip of layers δ and initial angle of borehole deflection α for the assumed function (7) there was defined coefficient c, which was equal to the first derivative at the origin of the coordinates system (Fig. 1), and which has been written in the form:

$$y'(0) = c = \mathrm{tg}(\alpha - \delta) \tag{12}$$

For technological reasons the objective function should have additional limitations on keeping admissible values by doglegs k_d over the entire length of the planned borehole profile (Lubinski, 1961, 1987). The curvature k of the plot in 2D is frequently described with the formula:

$$k = \frac{|y''|}{(1 + y'^2)^{\frac{3}{2}}} \tag{13}$$

Having assumed that $(|y'| < 1)$, the equation (13) can be simplified:

$$k = |y''| \tag{14}$$

After calculating the second derivative of function (5) we have an equation for the curvature of borehole axis in the form:

$$k = |6ax + 2b| \tag{15}$$

The curvature of the analyzed plot should meet the following dependence:

$$|6ax + 2b| \leq k_d \tag{16}$$

For $x \in [0, H]$ the local maximum of borehole axis occurs at the end of the interval. Thus, the following limitations were obtained from inequality (16) and coordinates of points P_1 and S_0:

$$|2b| - k_d \leq 0 \tag{17}$$

$$\left| \frac{6A}{H^2} - 4b - \frac{6c}{H} \right| - k_d \leq 0 \tag{18}$$

For determining the extremum of function (1), the Lagrange function F_1 was written in the form:

$$\begin{aligned} F_1 = & \sum_{j=1}^{m} (K_{tj} \cdot l_j) + \lambda_1 (2b - k_d) + \lambda_2 (-2b - k_d) + \\ & + \lambda_3 \left(\frac{6A}{H^2} - 4b - \frac{6c}{H} - k_d \right) + \lambda_4 \left(-\frac{6A}{H^2} + 4b + \frac{6c}{H} - k_d \right) \end{aligned} \tag{19}$$

where: $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ Lagrange multipliers.

For finding a conditional minimum from the objective function (19), the following derivatives were calculated from (19) after accounting for (10) and (11):

$$\frac{\partial F_1}{\partial b} = bu_1 + u_2 + 2\lambda_1 - 2\lambda_2 - 4\lambda_3 + 4\lambda_4 \tag{20}$$

$$\lambda_1\left(\frac{\partial F_1}{\partial \lambda_1}\right) = \lambda_1(2b - k_d) \tag{21}$$

$$\lambda_2\left(\frac{\partial F_1}{\partial \lambda_2}\right) = \lambda_2(-2b - k_d) \tag{22}$$

$$\lambda_3\left(\frac{\partial F_1}{\partial \lambda_3}\right) = \lambda_3\left(\frac{6A}{H^2} - 4b - \frac{6c}{H} - k_d\right) \tag{23}$$

$$\lambda_4\left(\frac{\partial F_1}{\partial \lambda_4}\right) = \lambda_4\left(-\frac{6A}{H^2} + 4b + \frac{6c}{H} - k_d\right) \tag{24}$$

where:

$$u_1 = \frac{1{,}8}{H^2}\left[\sum_{j=1}^{m}\left(K_j \cdot \Delta x_j^5\right)\right] - \frac{3}{H}\left[\sum_{j=1}^{m}\left(K_j \cdot \Delta x_j^4\right)\right] + \frac{4}{3}\left[\sum_{j=1}^{m}\left(K_j \cdot \Delta x_j^3\right)\right] \tag{25}$$

$$\begin{aligned} u_2 = {} & \frac{1{,}8}{H^3}\left(c - \frac{A}{H}\right)\left[\sum_{j=1}^{m}\left(K_j \cdot \Delta x_j^5\right)\right] + \frac{1{,}5}{H^2}\left(\frac{A}{H} - c\right)\left[\sum_{j=1}^{m}\left(K_j \cdot \Delta x_j^4\right)\right] - \\ & - \frac{c}{H}\left[\sum_{j=1}^{m}\left(K_j \cdot \Delta x_j^3\right)\right] + c\left[\sum_{j=1}^{m}\left(K_j \cdot \Delta x_j^2\right)\right] \end{aligned} \tag{26}$$

It follows from the necessary condition for the extremum that the first partial derivatives should be equal to zero; then we get a system of five equations:

$$bu_1 + u_2 + 2\lambda_1 - 2\lambda_2 - 4\lambda_3 + 4\lambda_4 = 0 \tag{27}$$

$$\lambda_1(2b - k_d) = 0 \tag{28}$$

$$\lambda_2(-2b - k_d) = 0 \tag{29}$$

$$\lambda_3\left(\frac{6A}{H^2} - 4b - \frac{6c}{H} - k_d\right) = 0 \tag{30}$$

$$\lambda_4\left(-\frac{6A}{H^2} + 4b + \frac{6c}{H} - k_d\right) = 0 \tag{31}$$

By solving a system of equations (27) – (31) we determine b, λ_1, λ_2, λ_3, λ_4. Then the coefficient a is calculated from equation (6) and we obtain the equation of curve (5) which realizes the objective function (1). This result is obtained for a profile in 2D.

Sometimes a spatial change of the azimuth of borehole axis is observed in the drilling practice. Therefore the further part of the paper is additionally devoted to the intensity of deflection of borehole axis in 3D. Then the specific parameters of the profile are determined from the bottom of the borehole to the beginning of the dogleg. With the given step, the function (4) is calculated for coordinated of $S_i(x_i, y_i, z_i)$. Then two coordinates of points in the first coordinates system are defined with the following formulae:

$$X_i = x_i \cos\delta - y_i \sin\delta \tag{32}$$

$$Y_i = x_i \sin\delta + y_i \cos\delta \tag{33}$$

The growing lengths of the borehole trajectory l_i in 2D between two successive points at small distances should be treated as a diagonal of a rectangular triangle. They can be calculated from the formula:

$$l_i = \sqrt{(X_{i-1} - X_i)^2 + (Y_{i-1} - Y_i)^2} \tag{34}$$

Otherwise, depending on the required accuracy, they should be calculated from equation (8) or (9). In 3D (OZ axis in fig. 1) the azimuth of the directional borehole axis is changed. From the practical point of view, the assumed change of the azimuth should preferably be $\Delta\beta$ per 10 borehole meters and the third coordinate z_i at the section l_i should be determined from the formula:

$$z_i = -0{,}1 \cdot l_i \cdot \mathrm{tg}\,\Delta\beta \tag{35}$$

Thus, the actual length L_i of borehole axis in 3D between two successive neighboring points is treated as a diagonal of cubicoid and its value is approximately:

$$L_i = \sqrt{l_i^2 + z_i^2} \tag{36}$$

The third coordinate of point S_i of profile in the coordinates system XYZ equals to:

$$Z_i = z_{i-1} + z_i \tag{37}$$

The following parameters of a directional borehole axis can be defined for selected coordinates:

- angle of deflection of borehole axis:

$$\alpha_i = \mathrm{arctg}\left(3ax_i^2 + 2bx_i + c\right) + \delta \tag{38}$$

- azimuth of borehoole axis:

$$\beta_i = \beta_{i-1} - 0{,}1\Delta\beta L_i \tag{39}$$

– curvature of borehole axis:

$$k_1 = \frac{\left|6ax_1 + 2b\right|}{\sqrt{\left[1 + \left(3ax_i^2 + 2bx_i + c\right)^2\right]^3}} \tag{40}$$

It should be emphasized that by using equation (39) successively, one should reach point P_1 (Fig. 1), where the initial azimuth is determined, and following which the first dogleg interval should be drilled.

Anticollision analysis of boreholes

Commonly more and more boreholes are being performed from one site or one drilling platform, which may result in their colliding. To avoid this it is advisable to determine the trajectory of the borehole axis and the zone in which the borehole axes will be present. This is a result of measurements of the angle of deflection, azimuth and length of the borehole, given with a definite level of uncertainty. It should be assumed for those three variables that every measurement point of the borehole axis location in 3D may be present in the so-called error ellipsoid (Fig. 1), the parameters of which depend on the accuracy of the applied measurement equipment. The analysis should also encompass new methodics of spatial positioning of borehole axes (Landmark, 2008). The method of minimum curvature and average angles is most commonly used. Finally, a spatial zone should be formed around each axis, with the borehole inside. Having a few boreholes already performed in a given area and a new one ahead, it is recommended to determine the zone of potential borehole axis location around each of them. Then the so-called separation coefficient should be determined This is a quotient of a distance of centers of ellipsoids located on the borehole axes at the checked depth and the sum of distances resulting from the error of measurement of existing and planned location of borehole axis projected on a horizontal plane cutting borehole axes at the closest measurement points. If the obtained separation coefficient is less than zero, the collision of boreholes is possible in practice. To lower the risk of collision of the neighboring boreholes it is recommended that the borehole trajectories were re-designed using changed parameters so that a suitable separation coefficient is obtained. It should be emphasized that the higher is its value the lower is the risk of collision. It is also purposeful to regularly monitor the actual position the drilled borehole and check it out with the assumed trajectory.

The entire procedure was described, e.g. in (Gonet et al., 2012). It should be mentioned that those calculations are time-consuming therefore the use of computer programs should be considered, e.g. package Compass Landmark by Halliburton, 2008. Such an approach to designing directional borehole axes allows for defining the level of risk of collision of borehole axes and checking it in when new boreholes are performed.

Conclusions

1. Systematically developing directional drilling finds newer and newer applications, which mainly stems from the new advances in the borehole drilling technique and technology.

2. Majority of directional boreholes are designed in 2D, and specific type of borehole trajectory, e.g. „J" or „S' type are assumed arbitrarily.

3. The presented method of designing directional boreholes is applicable to designing directional borehole axes in 2D and 3D. The objective function was assumed in the form of a minimum cost of performing the borehole and limiting factor related with the admissible curvature of the dogleg.

4. When a few boreholes are drilled from one place, which is the case on drilling platforms and planned shale gas drilling it is recommended that the trajectory of the borehole is determined and also zone of potential position of borehole axes is identified according to the accuracy level represented by equipment used for measuring the angle of deflection of the dogleg, azimuths and length. Thanks to this the real risk of boreholes collision can be minimized.

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References

COMPASS, 2008, Landmark Halliburton.

Gonet A., 1981. *Możliwości sterowania kątem odchylenia otworów wiertniczych przy pomocy stabilizatorów*. Archiwum Górnictwa, vol. 26, issue 4, str. 735- 745.

Gonet A., 1982. *Metodyka stabilizacji dolnej części przewodu wiertniczego*. Archiwum Górnictwa, vol. 27, issue 3, str. 225-245.

Gonet A., 1987. *Technologia stołowego wiercenia otworów kierunkowych w świetle badań modelowych i warunków geologicznych*. Zeszyty Naukowe AGH, Wiertnictwo, Nafta, Gaz, z. 3, Kraków.

Gonet A., Knez D., Stryczek S., 2012. *Analiza wpływu błędów pomiarowych na wyznaczanie trajektorii osi otworu wiertniczego*. Bezpieczeństwo Pracy i Ochrona Środowiska w Górnictwie, w druku.

Lubinski A., 1961. *Maximum permissible dog-legs in rotary boreholes*. Journal of Petroleum Technology, February.

Lubinski A., 1987. *Developments in Petroleum Engineering*, vol. 1, Stability of Tubulars, Deviation Control, ed. Stefan Miska, Houston, Gulf Publishing Company.

Miska S., 1979. *Drilling Optimization*, The University of Trondheim.

Miska S., Miska W., 2006. *Modeling of Complex Bottom Hole Assemblies in Curved and Straight Holes*. Archives of Mining Sciences, vol. 51, issue 1, p. 35-54

Mitchell R.F., Miska S. Z., 2011. *Fundamentals of Drilling Engineering*. Society of Petroleum Engineers, Richardson.

Matanović D., 2007. *Tehnika izrade bušotina*. Manualia Universitatis Studiorum Zagrabiensis Udžbenici Sveulista u Zagrebu.

Pinka J., Wittenberger G., Engel J., 2007. *Borehole Mining*. F. BERG. TU, Košice.

Wojtanowicz A.,1975. *Problemy optymalizacji procesu wiercenia*. Skrypt AGH, nr 460, Kraków.

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