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# Using Single-Beam Observations of Fish Echoes from Sidelobe in Fish Target Strength Estimation

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Fish target estimation when using single-beam echosounder data is based on solving integral equation by inverse technique with probability density functions (PDFs) of echo envelope and beam pattern as known variables. Typically - by setting threshold on echo level - only main-lobe observation are used due to difficulties in beam pattern PDF evaluation and signal-to-noise ratio constraints. This paper presents a method that uses side-lobe observations in the beam pattern probability density function and all collected echo peak values. These side-lobe echo observations indifferentiable when using single-beam system are combined with those from main-lobe. The smoothed Expectation Maximization method (EMS) is used to estimate densities and acoustic sizes of fish main-lobe with side-lobe observations.

### 1. Introduction

Target strength estimation when using single beam echosounder systems leads into inverse problem in which probability density function (PDF) of target strength is estimated from fish echoes. Due to hydroacoustic system characteristics the reconstruction is based on incomplete data. This kind of problems is an example of statistical linear inverse problem (SLIP), which is typically illconditioned and can be solved using direct inverse techniques based on regularization (i.e. Singular Value Decomposition - SVD) or iterative ones in which additional constraints are specified (i.e. Expectation, Maximization, Smoothing - EMS) [1] [2] [4]. The ill-conditioning originate from the kernel of SLIP, which in this case is determined by beam pattern probability density function. In most cases the observed data are restricted to the certain range limited by side-lobe level. This approach allows omitting the problem of ambiguity of beam pattern function [3]. Nowadays, when dynamic range of digital echosounders have been increased it

is possible that also echoes from sidelobes are included in data set.

# 2. Problem Formulation

The statistical linear inverse problems (SLIP) are often presented as a linear operator equation:

$$y(x) = (K f)(x) + n(x) \tag{1}$$

where: f - unknown function, y - observation, K - linear operator; n - noise. In case of fish target strength estimation observation y is represented as a echo level peak values PDF  $(p_E)$ , linear operator K is constructed from logarithmic beam pattern PDF  $(p_B)$  and unknown function represents fish target strength PDF  $(p_{TS})$ . The noise n can be treated as an error in echo level PDF, formed during histogram construction. The Eq. (1) represents the discretized version of single beam integral equation [1], being convolution like integral of the following form:

$$p_E(E) = \int_{B_-}^0 p_B(B) p_{TS}(E - B - G) dB$$
 (2)

where G represents system gain (E=TS+B+G) and  $B_{min}$  is lower threshold of logarithmic beam pattern fuction included in calculations.

For PDF calculation let us consider the ideal circular piston in a infinite baffle. Its one-way beam pattern function b is:

$$b(\theta) = \frac{2J_1(x)}{x} \tag{3}$$

where is x defined by  $x=x(\theta)=ka$   $sin\theta$  (k – wave number, a – transducer radius) and  $J_n()$  – Bessel function of first kind order n. Figure 1 presents the logarithmic version of two-way pattern which is derived by transform  $B(\theta)=10$  log  $b(\theta)^2=20$  log  $b(\theta)$ .

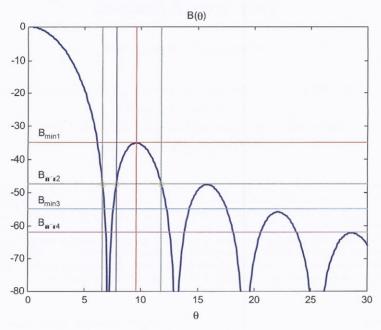


Fig 1. Two-way beam pattern function for ideal circular piston in a infinite baffle.

The kernel function of the inverse problem  $p_B()$  for two-way system is obtained from its absolute version  $p_b()$ , which may be expressed as a parametric function  $p_b(b){=}(b^2(\theta)$ ,  $p_b(\theta)$ ) with angle  $\theta$  as a parameter:

$$p_{\theta}(b) = \left( \left( \frac{2J_1(x)}{x} \right)^2, \frac{p_{\theta}(\theta) \operatorname{tg} \theta}{\left| \frac{8J_1(x)J_2(x)}{x} \right|} \right)$$
(4)

where  $p_{\theta}()$  is a probablity density function of random angular position of fish. Then using logarithmic transform of variables  $B(b)=20 \log b$  its PDF relation may be written as:

$$p_B(B) = \frac{\ln 10}{20} \left| 10^{\frac{B}{20}} \right| p_b(10^{\frac{B}{20}})$$
 (5)

Calculation of PDF of fish position  $p_{\theta}()$  is based on the assumption of uniform distribution of fish in water column (cartesian coordinates), which gives *sine* like distribution of angular position  $\theta$  [1]:

$$p_{\theta}(\theta) = \frac{1}{1 - \cos \theta_{\text{max}}} \sin \theta \tag{6}$$

where  $\theta_{max}$  is maximum angle of beam pattern involved in calculation.

When only main lobe of beam pattern is used the inverse of beam pattern function needed for  $p_b()$  computation is single-valued. Note, however, that when side-lobe are involved the inverse function is non single-valued (Fig.1) and then its PDF calculation must be made for all angles  $\theta$  having the same value B. This makes the Eq. (3) still valid but inverse function  $b^{-1}()$  needs to be computed to find all occurances of  $\theta$  for every b being now independent variable and followed the sumation of

PDF values for those angles. The inverse of beam pattern function now called  $\theta(b)$  may be calculated iteratively using Newton iteration  $x_{n+1}=x_n-f(x_n)/f'(x_n)$ , which for theoretical beam pattern leads to following rule for space frequency u determination:

$$x_{n+1} = x_n + \frac{J_1(x_n) + \frac{bx_n}{2}}{J_2(x_n)}$$
 (7)

followed by angle determination  $\theta$  from variable x:

$$\theta = \arcsin\frac{x}{ka} \tag{8}$$

The final result of this approach is presented in Figure 2, showing theoretical beam pattern PDF  $p_B$ .() with inclusion of three side-lobes. The components from sidelobes are presented as dotted lines. Note the theoretical left-sided infinities in PDF where sidelobes reaches its local maxima.

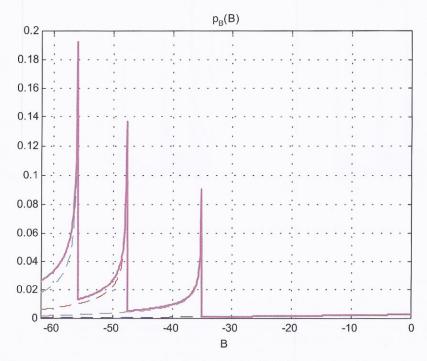


Fig. 2. Theoretical beam pattern probability density function (PDF) for ideal circular piston when three sidelobes are included.

To construct the kernel of inverse problem described by Eq. (1), the beam pattern PDF needs to be descretized. In most cases this leads to simple sampling of theoretical PDF, however in this case due to infinities and histogram nature of observed echo level PDF  $p_E()$  it must be done carefully. To check the behavour of descretized version of PDF another method of its determination is suggested. The proposed method is based on computer generation of assumed angular position distribution. The distribution desribed by PDF function from Eq. (6) can be generated using inverse of cumulative distribution function (CDF) method. In case of *sine* like distribution its CDF is *cosine* like function so angular position distribution may be generated

transforming uniform distibution by *arccos* function. To obtain so called distribution of beam pattern function Eq. (3) is applied to angular position distribution followed by logarithmic transform. The results of such approach are presented in Figure 3 in form of histogram chart with theoretical PDF as a solid curve for comparison purposes. Histogram in Figure 3a was generated using 10 000 realizations devided into 20 bins and histogram in Figure 3b – 100 000 devided into 40 bins. Note that the descrete approximation depends not only on bin size but also on bins position, especially when the bin contains theoretical infinity and low values alltogether. Additionally, the values in border bins may be inaccurate.

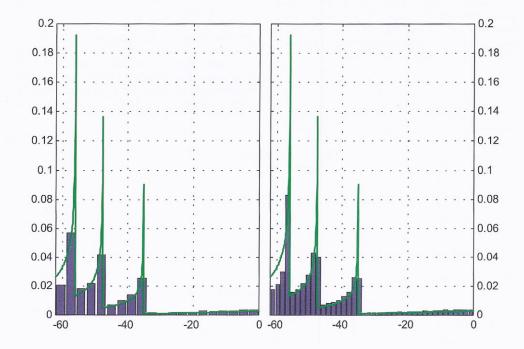


Fig.3. Randomly generated beam pattern probability density function for ideal circular piston a) 20 bins b) 40 bins.

#### 3. Simulations

Being aware of the problems of determining the discretized beam pattern PDF function with sidelobes, the next step is to verify the possibility of using it as the kernel of inverse method for target strength PDF estimation. The Expectation, Maximization, Smoothing EMS being the most robust iterative technique based on statistical approach was adopted to this problem and its performances were thoroughly tested [1]. To check its performance in case of sidelobe data the simulation data were generated. The fish backscattering crosssection  $\sigma_{BS}$  distribution was assumed to have Laplace distribution, which when transformed to target strength domain represents logarithmic Rayleigh distribution function. The histogram of simulated TS values is presented in Figure 4a. The results of simulation of beam pattern distribution (Figure 4b) described in previous section were used as base for random values of beam pattern B. The 3dB value of beam pattern was setup at 3° giving 6° of beam width and 23° was used as effective angular range, giving 62dB of logarithmic dynamic range. The system constant G was setup to 50dB. The resulting echo level PDF for this case is shown in Fig. 4c.

Figure 5 shows splined probability density function estimates respectively. Note, that if you want reconstruct 40 dB range of TS data (from – 70dB to –30dB) the rule of thumb in inverting the incomplete data says that you need to select 40dB range of observation and also 40dB range of kernel data. Otherwise, you run a risk of using a part of observation, which has not complete representation. As an example in Figure 5c the result of direct convolution of bolded parts of curves from Figures 5a and 5b, both having 40 dB range, are presented. Only the half of it matches the simulated echo level PDF, so 40dB range from –20dB to 20dB is valuable to reconstruction purposes.

Figure 6 shows inverse reconstruction performances. To make the simulation more realistic the random noise were added to observation data (Figure 6a). Figure 6b shows the naive reconstruction using SVD technique and Figure 6c EMS output compared to original TS PDF  $p_{TS}(TS)$ . Both outputs were used as a source data, and after running through direct process again, compared with a part of observation used in inverse process.

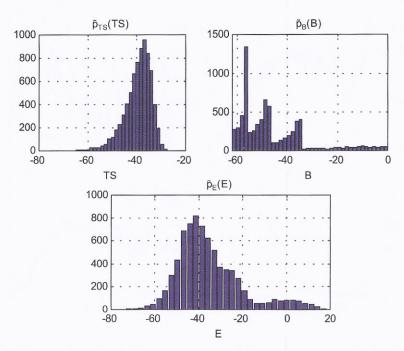


Fig. 4. Simulated probability density functions (PDFs) a) target strength PDF b) beam pattern PDF c) echo level PDF.

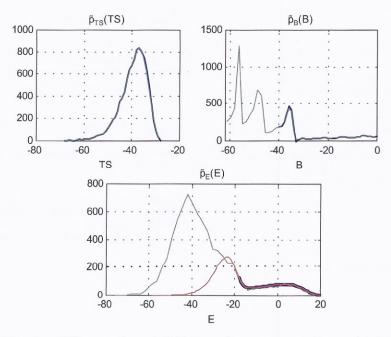


Fig. 5. Splined versions of histogram data – the bolded part of curves represents data used in inverse process a) target strength PDF b) beam pattern PDF c) echo level PDF.

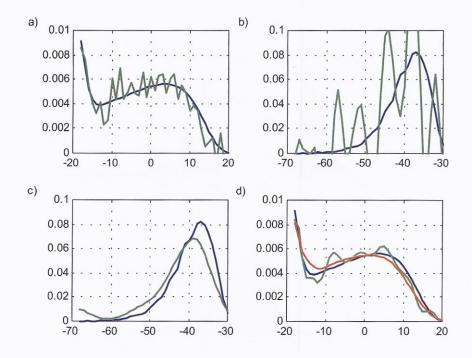


Fig. 6. Reconstruction a) source data with random noise b) naive reconstruction c) EMS reconstruction d) verification

# 4. Conclusion

The paper presents an idea of using sidelobe observation in the inverse process of reconstructing fish target strength densities from single beam data. It is especially valuable for new digital echosounders, which have increased dynamic properties. The theoretic approach to construction of probability density function of beam pattern with sidelobes is presented. The shape of this function, which has local infinities, needs careful treatment and robust inverse method. The smoothed Expectation and Maximization method was used as iterative inverse method and its usefulness was confirmed by tests run on simulated data.

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