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Complex technical systems safety prediction

Keywords

safety function, complex system, Semi-Markov process, system operation process

Abstract

There are presented general safety analytical models of complex multistate technical systems related to their operation processes. They are the integrated general models of complex technical systems, linking their multistate safety models and their operation processes models and considering variable at the different operation states safety structures and their components safety parameters. The conditional safety functions at the system particular operation states and independent of the system particular operation states the unconditional safety function and the risk function of the complex technical systems are defined. These joint models of the safety and the variable in time system operation processes are constructed for multistate series, parallel, "*m* out of *n*", consecutive "*m* out of *n*: F", series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-series systems. The joint models are applied to determining safety characteristics of these systems related to their varying in time safety structures and their components safety characteristics. Under the assumption that the considered systems are exponential, the unconditional safety functions of these systems are determined.
The proposed models and methods are applied to the safety analysis, evaluation and prediction of the one subsystem of the port grain transportation system related to varying in time their operation processes, structures and components safety parameters.

1. Introduction

Most real technical systems are structurally very complex and they often have complicated operation processes. Large numbers of components and subsystems and their operating complexity cause that the evaluation and prediction of their safety is difficult. The time dependent interactions between the systems' operation processes operation states changing and the systems' structures and their components safety states changing processes are evident features of most real technical systems. The common safety and operation analysis of these complex technical systems is of great value in the industrial practice. The convenient tools for analyzing this problem are the multistate system's safety modeling [8], [15-17] commonly used with the semi-Markov modeling [1-3], [4], [5], [9], [10] of the systems operation processes, leading to the construction the joint general safety models of the complex technical systems related to their operation process [6], [8], [7], [11], [13]. The main objective of this chapter is to present recently developed, the general safety analytical models of complex non-repairable and repairable multistate technical systems related to their operation processes [6], [7], [8] and to apply them practically to real industrial systems and processes [8], [12], [11], [13]. In the case of large systems, the determination of the exact safety functions of the systems and the system risk functions leads us to very complicated formulae that are often useless for safety practitioners. One of the important techniques in this situation is the asymptotic approach [5], [8] to system safety and safety evaluation. This aspect of complex technical systems is also shortly discussed in this paper.

2. System operation at variable conditions

We assume that the system during its operation process is taking $v, v \in N$, different operation states $z_1, z_2, \ldots, z_v$. Further, we define the system operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_v\}$. Moreover, we assume that the system operation process $Z(t)$ is a semi-Markov process [4], [7], [8], [11]-[14] with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,...,v,$ $b \neq l.$ Under these assumptions, the system operation process may be described by:

- the vector $[p_b(0)]_{1\times v}$ of the initial probabilities $p_b(0) = P(Z(0) = z_b),$ $b = 1,2,...,v,$ of the system operation process $Z(t)$ staying at particular operation states at the moment $t = 0$;
- the matrix $[p_{bl}]_{v\times v}$ of probabilities p_{bl}, $b, l = 1,2,...,v,$ $b \neq l,$ of the system operation process $Z(t)$ transitions between the operation states z_b and z_l;
- the matrix $[H_{bl}(t)]_{v\times v}$ of conditional distribution functions $H_{bl}(t) = P(\theta_{bl} < t)$, $t \geq 0,$ $b, l = 1,2,...,v,$ $b \neq l,$ of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states.

As the mean values $E[\theta_{bl}]$ of the conditional sojourn times θ_{bl} are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t), \ b,l = 1,2,...,v, \ b \neq l, \quad (1)$$

then from the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ_b, $b = 1,2,...,v,$ of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,...,v,$ are given by [8], [11]-[14]

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ t \geq 0, \ b = 1,2,...,v.$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,...,v,$ at the operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \ b = 1,2,...,v, \quad (2)$$

where M_{bl} are defined by the formula (1).

The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states $p_b(t) = P(Z(t) = z_b)$, $t \in <0,+\infty),$ $b = 1,2,...,v,$ are given by [8], [11]-[14]

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ b = 1,2,...,v, \quad (3)$$

where M_b, $b = 1,2,...,v,$ are given by (2), while the steady probabilities π_b of the vector $[\pi_b]_{1\times v}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \quad (4)$$

3. Safety of multistate systems at variable operation conditions

We assume that the changes of the operation states of the system operation process $Z(t)$ have an influence on the system multistate components E_i, $i = 1,2,...,n,$ safety and the system safety structure as well. Consequently, we denote the system multistate component E_i, $i = 1,2,...,n,$ conditional lifetime in the safety state subset $\{u, u+1,..., z\}$ while the system is at the operation state z_b, $b = 1,2,...,v,$ by $T_i^{(b)}(u)$ and its conditional safety function by the vector

$$[S_i(t,\cdot)]^{(b)} = [1, [S_i(t,1)]^{(b)}, ..., [S_i(t,z)]^{(b)}], \quad (4)$$

with the coordinates defined by

$$[S_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t \mid Z(t) = z_b) \quad (5)$$

for $t \in <0,\infty),$ $u = 1,2,..., z,$ $b = 1,2,...,v.$

The safety function $[S_i(t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the safety state subset $\{u, u+1,..., z\}$ is greater than t, while the system operation process $Z(t)$ is at the operation state z_b. (6)

Similarly, we denote the system conditional lifetime in the safety state subset $\{u, u+1,..., z\}$ while the system is at the operation state z_b, $b = 1,2,...,v,$ by $T^{(b)}(u)$ and the conditional safety function of the system by the vector

$$[\boldsymbol{S}(t,\cdot)]^{(b)} = [1, [\boldsymbol{S}(t,1)]^{(b)}, ..., [\boldsymbol{S}(t,z)]^{(b)}], \quad (6)$$

with the coordinates defined by

$$[\boldsymbol{S}(t,u)]^{(b)} = P(T^{(b)}(u) > t \mid Z(t) = z_b) \quad (7)$$

for $t \in <0,\infty),$ $u = 1,2,..., z,$ $b = 1,2,...,v.$

The safety function $[\boldsymbol{S}(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the safety state subset $\{u, u+1, ..., z\}$ is greater than t, while the system operation process $Z(t)$ is at the operation state z_b.

Thus, the system conditional lifetimes in the safety states subset $\{u, u+1, ..., z\}$ at the operational state z_b

$$T^{(b)}(u) = T(T_1^{(b)}(u), T_2^{(b)}(u), ..., T_n^{(b)}(u))$$

defined for $u = 1,2,...,z,\ b = 1,2,...,\nu,\ n \in N$, are dependent on the components conditional lifetimes $T_1^{(b)}(u),\ T_2^{(b)}(u),\ ...,\ T_n^{(b)}(u)$, in the safety states subset $\{u, u+1, ..., z\}$ at the operation state z_b and the coordinates of the system conditional safety function at the operation state z_b

$$[\boldsymbol{S}(t,u)]^{(b)}$$

$$= \boldsymbol{S}([S_1(t,u)]^{(b)}, [S_2(t,u)]^{(b)}, ..., [S_n(t,u)]^{(b)})$$

defined for $t \in <0, \infty),\ u = 1,2,...,z,\ b = 1,2,...,\nu,\ n \in N$, are dependent on the coordinates $[S_1(t,u)]^{(b)}$, $[S_2(t,u)]^{(b)}$, ..., $[S_n(t,u)]^{(b)}$, of the components conditional safety functions at the operation state z_b.

Further, we denote the system unconditional lifetime in the safety state subset $\{u, u+1, ..., z\}$ by $T(u)$ and the unconditional safety function of the system by the vector

$$\boldsymbol{S}(t,\cdot) = [1,\ \boldsymbol{S}(t,1), ...,\ \boldsymbol{S}(t,z)\ ], \quad (8)$$

with the coordinates defined by

$$\boldsymbol{S}(t,u) = P(T(u) > t)$$

for $t \in <0, \infty),\ u = 1,2,...,z$.

In the case when the system operation time θ is large enough, the coordinates of the unconditional safety function of the system defined by (8) are given by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \geq 0,\ u = 1,2,...,z, \quad (9)$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,...,z,\ b = 1,2,...,\nu$, are the coordinates of the system conditional safety functions defined by (6)-(7) and $p_b, b = 1,2,...,\nu$, are the system operation process limit transient probabilities given by (3).

The mean value of the system unconditional lifetime $T(u)$ in the safety state subset $\{u, u+1, ..., z\}$ is given by [7], [8], [11]-[14]

$$\mu(u) \cong \sum_{b=1}^{\nu} p_b \mu_b(u),\ u = 1,2,...,z, \quad (10)$$

where $\mu_b(u)$ are the mean values of the system conditional lifetimes $T^{(b)}(u)$ in the safety state subset $\{u, u+1, ..., z\}$ at the operation state z_b, $b = 1,2,...,\nu$, given by

$$\mu_b(u) = \int_0^{\infty} [\boldsymbol{S}(t,u)]^{(b)} dt,\ u = 1,2,...,z, \quad (11)$$

$[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,...,z,\ b = 1,2,...,\nu$, are defined by (6)-(7) and p_b are given by (3). Whereas, the variance of the system unconditional lifetime $T(u)$ is given by

$$\sigma^2(u) = 2\int_0^{\infty} t\ \boldsymbol{S}(t,u)dt - [\mu(u)]^2,\quad u = 1,2,...,z, \quad (12)$$

where $\boldsymbol{S}(t,u)$, $u = 1,2,...,z$, are given by (8)-(9) and $\mu(u),\ u = 0,1,...,z$, are given by (10)-(11).

Hence, according to (1.19) [8], we get the following formulae for the mean values of the unconditional lifetimes of the system in particular safety states

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ u = 0,1,...,z-1,$$

$$\bar{\mu}(z) = \mu(z), \quad (13)$$

where $\mu(u),\ u = 0,1,...,z$, are given by (10)-(11).

Moreover, according to (1.20)-(1.21) [8], if r is the system critical safety state, then the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r),\ t \in <0, \infty), \quad (14)$$

where $\boldsymbol{S}(t,r)$ is the coordinate of the system unconditional safety function given by (9) for $u = r$ and if τ is the moment when the system risk function exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \quad (15)$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (14).

Further, we assume that the system components E_i, $i = 1,2,...,n$, at the system operation states

z_b, $b = 1,2,...,\nu$, have the exponential safety functions, i.e. their coordinates are given by

$$[S_i\ (t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b)$$

$$= \exp[-[\lambda_i\ (u)]^{(b)} t]$$

for $t \in <0,\infty)$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, and we have

$$[\boldsymbol{S}(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b)$$

$$= [\boldsymbol{S}([S_1\ (t,u)]^{(b)}, [S_2\ (t,u)]^{(b)}, ..., [S_n\ (t,u)]^{(b)})]^{(b)}$$

$$= [\boldsymbol{S}(\exp[-[\lambda_1\ (u)]^{(b)} t], \exp[-[\lambda_2\ (u)]^{(b)} t], ...,$$

$$\exp[-[\lambda_n\ (u)]^{(b)} t])]^{(b)}$$

for $t \in <0,\infty)$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, $n \in N$.

The reason for this strong assumption on the system components is that the exponential distribution has "no memory" expressed in the following property

$$[S_i\ (t_0 + t,u)]^{(b)}$$

$$= P(T_i^{(b)}(u) > t_0 + t / T_i^{(b)}(u) > t_0 | Z(t) = z_b)$$

$$= P(T_i^{(b)}(u) > t_0 + t \cap T_i^{(b)}(u) > t_0 | Z(t) = z_b)$$

$$/ P(T_i^{(b)}(u) > t_0 | Z(t) = z_b)$$

$$= P(T_i^{(b)}(u) > t_0 + t | Z(t) = z_b)\ / P(T_i^{(b)}(u) > t_0 | Z(t) = z_b)$$

$$= \exp[-[\lambda_i\ (u)]^{(b)} (t + t_0) / \exp[-[\lambda_i\ (u)]^{(b)} t_o]$$

$$= \exp[-[\lambda_i\ (u)]^{(b)} t]$$

$$= P(T_i^{(b)}(u) > t | Z(t) = z_b)\ = [S_i\ (t,u)]^{(b)}$$

for $t_0 \in <0,\infty)$ and $t \in <0,\infty)$, $u = 1,2,...,z$.

Both of them, the assumption about the exponential safety functions of the system components and the above property, justify the following form of the formula (9)

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{S}(t,u)]^{(b)}$$

$$= \sum_{b=1}^{\nu} p_b [\boldsymbol{R}(\exp[-[\lambda_1\ (u)]^{(b)} t], \exp[-[\lambda_2\ (u)]^{(b)} t]$$

$$, ..., \exp[-[\lambda_n\ (u)]^{(b)} t])]^{(b)}$$

for $t \geq 0$, $u = 1,2,...,z$.

The application of the above formula and the results given in Chapter 1 [8] yield the following results formulated in the form of the following proposition.

Proposition 1

If components of the multi-state system at the operation state z_b, $b = 1,2,...,\nu$, have the exponential safety functions given by

$$[S_i\ (t,\cdot)]^{(b)} = [1, [S_i\ (t,1)]^{(b)}, \ldots, [S_i\ (t,z)]^{(b)}], \qquad (16)$$

$$t \in (-\infty,\infty),\ b = 1,2,...,\nu,$$

where

$$[S_i\ (t,u)]^{(b)} = 1 \text{ for } t < 0,$$

$$[S_i\ (t,u)]^{(b)} = \exp[-[\lambda_i\ (u)]^{(b)} t] \text{ for } t \geq 0,$$

$$[\lambda_i\ (u)]^{(b)} > 0, \qquad (17)$$

$$i = 1,2,...,n,\ u = 1,2,\ldots,z,\ b = 1,2,...,\nu,$$

in the case of series, parallel, "*m* out of *n*", consecutive "*m* out of *n*: F" systems and respectively by

$$[S_{ij}\ (t,\cdot)]^{(b)} = [1, [S_{ij}\ (t,1)]^{(b)}, \ldots, [S_{ij}\ (t,z)]^{(b)}], \qquad (18)$$

$$t \in (-\infty,\infty),\ b = 1,2,...,\nu,$$

where

$$[S_{ij}\ (t,u)]^{(b)} = 1 \text{ for } t < 0,$$

$$[S_{ij}\ (t,u)]^{(b)} = \exp[-[\lambda_{ij}\ (u)]^{(b)} t] \text{ for } t \geq 0,$$

$$[\lambda_{ij}\ (u)]^{(b)} > 0, \qquad (19)$$

$$i = 1,2,...,k,\ j = 1,2,...,l_i,\ u = 1,2,\ldots,z,\ b = 1,2,...,\nu,$$

in the case of series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-series systems and the system operation time θ is large enough, then its multistate unconditional safety function is given by the vector:

i) for a series system

$$\bar{\boldsymbol{S}}_n(t,\cdot) = [1, \bar{\boldsymbol{S}}_n(t,1), ..., \bar{\boldsymbol{S}}_n(t,z)], \quad (20)$$

where

$$\bar{\boldsymbol{S}}_n(t,u) = 1 \text{ for } t < 0,$$

$$\bar{\boldsymbol{S}}_n(t,u) \cong \sum_{b=1}^{v} p_b \exp[-\sum_{i=1}^{n}[\lambda_i(u)]^{(b)} t] \quad (21)$$

for $t \ge 0$, $u = 1,2,...,z$;

ii) for a parallel system

$$\boldsymbol{S}_n(t,\cdot) = [1, \boldsymbol{S}_n(t,1), ..., \boldsymbol{S}_n(t,z)], \quad (22)$$

where

$$\boldsymbol{S}_n(t,u) \cong 1 - \sum_{b=1}^{v} p_b \prod_{i=1}^{n}[1 - \exp[-[\lambda_i(u)]^{(b)} t]] \quad (23)$$

for $t \ge 0$, $u = 1,2,...,z$;

iii) for a "*m* out of *n*" system

$$\boldsymbol{S}_n^m(t,\cdot) = [1, \boldsymbol{S}_n^m(t,1), ..., \boldsymbol{S}_n^m(t,z)], \quad (24)$$

where

$$\boldsymbol{S}_n^m(t,u) = 1 \text{ for } t < 0,$$

$$\boldsymbol{S}_n^m(t,u)$$

$$\cong 1 - \sum_{b=1}^{v} p_b \sum_{\substack{r_1, r_2, ..., r_n = 0 \\ r_1 + r_2 + ... + r_n \le m-1}}^{1} \prod_{i=1}^{n} \exp[-r_i[\lambda_i(u)]^{(b)} t]$$

$$[1 - \exp[-[\lambda_i(u)]^{(b)} t]]^{1-n} \quad (25)$$

for $t \ge 0$, $u = 1,2,...,z$,

or

$$\bar{\boldsymbol{S}}_n^{\bar{m}}(t,\cdot) = [1, \bar{\boldsymbol{S}}_n^{\bar{m}}(t,1), ..., \boldsymbol{S}_n^{\bar{m}}(t,z)], \quad (26)$$

where

$$\boldsymbol{S}_n^{\bar{m}}(t,u) = 1 \text{ for } t < 0,$$

$$\bar{\boldsymbol{S}}_n^{\bar{m}}(t,u) \cong \sum_{b=1}^{v} p_b \sum_{\substack{r_1, r_2, ..., r_n = 0 \\ r_1 + r_2 + ... + r_n \le \bar{m}}}^{1} \prod_{i=1}^{n}[1 - \exp[-[\lambda_i(u)]^{(b)} t]]^{r_i}$$

$$\exp[-(1 - r_i)[\lambda_i(u)]^{(b)} t] \quad (27)$$

for $t \ge 0$, $\bar{m} = n - m$, $u = 1,2,...,z$;

iv) for a consecutive "*m* out of *n*: F" system

$$\boldsymbol{CS}_n^m(t,\cdot) = [1, \boldsymbol{CS}_n^m(t,1), ..., \boldsymbol{CS}_n^m(t,z)], \quad (28)$$

where

$$\boldsymbol{CS}_n^m(t,u) = 1 \text{ for } t < 0,$$

$$\boldsymbol{CS}_n^m(t,u) \cong \begin{cases} 1 & \text{for } n < m, \\ 1 - \sum_{b=1}^{v} p_b \prod_{i=1}^{n}[1 - \exp[-[\lambda_i(u)]^{(b)} t]] & \text{for } n = m, \\ \sum_{b=1}^{v} p_b \exp[-[\lambda_n(u)]^{(b)} t][\boldsymbol{CS}_{n-1}^m(t,u)]^{(b)} & \\ + \sum_{i=1}^{m-1} \exp[-[\lambda_{n-i}(u)]^{(b)} t][\boldsymbol{CS}_{n-i-1}^m(t,u)]^{(b)} & \\ \prod_{j=n-i+1}^{n}[1 - \exp[-[\lambda_j(u)]^{(b)} t]] & \text{for } n > m, \end{cases} \quad (29)$$

for $t \ge 0$, $u = 1,2,...,z$;

v) for a series-parallel system

$$\boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,\cdot)$$

$$= [1, \boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,1), ..., \boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,z)], \quad (30)$$

where

$$\boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,u) = 1 \text{ for } t < 0,$$

$$\boldsymbol{S}_{k;l_1,l_2,...,l_k}(t,u)$$

$$\cong 1 - \sum_{b=1}^{v} p_b \prod_{i=1}^{k}[1 - \exp[-\sum_{j=1}^{l_i}[\lambda_{ij}(u)]^{(b)} t]] \quad (31)$$

for $t \ge 0$, $u = 1,2,...,z$;

vi) for a parallel-series system

$$\bar{\boldsymbol{S}}_{k;l_1,l_2,...,l_k}(t,\cdot)$$

$$= [1, \bar{\boldsymbol{S}}_{k;l_1,l_2,...,l_k}(t,1), ..., \bar{\boldsymbol{S}}_{k;l_1,l_2,...,l_k}(t,z)] \qquad (32)$$

where

$$\bar{\boldsymbol{S}}_{k;l_1,l_2,...,l_k}(t,u) = 1 \text{ for } t < 0,$$

$$\bar{\boldsymbol{S}}_{k;l_1,l_2,...,l_k}(t,u)$$

$$\cong \sum_{b=1}^{v} p_b \prod_{i=1}^{k} [1 - \prod_{j=1}^{l_i} [1 - \exp[-[\lambda_{ij}(u)]^{(b)} t]]] \qquad (33)$$

for $t \geq 0$, $u = 1,2,..., z$;

vii) for a series-"*m* out of *k*" system

$$\boldsymbol{S}^{m}_{k;l_1,l_2,...,l_k}(t,\cdot)$$

$$= [1, \boldsymbol{S}^{m}_{k;l_1,l_2,...,l_k}(t,1), ..., \boldsymbol{S}^{m}_{k;l_1,l_2,...,l_k}(t,z)], \qquad (34)$$

where

$$\boldsymbol{S}^{m}_{k;l_1,l_2,...,l_k}(t,u) = 1 \text{ for } t < 0,$$

$$\boldsymbol{S}^{m}_{k;l_1,l_2,...,l_k}(t,u)$$

$$\cong 1 - \sum_{b=1}^{v} p_b \sum_{\substack{r_1,r_2,...,r_k = 0 \\ r_1+r_2+...+r_k \leq m-1}}^{1} \prod_{i=1}^{k} [[\prod_{j=1}^{l_i} \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{r_i}$$

$$\cdot [1 - \prod_{j=1}^{l_i} \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{1-r_i} \qquad (35)$$

for $t \geq 0$, $u = 1,2,..., z$,

or

$$\bar{\boldsymbol{S}}^{\bar{m}}_{k;l_1,l_2,...,l_k}(t,\cdot)$$

$$= [1, \bar{\boldsymbol{S}}^{\bar{m}}_{k;l_1,l_2,...,l_k}(t,1), ..., \bar{\boldsymbol{S}}^{\bar{m}}_{k;l_1,l_2,...,l_k}(t,z)], \qquad (36)$$

where

$$\bar{\boldsymbol{S}}^{\bar{m}}_{k;l_1,l_2,...,l_k}(t,u) = 1 \text{ for } t < 0,$$

$$\bar{\boldsymbol{S}}^{\bar{m}}_{k;l_1,l_2,...,l_k}(t,u)$$

$$\cong \sum_{b=1}^{v} p_b \sum_{\substack{r_1,r_2,...,r_k = 0 \\ r_1,r_2,...+r_k \leq \bar{m}}}^{1} \prod_{i=1}^{k} [1 - [\prod_{i=1}^{l_i} \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{r_i}$$

$$\cdot [\prod_{j=1}^{l_i} \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{1-r_i} \qquad (37)$$

for $t \geq 0$, $\bar{m} = k - m$, $u = 1,2,..., z$;

viii) for a "m_i out of l_i"-series system

$$\overline{\boldsymbol{S}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}}(t,\cdot)$$

$$= [1, \overline{\boldsymbol{S}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}}(t,1), ..., \overline{\boldsymbol{S}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}}(t,z)], \qquad (38)$$

where

$$\overline{\boldsymbol{S}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}}(t,u) = 1 \text{ for } t < 0,$$

$$\overline{\boldsymbol{S}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}}(t,u)$$

$$\cong \sum_{b=1}^{v} p_b \prod_{i=1}^{k} [1 - \sum_{\substack{r_1,r_2,...,r_{l_i} = 0 \\ r_1+r_2+...+r_{l_i} \leq m_i - 1}}^{1} \prod_{j=1}^{l_i} \exp[-r_j [\lambda_{ij}(u)]^{(b)} t]]$$

$$\cdot [1 - \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{1-r_j}] \qquad (39)$$

for $t \geq 0$, $u = 1,2,..., z$,

or

$$\overline{\bar{\boldsymbol{S}}^{\bar{m}_1,\bar{m}_2,...,\bar{m}_k}_{k;l_1,l_2,...,l_k}}(t,\cdot)$$

$$= [1, \overline{\bar{\boldsymbol{S}}^{\bar{m}_1,\bar{m}_2,...,\bar{m}_k}_{k;l_1,l_2,...,l_k}}(t,1), ..., \overline{\bar{\boldsymbol{S}}^{\bar{m}_1,\bar{m}_2,...,\bar{m}_k}_{k;l_1,l_2,...,l_k}}(t,z)], \qquad (40)$$

where

$$\overline{\bar{\boldsymbol{S}}^{\bar{m}_1,\bar{m}_2,...,\bar{m}_k}_{k;l_1,l_2,...,l_k}}(t,u) = 1 \text{ for } t < 0,$$

$$\overline{\bar{\boldsymbol{S}}^{\bar{m}_1,\bar{m}_2,...,\bar{m}_k}_{k;l_1,l_2,...,l_k}}(t,u)$$

$$\cong \sum_{b=1}^{v} p_b \prod_{i=1}^{k} [\sum_{\substack{r_1,r_2,...,r_{l_i} = 0 \\ r_1+r_2+...+r_{l_i} \leq \bar{m}_i}}^{1} \prod_{j=1}^{l_i} [1 - \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{r_j}$$

$$\cdot \exp[-(1 - r_j)[\lambda_{ij}(u)]^{(b)} t]] \qquad (41)$$

for $t \geq 0$, $\bar{m}_i = l_i - m_i$, $i = 1,2,..., k$, $u = 1,2,..., z$;

ix) for a series-consecutive "*m* out of *k*: F" system

$$\boldsymbol{CS}^{m}_{k;l_1,l_2,...,l_k}(t,\cdot)$$

$$= [1, \boldsymbol{CS}^{m}_{k;l_1,l_2,...,l_k}(t,1), ..., \boldsymbol{CS}^{m}_{k;l_1,l_2,...,l_k}(t,z)], \quad (42)$$

where

$$\boldsymbol{CS}^{m}_{k;l_1,l_2,...,l_k}(t,u) = 1 \text{ for } t < 0,$$

$$\boldsymbol{CS}^{m}_{k;l_1,l_2,...,l_k}(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{CS}^{m}_{k;l_1,l_2,...,l_k}(t,u)]^{(b)} \quad (43)$$

for $t \geq 0$, $u = 1,2,...,z$,

and $[\boldsymbol{CS}^{m}_{k;l_1,l_2,...,l_k}(t,u)]^{(b)}$, $b = 1,2,...,\nu$, are given by

$$[\boldsymbol{CS}^{m}_{k;l_1,l_2,...,l_k}(t,u)]^{(b)}$$

$$= \begin{cases} 1 & \text{for } k < m, \\ 1 - \prod_{i=1}^{k}[1 - \exp[-\sum_{j=1}^{l_i} \lambda_{ij}(u)t]] & \text{for } k = m, \\ \exp[-\sum_{j=1}^{l_k}[\lambda_{kj}(u)]^{(b)} t][\boldsymbol{CS}^{m}_{k-1;l_1,l_2,...,l_k}(t,u)]^{(b)} & \\ + \sum_{j=1}^{m-1}[\exp[-\sum_{v=1}^{l_{k-j}}[\lambda_{k-jv}(u)]^{(b)} t]][\boldsymbol{CS}^{m}_{k-j-1;l_1,l_2,...,l_k}(t,u)]^{(b)} & \\ \cdot \prod_{i=k-j+1}^{k}[1 - \exp[-\sum_{v=1}^{l_i}[\lambda_{iv}(u)]^{(b)} t]] & \text{for } k > m, \end{cases} \quad (44)$$

for $t \geq 0$, $u = 1,2,...,z$;

x) for a consecutive "m_i out of l_i: F"-series system

$$\overline{\boldsymbol{CS}}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}(t,\cdot)$$

$$= [1, \overline{\boldsymbol{CS}}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}(t,1), ..., \overline{\boldsymbol{CS}}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}(t,z)], \quad (45)$$

where

$$\overline{\boldsymbol{CS}}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}(t,u) = 1 \text{ for } t < 0,$$

$$\overline{\boldsymbol{CS}}^{m_1,m_2,...,m_k}_{k;l_1,l_2,...,l_k}(t,u) \cong \sum_{b=1}^{\nu} p_b \prod_{i=1}^{k}[\boldsymbol{CS}^{m_i}_{i,l_i}(t,u)]^{(b)} \quad (46)$$

for $t \geq 0$, $u = 1,2,...,z$,

and $[\boldsymbol{CS}^{m_i}_{i,l_i}(t,u)]^{(b)}$, $i = 1, 2, \ldots, k$, $b = 1,2,...,\nu$, are given by

$$[\boldsymbol{CS}^{m_i}_{i,l_i}(t,u)]^{(b)}$$

$$= \begin{cases} 1 & \text{for } l_i < m_i, \\ 1 - \prod_{j=1}^{l_i}[1 - \exp[-[\lambda_{ij}(u)]^{(b)} t]] & \text{for } l_i = m_i, \\ \exp[-[\lambda_{il_i}(u)]^{(b)} t][\boldsymbol{CS}^{m_i}_{i,l_i-1}(t,u)]^{(b)} & \\ + \sum_{j=1}^{m_i-1} \exp[-[\lambda_{il_i-j}(u)]^{(b)} t][\boldsymbol{CS}^{m_i}_{i,l_i-j-1}(t,u)]^{(b)} & \\ \cdot \prod_{v=l_i-j+1}^{l_i}[1 - \exp[-[\lambda_{iv}(u)]^{(b)} t]] & \text{for } l_i > m_i, \end{cases} \quad (47)$$

for $t \geq 0$, $u = 1,2,...,z$.

Remark 1

The formulae for the safety functions stated in *Proposition 1* are valid for the considered systems under the assumption that they do not change their structures at diferrent operation states z_b, $b = 1,2,...,\nu$. This limitation can be simply omitted by the replacement in these formulae the system's structure shape constant parameters n, m, k, m_i, l_i, espectively by their changing at diferrent operation states z_b, $b = 1,2,...,\nu$, equivalent structure shape parameters $n^{(b)}$, $m^{(b)}$, $k^{(b)}$, $m_i^{(b)}$, $l_i^{(b)}$, $b = 1,2,...,\nu$. For the exponential complex technical systems, considered in *Proposition 1*, we determine the mean values $\mu(u)$ and the standard deviations $\sigma(u)$ of the unconditional lifetimes of the system in the safety state subsets $\{u, u+1,..., z\}$, $u = 1,2,...,z$, the mean values $\bar{\mu}(u)$ of the unconditional lifetimes of the system in the particular safety states u, $u = 1,2,...,z$, the system risk function $\boldsymbol{r}(t)$ and the moment τ when the system risk function exceeds a permitted level δ respectively defined by (10)-(15), after substituting for $\boldsymbol{S}(t,u)$, $u = 1,2,...,z$, the coordinates of the unconditional safety functions given respectively by (20)-(47).

4. Asymptotic approach to safety of large multistate systems at variable operation conditions

In the case of large systems, the determination of the exact safety functions of the complex systems and the system risk functions, sometimes, leads us to very complicated formulae that are often useless for safety practitioners. One of the important techniques that can be useful in this situation is the asymptotic approach [5], [8] to system safety evaluation. In this approach, instead of the preliminary complex formula for the system safety function, after assuming that the number of system components tends to infinity and finding the limit safety of the system, we can obtain its simplified form. Moreover, in the case of large systems, the possibility of combining the results of the safety joint models of complex technical systems and the results concerning the limit safety functions of the considered systems is possible. This way, the results concerned with asymptotic approach to estimation of non-repairable multi-state systems at variable operation conditions may be obtained. Main results concerning asymptotic approach to multi-state large system safety with ageing components in the constant operation conditions are comprehensively presented in the work [5], [8] and some of these results' extentions to the systems operating at the variable conditions can be found in [8].

In order to combine the results on the safety of multi-state systems related to their operation processes and the results concerning the limit safety functions of the multistate systems, and to obtain the results on the asymptotic approach to the evaluation of the large multi-state systems safety at the variable operation conditions, we assume the following definition [8].

Definition 1

A safety function

$$\boldsymbol{\mathcal{S}}(t,\cdot) = [1, \boldsymbol{\mathcal{S}}(t,1),..., \boldsymbol{\mathcal{S}}(t,z)],\ t \in (-\infty,\infty), \qquad (48)$$

where

$$\boldsymbol{\mathcal{S}}(t,u) = \sum_{b=1}^{v} p_b [\boldsymbol{\mathcal{S}}(t,u)]^{(b)},\ \ u = 1,2,...,z, \qquad (49)$$

is called a limit safety function of a complex multistate system with the safety function sequence

$$\boldsymbol{S}_n(t,\ \cdot) = [1, \boldsymbol{S}_n(t,1),..., \boldsymbol{S}_n(t,z)],\ t \in (-\infty,\infty)\ , \qquad (50)$$

$n \in N$,

where

$$\boldsymbol{S}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}_n(t,u)]^{(b)},\ u = 1,2,...,z, \qquad (51)$$

if there exist normalizing constants

$$a_n^{(b)}(u) > 0,\ \ b_n^{(b)}(u) \in (-\infty,\infty),$$

$$u = 1,2,...,z,\ b = 1,2,...,v,$$

such that

$$\lim_{n\to\infty} [\boldsymbol{S}_n(a_n^{(b)}(u)t + b_n^{(b)}(u),u)]^{(b)} = [\boldsymbol{\mathcal{S}}(t,u)]^{(b)}$$

for all t from the sets of continuity points $C_{[\boldsymbol{\mathcal{R}}(u)]^{(b)}}$ of the functions $[\boldsymbol{\mathcal{S}}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,v$.

Hence, for sufficiently large n, the following approximate formulae are valid

$$\boldsymbol{S}_n(t,\ \cdot) = [1, \boldsymbol{S}_n(t,1),..., \boldsymbol{S}_n(t,z)],\ t \in (-\infty,\infty), \qquad (52)$$

where

$$\boldsymbol{S}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{\mathcal{S}}(\frac{t - b_n^{(b)}(u)}{a_n^{(b)}(u)}, u)]^{(b)}, \qquad (53)$$

$t \in (-\infty,\infty)$, $u = 1,2,...,z$.

The following propositions concerned with the large series-parallel and parallel-series exponential systems operating at the variable operation states are exemplary results that can be worked out on the basis of the results included in [5], [8] for the considered in the paper large systems.

Proposition 2

If components of the multistate series-parallel regular system at the operation states z_b, $b = 1,2,...,v$, i.e., the system with the structure shape parameters such that

$k = k_n^{(b)}$, $l_1 = l_2 = ... = l_k = l_n^{(b)}$, $b = 1,2,...,v$, $n \in N$,

have the exponential safety functions given by (18)-(19) are homogeneous, i.e.,

$$[\lambda_{ij}(u)]^{(b)} = [\lambda(u)]^{(b)},\ \ i = 1,2,...,k_n^{(b)},$$

$$j = 1,2,...,l_n^{(b)},\ \ b = 1,2,...,v,$$

then the system unconditional multistate safety function is given by the approximate formulae, respectively in the following cases of the system structure shape at the particular operation states:

i) $k_n^{(b)} = n$, $l_n^{(b)} > 0$,

$$\boldsymbol{S}t,\cdot) = [1, \boldsymbol{S}(t,1),..., \boldsymbol{S}(t,z)]$$

where

$$\boldsymbol{S}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \exp[-n \exp[-[\lambda(u)]^{(b)} l_n^{(b)} t]] \qquad (54)$$

for $t \in (-\infty,\infty)$, $u = 1,2,...,z$;

$k_n^{(b)} \to k^{(b)}$, $l_n^{(b)} \to \infty$,

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),..., \boldsymbol{S}(t,z)]$$

where

$$\boldsymbol{S}(t,u) \cong \begin{cases} 1 & \text{for } t < 0, \\ 1 - \sum_{b=1}^{v} p_b [1 - \exp[-[\lambda(u)]^{(b)} l_n^{(b)} t]]^{k^{(b)}} & \text{for } t \geq 0, \end{cases} \quad (55)$$

$u = 1,2,...,z.$

Proposition 3

If components of the multistate parallel-series regular system at the operation states z_b, $b = 1,2,...,v$, i.e., the system with the structure shape parameters such that

$k = k_n^{(b)}$, $l_1 = l_2 = ... = l_k = l_n^{(b)}$, $b = 1,2,...,v$, $n \in N$,

have the exponential safety functions given by (18)-(19) are homogeneous, i.e.,

$$[\lambda_{ij}(u)]^{(b)} = [\lambda(u)]^{(b)}, \quad i = 1,2,...,k_n^{(b)},$$

$$j = 1,2,...,l_n^{(b)}, \quad b = 1,2,...,v,$$

then the system unconditional multi-state safety function is given by the approximate formulae, respectively in the following cases of the system structure shapes at the particular operation states:

i) $k_n^{(b)} = n$, $l_n^{(b)} \to l^{(b)}$, $l^{(b)} > 0$,

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)]$$

where

$$\boldsymbol{S}(t,u) \cong \begin{cases} 1 & \text{for } t < 0, \\ \sum_{b=1}^{v} p_b \exp[-n([\lambda(u)]^{(b)} t)^{l^{(b)}}] & \text{for } t \geq 0, \end{cases} \quad (56)$$

$u = 1,2,...,z.$

ii) $k_n^{(b)} \to k^{(b)}$, $l_n^{(b)} \to \infty$,

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)]$$

where

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [1 - \exp[-l_n^{(b)} \exp[-[\lambda(u)]^{(b)} t]]]^{k^{(b)}} \quad (57)$$

for $t \in (-\infty,\infty)$, $u = 1,2,...,z.$

It is possible to obtain similar and more general results for other considered in the paper multistate systems after some modification of the results included in [5], [8].

5. Application

As an example we will analyse the safety of one of the subsystems of the port grain elevator in its operation process. The considered system is composed of four multi-state non-homogeneous series-parallel transportation subsystems and it is the basic structure in the Baltic Grain Terminal of the Port of Gdynia assigned to handle and clearing of exported and imported grain. One of the basic elevator functions is loading railway trucks with grain. The railway truck loading is performed in the following successive grain transportation system steps:

- gravitational passing of grain from the storage placed on the 8th elevator floor through 45 hall to horizontal conveyors placed in the elevator basement,
- transport of grain through horizontal conveyors to vertical bucket elevators transporting grain to the main distribution station placed on the 9th floor,
- gravitational dumping of grain through the main distribution station to the balance placed on the 6th floor,
- dumping weighed grain through the complex of flaps placed on the 4th floor to horizontal conveyors placed on the 2nd floor,
- dumping of grain from horizontal conveyors to worm conveyors,
- dumping of grain from worm conveyors to railway trucks.

In loading the railway trucks with grain the following presented in *Figure 1* transportation subsystems take part:

S_1 – horizontal conveyors of the first type,
S_2 – vertical bucket elevators,
S_3 – horizontal conveyors of the second type,
S_4 – worm conveyors,
the main distribution station and the balance.

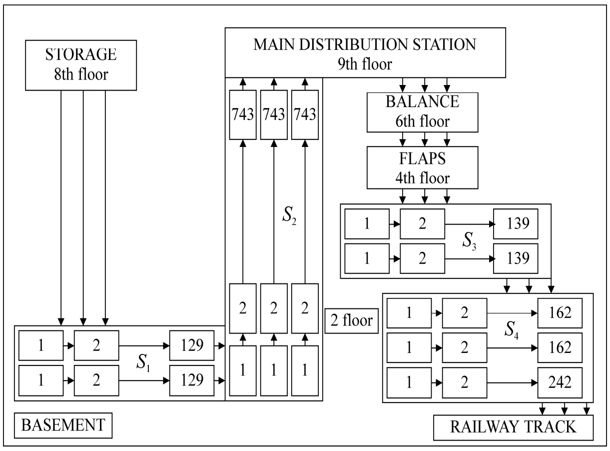

Figure 1. The scheme of the port grain transportation system structure

We will analyse the safety of the subsystem S_4 only. Taking into account the operation process of the considered transportation system, described by its operators, we distinguish its following $\nu = 3$ operation states:

z_1 – the system operation with the largest efficiency when all components of the subsystems S_1, S_2, S_3 and S_4 are used,

z_2 – the system operation with less efficiency system when the first conveyor of subsystem S_1, the first and second elevators of subsystem S_2, the first conveyor of subsystem S_3 and the first and second conveyors of subsystem S_4 are used,

z_3 – the system operation with least efficiency when only the first conveyor of subsystem S_1, the first elevator of subsystem S_2, the first conveyor of subsystem S_3 and the first conveyor of subsystem S_4 are used.

This way, the changes of the grain transportation system safety structure at different operation states are defined.

Considering the system operators opinion, we assume the vector of approximate values of the initial probabilities $p_b(0)$, $b = 1, 2, 3$,

$$[p_b(0)]_{1x3} = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$$

of the system operation process staying at the particular states z_b at the time $t = 0$ and the matrix of the probabilities of transitions between the states are given by

$$[p_{bl}]_{3x3} = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{4}{9} & 0 & \frac{5}{9} \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix} \qquad (58)$$

Moreover, we assume the following matrix of the conditional distribution functions of the system sojourn times θ_{bl}, $b, l = 1, 2, 3$,

$$[H_{bl}(t)]_{3x3} = \begin{bmatrix} 0 & 1 - e^{-5t} & 1 - e^{-10t} \\ 1 - e^{-40t} & 0 & 1 - e^{-50t} \\ 1 - e^{-10t} & 1 - e^{-20t} & 0 \end{bmatrix}$$

Further, according (1) we fix the conditional mean values $M_{bl} = E[\theta_{bl}]$, $b, l = 1,2,3,4,$ of the system sojourn times at the particular operation states as follows:

$$M_{12} = 0.20 \quad M_{13} = 0.10$$

$$M_{21} = 0.025 \quad M_{23} = 0.020 \qquad (59)$$

$$M_{31} = 0.10 \quad M_{32} = 0.05.$$

This way, the exemplary system operation process is defined and we may find its main characteristics. Namely, applying (2), (58) and (59) unconditional mean sojourn times at the particular operation states are given by:

$$M_1 = E[\theta_1] = p_{12}M_{12} + p_{13}M_{13} + p_{14}M_{14}$$

$$= \frac{1}{3}0.20 + \frac{2}{3}0.10 = 0.133, \qquad (60)$$

$$M_2 = E[\theta_2] = p_{21}M_{21} + p_{23}M_{23}$$

$$= \frac{4}{9}0.025 + \frac{5}{9}0.020 = 0.022, \qquad (61)$$

$$M_3 = E[\theta_3] = p_{31}M_{31} + p_{32}M_{32}$$

$$= \frac{1}{3}0.10 + \frac{2}{3}0.05 = 0.067. \qquad (62)$$

Further, according to (4), the system of equations

$$\begin{cases}[\pi_1, \pi_2, \pi_3] = [\pi_1, \pi_2, \pi_3][p_{bl}]_{3x3} \\ \pi_1 + \pi_2 + \pi_3 = 1,\end{cases}$$

after considering (58), takes the form

$$\begin{cases}\pi_1 = \frac{4}{9}\pi_2 + \frac{1}{3}\pi_3 \\ \pi_2 = \frac{1}{3}\pi_1 + \frac{2}{3}\pi_3 \\ \pi_3 = \frac{2}{3}\pi_1 + \frac{5}{9}\pi_2 \\ \pi_1 + \pi_2 + \pi_3 = 1.\end{cases}$$

The approximate solutions of the above system of equations are:

$$\pi_1 \cong 0.279, \ \pi_2 \cong 0.344, \ \pi_3 \cong 0.377. \qquad (63)$$

According to (3), the limit values of the system operation process transient probabilities $p_b(t)$ at the operation states z_b are given by

$$p_1 \cong 0.530, \ p_2 \cong 0.109, \ p_3 \cong 0.361. \qquad (64)$$

Taking into account the efficiency of the considered port grain transportation system we distinguish the following three safety states of the systems and its components:
state 2 – the state ensuring the largest efficiency of the system and its conveyors,
state 1 – the state ensuring less efficiency of the system caused by throwing grain off the system conveyors,
state 0 – the state involving failure of the system.
We assume that the system safety structure and its subsystems and components safety depend on its changing in time operation states. Considering the assumptions and agreements of these sections, we assume that its subsystems S_υ, $\upsilon = 1,2,3.4,$ are composed of three-state, i.e. z = 3, components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,3,4,$ having the conditional safety functions given by the vector

$$[S_{ij}^{(\upsilon)}(t,\cdot)]^{(b)} = [1, [S_{ij}^{(\upsilon)}(t,1)]^{(b)}, [S_{ij}^{(\upsilon)}(t,2)]^{(b)}],$$

$$b = 1,2,3,$$

with the exponential co-ordinates

$$[S_{ij}^{(\upsilon)}(t,1)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}],$$

$$[S_{ij}^{(\upsilon)}(t,2)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}],$$

different at various operation states z_b, $b = 1,2,3,$ and with the intensities of departure from the safety state subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}, \ [\lambda_{ij}^{(\upsilon)}(2)]^{(b)}, \ b = 1,2,3.$$

The influence of the system operation states changing on the changes of the system safety structure and its components safety functions is as follows. Next will analyse the safety of the subsystem S_4 only.
At the system operation state z_1, the subsystem S_4 consists of three chain conveyors forming series subsystems ($k^{(1)} = 3$), each composed of a wheel driving the belt, a reversible driving wheel and 160, 160 and 240 links respectively. Thus, two conveyors have 162 components and the remaining one has 242 components ($l_1^{(1)} = 162$, $l_2^{(1)} = 162$, $l_3^{(1)} = 242$) what means that the subsystem is a non-homogeneous non-regular three-state series-parallel system. with the exponential safety functions. In two series subsystems of the subsystem S_4 there are respectively:
- 2 two driving wheels marked by $E_{ij}^{(4)}$, $i = 1,2,$ $j = 1,2,$ with a safety function co-ordinates

$$[S_{ij}^{(4)}(t,1)]^{(1)} = \exp[-0.005t],$$

$$[S_{ij}^{(4)}(t,2)]^{(1)} = \exp[-0.006t], \ t \geq 0, \ i = 1,2, \ j = 1,2;$$

- 160 links marked by $E_{ij}^{(4)}$, $i = 1,2,$ $j = 3,4,...,162,$ with a safety function co-ordinates

$$[S_{ij}^{(4)}(t,1)]^{(1)} = \exp[-0.012t],$$

$$[S_{ij}^{(4)}(t,2)]^{(1)} = \exp[-0.014t], \ t \geq 0,$$

$$i = 1,2, \ j = 3,4,...,162.$$

In the third series subsystems of the subsystem S_4 there are respectively:
- 2 two driving wheels marked by $E_{ij}^{(4)}$, $i = 3,$ $j = 1,2,$ with a safety function co-ordinates

$$[S_{ij}^{(4)}(t,1)]^{(1)} = \exp[-0.022t],$$

$$[S_{ij}^{(4)}(t,2)]^{(1)} = \exp[-0.024t], \ t \geq 0, \ i = 3, \ j = 1,2;$$

- 240 links marked by $E_{ij}^{(4)}$, $i = 3$, $j = 3,4,...,242$, with a safety function co-ordinates

$[S_{ij}^{(4)}(t,1)]^{(1)} = \exp[-0.034t]$,

$[S_{ij}^{(4)}(t,2)]^{(1)} = \exp[-0.040t]$, $t \geq 0$,

$i = 3$, $j = 3,4,...,242$.

Thus, at the operation state z_1, the subsystem S_4 is a three-state series-parallel system with its structure shape parameters $k^{(1)} = 3$, $l_1^{(1)} = 162$, $l_2^{(1)} = 162$, $l_3^{(1)} = 242$, and according to the formulae appearing after Definition 3.11 in [8] and (30)-(31) its conditional safety function is given by

$$[\boldsymbol{S}^{(4)}(t,\cdot)]^{(1)} = [1, [\boldsymbol{S}^{(4)}(t,1)]^{(1)}, [\boldsymbol{S}^{(4)}(t,2)]^{(1)}], \quad (65)$$

$t \geq 0$,

where

$$[\boldsymbol{S}^{(4)}(t,1)]^{(1)} = \boldsymbol{S}_{3;162,162,242}(t,1)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{162}[R_{ij}^{(4)}(t,1)]^{(1)}][1 - \prod_{j=1}^{242}[R_{3j}^{(4)}(t,1)]^{(1)}]$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{162}[\lambda_{ij}^{(4)}(1)]^{(1)} t]][1 - \exp[-\sum_{j=1}^{242}\lambda_{3j}^{(4)}(1)]]$$

$$= 1 - [1 - \exp[-[0.005 \cdot 2 + 0.012 \cdot 160]t]^2$$

$$\cdot [1 - \exp[-[0.022 \cdot 2 + 0.034 \cdot 240]t]$$

$$= 1 - [1 - \exp[-1.930t]]^2 \cdot [1 - \exp[-8.204t]$$

$$= 2\exp[-1.930t] - 2\exp[-10.134t] - \exp[-3.860t]$$

$$+ \exp[-12.064t] + \exp[-8.204t], \quad (66)$$

$$[\boldsymbol{S}^{(4)}(t,2)]^{(1)} = \boldsymbol{S}_{3;162,162,242}(t,2)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{162}[S_{ij}^{(4)}(t,2)]^{(1)}][1 - \prod_{j=1}^{242}[S_{3j}^{(4)}(t,2)]^{(1)}]$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{162}[\lambda_{ij}^{(4)}(2)]^{(1)} t]][1 - \exp[-\sum_{j=1}^{242}\lambda_{3j}^{(4)}(2)]]$$

$$= 1 - [1 - \exp[-[0.006 \cdot 2 + 0.014 \cdot 160]t]^2$$

$$\cdot [1 - \exp[-[0.024 \cdot 2 + 0.040 \cdot 240]t]$$

$$= 1 - [1 - \exp[-2.252t]]^2 \cdot [1 - \exp[-9.648t]$$

$$= 2\exp[-2.252t] - 2\exp[-11.900t] - \exp[-4.504t]$$

$$+ \exp[-14.152t] + \exp[-9.648t]. \quad (67)$$

The expected values of the subsystem S_4 conditional lifetimes in the safety state subsets $\{1,2\}, \{2\}$ at the operation state z_1, calculated from the results given by (66)-(67), according to (11)-(12), respectively are:

$$\mu_1(1) \cong 0.785 \ \mu_1(2) \cong 0.672 \text{ month}. \quad (68)$$

At the system operation state z_2, the subsystem S_4, consists of three identical chain conveyors forming series subsystems ($k^{(2)} = 2$), each composed of a wheel driving the belt, a reversible driving wheel and 160 links ($l_1^{(2)} = 162$, $l_2^{(2)} = 162$) what means that the subsystem is a non-homogeneous regular three-state series-parallel system with the exponential safety functions. In the series subsystems of the subsystem S_4 there are respectively:

- 2 two driving wheels marked by $E_{ij}^{(4)}$, $i = 1,2$, $j = 1,2$, with a safety function co-ordinates

$[S_{ij}^{(4)}(t,1)]^{(2)} = \exp[-0.002t]$,

$[S_{ij}^{(4)}(t,2)]^{(2)} = \exp[-0.004t]$, $t \geq 0$, $i = 1,2$, $j = 1,2$;

- 160 links marked by $E_{ij}^{(4)}$, $i = 1,2$, $j = 3,4,...,162$, with a safety function co-ordinates

$[S_{ij}^{(4)}(t,1)]^{(2)} = \exp[-0.008t]$,

$[S_{ij}^{(4)}(t,2)]^{(2)} = \exp[-0.010t]$, $t \geq 0$,

$i = 1,2$, $j = 3,4,...,162$.

Thus, at the operation state z_2, the subsystem S_4 is a three-state series-parallel system with its structure shape parameters $k^{(2)} = 2$, $l_1^{(2)} = 162$, $l_2^{(2)} = 162$, and according to the formulae appearing after Definition 3.11 in [8] and (30)-(31) its conditional safety function is given by

$$[\boldsymbol{S}^{(4)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{S}^{(4)}(t,1)]^{(2)}, [\boldsymbol{S}^{(4)}(t,2)]^{(2)}], \quad (69)$$

$t \geq 0$,

where

$$[\boldsymbol{S}^{(4)}(t,1)]^{(2)} = \boldsymbol{S}_{2;162,162}(t,1)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{162}[S_{ij}^{(4)}(t,1)]^{(2)}]$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{162}[\lambda_{ij}^{(4)}(1)]^{(2)} t]]$$

$$= 1 - [1 - \exp[-[0.002 \cdot 2 + 0.008 \cdot 160]t]^2$$

$$= 1 - [1 - \exp[-1.284t]]^2$$

$$= 2\exp[-1.284t] - \exp[-2.5680t], \quad (70)$$

$$[\boldsymbol{S}^{(4)}(t,2)]^{(2)} = \boldsymbol{S}_{2;162,162}(t,2)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{162}[S_{ij}^{(4)}(t,2)]^{(2)}]$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{162}[\lambda_{ij}^{(4)}(2)]^{(2)} t]]$$

$$= 1 - [1 - \exp[-[0.004 \cdot 2 + 0.010 \cdot 160]t]^2$$

$$= 1 - [1 - \exp[-1.608t]]^2$$

$$= 2\exp[-1.608t] - \exp[-3.2160t]. \quad (71)$$

The expected values of the subsystem S_4 conditional lifetimes in the safety state subsets $\{1,2\}, \{2\}$ at the operation state z_2, calculated from the results given by (70)-(71), according to (11)-(12), respectively are:

$$\mu_2(1) \cong 1.168 \ \mu_2(2) \cong 0.933 \text{ month}. \quad (72)$$

At the system operational state z_3, The subsystem S_4, consists of one chain conveyor forming a series system ($k^{(3)} = 1$), composed of a wheel driving the belt, a reversible driving wheel and 160 links ($l_1^{(3)} = 162$) with the exponential safety functions. In the series system of the subsystem S_4 there are respectively:

- 2 two driving wheels marked by $E_{ij}^{(4)}$, $i = 1, \ j = 1,2,$ with a safety function co-ordinates

$$[S_{ij}^{(4)}(t,1)]^{(3)} = \exp[-0.001t],$$

$$[S_{ij}^{(4)}(t,2)]^{(3)} = \exp[-0.003t], \ t \ge 0, \ i = 1, \ j = 1,2;$$

- 160 links marked by $E_{ij}^{(4)}$, $i = 1, \ j = 3,4,...,162$, with a safety function co-ordinates

$$[S_{ij}^{(4)}(t,1)]^{(3)} = \exp[-0.007t],$$

$$[S_{ij}^{(4)}(t,2)]^{(3)} = \exp[-0.009t], \ t \ge 0,$$

$$i = 1, \ j = 3,4,...,162.$$

Thus, at the operation state z_3, the subsystem S_4 is a three-state series-parallel system (a series system) with its structure shape parameters $k^{(3)} = 1$, $l_1^{(3)} = 162$, and according to the formulae appearing after Definition 3.11 in [8] and (30)-(31) its conditional safety function is given by

$$[\boldsymbol{S}^{(4)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{S}^{(4)}(t,1)]^{(3)}, [\boldsymbol{S}^{(4)}(t,2)]^{(3)}], \quad (73)$$

$$t \ge 0,$$

where

$$[\boldsymbol{S}^{(4)}(t,1)]^{(3)} = \boldsymbol{S}_{1;162}(t,1) = 1 - \prod_{i=1}^{1}[1 - \prod_{j=1}^{162}[S_{ij}^{(4)}(t,1)]^{(3)}]$$

$$= \prod_{j=1}^{162}[S_{1j}^{(4)}(t,1)]^{(3)} = \exp[-\sum_{j=1}^{162}[\lambda_{1j}^{(3)}(1)]^{(3)} t]$$

$$= \exp[-[0.001 \cdot 2 + 0.007 \cdot 160]t]$$

$$= \exp[-1.122t] \quad (74)$$

$$[\boldsymbol{S}^{(4)}(t,2)]^{(3)} = \boldsymbol{S}_{1;162}(t,2)$$

$$= 1 - \prod_{i=1}^{1}[1 - \prod_{j=1}^{162}[S_{ij}^{(4)}(t,2)]^{(3)}]$$

$$= \prod_{j=1}^{162}[S_{1j}^{(4)}(t,2)]^{(3)} = \exp[-\sum_{j=1}^{162}[\lambda_{1j}^{(3)}(2)]^{(3)} t]$$

$$= \exp[-[0.003 \cdot 2 + 0.009 \cdot 160]t]$$

$$= \exp[-1.446t] \quad (75)$$

The expected values of the subsystem S_4 conditional lifetimes in the safety state subsets $\{1,2\}, \{2\}$ at the operation state z_3, calculated from the results given by (74)-(75), according to (11)-(12), respectively are:

$$\mu_3(1) \cong 0.891 \ \mu_3(2) \cong 0.692 \text{ month}. \quad (76)$$

In the case when the subsystem S_4 operation time is large enough its unconditional four-state safety function is given by the vector

$$\boldsymbol{S}^{(4)}(t,\cdot) = [1, \boldsymbol{S}^{(4)}(t,1), \boldsymbol{S}^{(4)}(t,2)] \ t \geq 0, \quad (77)$$

where according to (9) and considering the system operation process transient probabilities at the operation states determined by (64), the vector co-ordinates are given respectively by

$$\boldsymbol{S}^{(4)}(t,1) = p_1[\boldsymbol{S}^{(4)}(t,1)]^{(1)} + p_2[\boldsymbol{S}^{(4)}(t,1)]^{(2)}$$

$$+ p_3[\boldsymbol{S}^{(4)}(t,1)]^{(3)}$$

$$= 0.530 \cdot [\boldsymbol{S}^{(4)}(t,1)]^{(1)} + 0.109 \cdot [\boldsymbol{S}^{(4)}(t,1)]^{(2)}$$

$$+ 0.361 \cdot [\boldsymbol{S}^{(4)}(t,1)]^{(3)} \text{ for } t \geq 0, \quad (78)$$

$$\boldsymbol{S}^{(4)}(t,2) = p_1[\boldsymbol{S}^{(4)}(t,2)]^{(1)} + p_2[\boldsymbol{S}^{(4)}(t,2)]^{(2)}$$

$$+ p_3[\boldsymbol{S}^{(4)}(t,2)]^{(3)}$$

$$= 0.530 \cdot [\boldsymbol{S}^{(4)}(t,2)]^{(1)} + 0.109 \cdot [\boldsymbol{S}^{(4)}(t,2)]^{(2)}$$

$$+ 0.361 \cdot [\boldsymbol{S}^{(4)}(t,2)]^{(3)} \text{ for } t \geq 0, \quad (79)$$

where the safety functions $[\boldsymbol{S}^{(4)}(t,1)]^{(1)}$, $[\boldsymbol{S}^{(4)}(t,1)]^{(2)}$, $[\boldsymbol{S}^{(4)}(t,1)]^{(3)}$ are given by (66), (70), (74) and $[\boldsymbol{S}^{(4)}(t,2)]^{(1)}$, $[\boldsymbol{S}^{(4)}(t,2)]^{(2)}$, $[\boldsymbol{S}^{(4)}(t,2)]^{(3)}$ are given by (67), (71), (75).

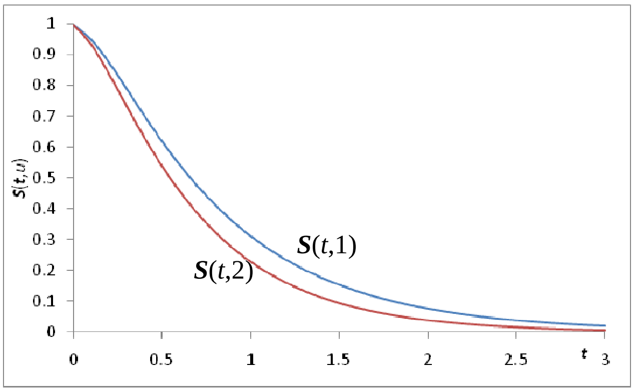

Figure 2. The graph of the subsystem S_4 safety function $\boldsymbol{S}^{(4)}(t,\cdot)$ coordinates.

The expected value of the subsystem S_4 unconditional lifetime in the safety state subset $\{1,2\}$, calculated according to (10) from the results given by (68), (72), (76) and (64), is

$$\mu(1) = p_1\mu_1(1) + p_2\mu_2(1) + p_3\mu_3(1)$$

$$= 0.530 \cdot 0.785 + 0.109 \cdot 1.168 + 0.361 \cdot 0.891$$

$$\cong 0.865 \text{ month.} \quad (80)$$

The expected value of the system unconditional lifetime in the safety state subset $\{2\}$, calculated according to (10) from the results given by (68), (72), (76) and (64), is

$$\mu(2) = p_1\mu_1(2) + p_2\mu_2(2) + p_3\mu_3(2)$$

$$= 0.530 \cdot 0.672 + 0.109 \cdot 0.933 + 0.361 \cdot 0.692$$

$$\cong 0.708 \text{ month.} \quad (81)$$

Further, considering (80) and (81) and applying (13), the mean values of the unconditional lifetimes in the particular safety states 1, 2, respectively are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.157 \text{ month,}$$

$$\bar{\mu}(2) = \mu(2) = 0.708 \text{ month.} \quad (82)$$

Since the critical safety state is $r = 1$, then the subsystem S_4 risk function, according to (14) and (64), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}^{(4)}(t,1)$$

$$= 1 - [0.530 \cdot [\boldsymbol{S}^{(4)}(t,1)]^{(1)} + 0.109 \cdot [\boldsymbol{S}^{(4)}(t,1)]^{(2)}$$
$$+ 0.361 \cdot [\boldsymbol{S}^{(4)}(t,1)]^{(3)}] \text{ for } t \geq 0, \quad (83)$$

where the safety functions $[\boldsymbol{S}^{(4)}(t,1)]^{(1)}$, $[\boldsymbol{S}^{(4)}(t,1)]^{(2)}$, $[\boldsymbol{S}^{(4)}(t,1)]^{(3)}$ are given by (66), (70), (74).

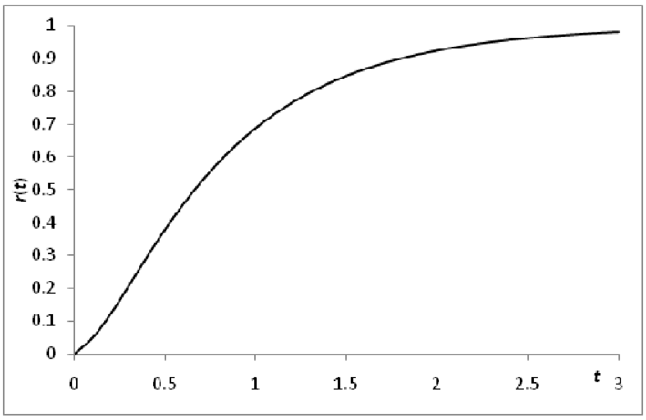

Figure3. The graph of the subsystem S_4 risk function $\boldsymbol{r}(t)$

Hence, by (15), the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.102 \qquad (84)$$

6. Conclusions

The integrated general model of complex systems' safety, linking their safety models and their operation processes models and considering variable at different operation states their safety structures and their components safety parameters is constructed. The material given in this chapter delivers the procedures and algorithms that allow to find the main an practically important safety characteristics of the complex technical systems at the variable operation condition. Next the results are applied to the safety evaluation of the one subsystem of the port grain transportation system. The predicted safety characteristics of the exemplary system operating at the variable conditions are different from those determined for this system operating at constant conditions. This fact justifies the sensibility of considering real systems at the variable operation conditions that is appearing out in a natural way from practice. This approach, upon the good accuracy of the systems' operation processes and the systems' components safety parameters identification, makes their safety prediction more precise.

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