dr inż. Piotr Stanisław FRĄCZAK[a], dr n. tech. Andrzej Antoni CZAJKOWSKI[b]

[a] Technical and Mechanical Engineering School of prof. Maksymilian Tytus Huber in Szczecin
Technikum Mechaniczno-Energetyczne im. prof. Maksymiliana Tytusa Hubera w Szczecinie

[b] Higher School of Technology and Economics in Szczecin, Faculty of Automotive Systems
Wyższa Szkoła Techniczno-Ekonomiczna w Szczecinie, Wydział Systemów Automotive

DIFFERENTIAL EQUATION IN DESCRIPTION OF TRANSIENT STATE IN THE *RLC* CIRCUITS AT CONSTANT VOLTAGE EXCITATION - WITH APPLICATION OF MATHEMATICA PROGRAM

Abstract

Introduction and aim: Some description and simulation of the transient in the RLC circuit have been presented in this paper. Also has been shown the application of the Laplace transform to solve the differential equation.
Material and methods: By using the Laplace transformation to the option of the transition from linear differential equations of the second order with constant coefficients to the algebraic equations. In numerical analysis, a reversed Laplace transform was applied by using the *Mathematica* program.
Results: It has been obtained the same curve shape of the transient current at the determination by the second-order differential equation (classical solution) and the different-integral equation by using the inverse Laplace transform.
Conclusion: By applying both the Laplace transform method and the analytical method, the same transient currents are obtained as a function of time.

Keywords: Circuits, transient states, differential equations, Laplace transform, numerical simulation, *Mathematica.*

(Received: 01.10.2017; Reviewed: 15.10.2017; Accepted: 30.10.2017)

RÓWNANIE RÓŻNICZKOWE W OPISIE STANU NIEUSTALONEGO W OBWODACH RLC PRZY WYMUSZENIU NAPIĘCIEM STAŁYM - Z ZASTOSOWANIEM PROGRAMU MATHEMATICA

Streszczenie

Wstęp i cel: *W pracy przedstawiono opis i symulacje stanu nieustalonego w obwodzie elektrycznym RLC. Pokazano zastosowanie przekształcenia Laplace'a do rozwiązywania równania różniczkowego.*
Materiał i metody: *W wyniku zastosowania przekształceń Laplace'a wskazano na możliwość przejścia od równań różniczkowych liniowych drugiego rzędu o stałych współczynnikach do równań algebraicznych. W analizie numerycznej zastosowano odwrtoną transformatę Laplace'a wykorzystując program Mathematica.*
Wyniki: *Otrzymano jednakowy kształt przebiegu krzywej prądu nieustalonego przy wyznaczaniu równaniem różniczkowym drugiego rzędu (rozwiązanie klasyczne) i równaniem różniczkowo-całkowym z wykorzystaniem przekształcenia odwrotnego Laplace'a.*
Wniosek: *Stosując zarówno metodę przekształceń Laplace'a i metodę analityczną otrzymuje się jednakowe przebiegi prądu nieustalonego w funkcji czasu.*

Słowa kluczowe: *Obwody elektryczne, stany nieustalone, równania różniczkowe, przekształcenie Laplace'a, symulacja numeryczna, Mathematica.*

(Otrzymano: 01.10.2017; Zrecenzowano: 15.10.2017; Zaakceptowano: 30.10.2017)

1. Transients in *RLC* circuits

A transient is a transient between two pre-set circuit states due to structure change or change in circuit parameters. The condition of the circuit is described by means of differential equations. A transient is a transient between two pre-set circuit states due to structure change or change in circuit parameters. The condition of the circuit is described by means of differential equations. For a linear, stationary circuit with clustered parameters, these are ordinary differential heterogeneous linear equations with constant coefficients [2]-[7].

With regard to linear electric circuits, the initial conditions imply from the commutation conditions[1], which in turn implies demands that the energy change in a given element of the system does not take place took place by the leaps. For each coil (linear and unconjugated with another) the current continuity condition is met, and for each capacitor the voltage continuity condition is met, in other words, these values can never be change by the leaps.

The solution of the differential equation is conveniently presented in the form of a sum of two components: transient (free) and fixed (forced). From the mathematical point of view, these components correspond to a general integral of the homogeneous equation and a particular integral of the heterogeneous equation [2]-[7].

The form of the fixed component depends both on the design and parameters of the circuit as well as on the forcing. In turn, the form of the transition component does not depend on the type of force, but only on the circuit's construction and its parameters. These components are determined from the initial conditions. It should be added that in real circuits (with losses), the transient component disappears after a certain time depending on the design and parameters of the circuit [2]-[7].

2. Theoretical analysis of the serial switching *RLC* of the circuit for constant voltage

Figure 1 presents the simulation of switching the RLC circuit to constant voltage E. In the circuit at time t=0, the circuit breaker W is closed. The current waveform should be determined.

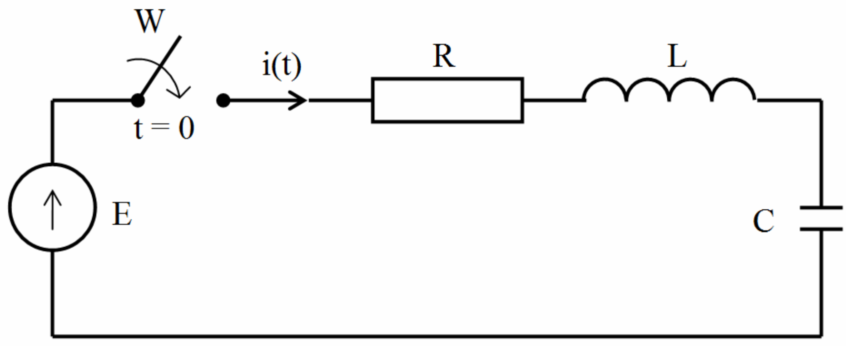

Fig. 1. Substitute circuit of the *RLC* serial circuit (transient state i = 0 for t = 0)

Source: Elaboration of the Authors

According to Kirchhoff's second law, we get dependence [2]-[7]:

$$u_L(t) + u_R(t) + u_C(t) = E. \tag{1}$$

Substituting for the equation (1) voltage drops $u_L(t)$, $u_R(t)$, $u_C(t)$ define by the formulas:

$$u_R(t) = L\frac{di(t)}{dt}, \tag{2}$$

[1] That is, switching conditions.

$$u_R(t) = R \cdot i(t), \tag{3}$$

$$u_C(t) = u_C(0) + \frac{1}{C} \cdot \int_0^t i(\tau)\, d\tau \tag{4}$$

we get the following differential-integral equation:

$$L\frac{di(t)}{dt} + R \cdot i(t) + u_C(0) + \frac{1}{C}\int_0^t i(\tau)\, d\tau = E. \tag{5}$$

The current i(t) at time t=0 equals zero. The first derivative of the function i =i (t) at time t=0 equals E / L. Therefore, the following initial conditions are accepted

$$i(t = 0) = 0, \tag{6}$$

$$\left.\frac{di}{dt}\right|_{(t=0)} = \frac{E}{L}. \tag{7}$$

and at the same time

$$u_C(t = 0) = 0. \tag{8}$$

Differentiating the equation (5) with respect to the variable t, we obtain a second order differential equation with constant coefficients, which has the following form:

$$L\frac{d^2 i(t)}{dt^2} + R \cdot \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0. \tag{9}$$

Equation (9) describes the phenomenon of discharging capacitor with capacity C through the circuit with resistance R and the coefficient of self-induction L, where C, R and L are assumed to be constant. An unknown function is the voltage i=i(t) between the covers of the capacitor.

For the equation (9) we assume that the particular integrals have the form:

$$i(t) = e^{\lambda t}. \tag{10}$$

Then the first and second derivative of the function (10) with respect to the variable t have the form:

$$\frac{di}{dt} = \lambda e^{\lambda t}, \tag{11}$$

$$\frac{d^2 i}{dt^2} = \lambda^2 e^{\lambda t}. \tag{12}$$

After substituting (10)-(12) into equation (9) and ordering, the characteristic equation of the differential equation (9) has the following form:

$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0. \tag{13}$$

The square root of discriminant of the characteristic equation (13) is defined by the formula:

$$\sqrt{\Delta} = \sqrt{\frac{R^2}{L^2} - 4\frac{1}{LC}}. \tag{14}$$

• Case 1. *Aperiodic case.*

Two different real roots of the equation (13).

The characteristic equation (13) has two different real roots when:

$$\frac{R^2}{L^2} - 4\frac{1}{LC} > 0 \quad \Leftrightarrow \quad R^2 - 4\frac{L}{C} > 0. \tag{15}$$

Then these roots have the following form:

$$\lambda_1 = -\frac{R}{2L} - \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 - 4\frac{1}{LC}}\ , \tag{16}$$

$$\lambda_2 = -\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 - 4\frac{1}{LC}}\ . \tag{17}$$

Therefore, we predict the particular integrals of the equation (9) in the following form:

$$i_1(t) = e^{\lambda_1 t}, \tag{18}$$

$$i_2(t) = e^{\lambda_2 t}. \tag{19}$$

The general integral of equation (9) has the following form:

$$i(t) \equiv A_1 i_1(t) + A_2 i_2(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}, \tag{20}$$

where $A_1, A_2 \in \Re$.

In turn, by differentiating the equation (20) with respect to the variable t, we obtain:

$$\frac{di(t)}{dt} = A_1\lambda_1 e^{\lambda_1 t} + A_2\lambda_2 e^{\lambda_2 t}. \tag{21}$$

The integration constants A_1 and A_2, considering the conditions (6) and (7) in equations (20) and (21), are determined from the system:

$$\begin{cases} i(0) = A_1 + A_2 = 0 \\ \left.\dfrac{di}{dt}\right|_{(t=0)} = A_1\lambda_1 + A_2\lambda_2 = \dfrac{E}{L}. \end{cases} \tag{22}$$

The solution of the system (22) has the form:

$$\begin{cases} A_1 = -\dfrac{1}{(\lambda_2 - \lambda_1)}\dfrac{E}{L} \\ A_2 = +\dfrac{1}{(\lambda_2 - \lambda_1)}\dfrac{E}{L}. \end{cases} \tag{23}$$

By inserting the received integration constants defined by formulas (23), to the general solution (21) the particular integral of the equation (9) under the initial conditions (6) and (7) takes the form:

$$i(t) = -\frac{1}{(\lambda_2 - \lambda_1)}\frac{E}{L}e^{\lambda_1 t} + \frac{1}{(\lambda_2 - \lambda_1)}\frac{E}{L}e^{\lambda_2 t}, \tag{24}$$

$$i(t) = \frac{1}{(\lambda_2 - \lambda_1)}\frac{E}{L}(e^{\lambda_2 t} - e^{\lambda_1 t})\ . \tag{25}$$

Finally, after entering formulas (16) and (17) to solution (25), we get:

$$i(t)=\frac{E}{L\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}}\left\{\exp\left[\left(-\frac{R}{2L}+\frac{1}{2}\sqrt{\frac{R^2}{L^2}-4\frac{1}{LC}}\right)t\right]-\exp\left[\left(-\frac{R}{2L}-\frac{1}{2}\sqrt{\frac{R^2}{L^2}-4\frac{1}{LC}}\right)t\right]\right\}. \quad (26)$$

or after transforming:

$$i(t)=\frac{E\cdot\exp\left[\left(-\frac{R}{2L}\right)t\right]}{L\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}}\left\{\exp\left[\left(\frac{1}{2}\sqrt{\frac{R^2}{L^2}-4\frac{1}{LC}}\right)t\right]-\exp\left[\left(-\frac{1}{2}\sqrt{\frac{R^2}{L^2}-4\frac{1}{LC}}\right)t\right]\right\}. \quad (27)$$

Using the hyperbolic sine definition [2] the general solution (27) receives the form:

$$i(t)\equiv i1(t)=\frac{2\cdot E\cdot\exp\left(-\frac{R}{2L}\cdot t\right)}{L\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}}\cdot\sinh\left[\left(\frac{1}{2}\sqrt{\frac{R^2}{L^2}-4\frac{1}{LC}}\right)\cdot t\right], \quad (28)$$

where $\left(\frac{R}{2L}\right)^2-\frac{1}{LC}>0$.

• Case 2. *Critical aperiodic case.*

One double real root of the equation (13).

A characteristic equation (13) has one double real root when:

$$\frac{R^2}{L^2}-4\frac{1}{LC}=0 \quad\Leftrightarrow\quad R^2-4\frac{L}{C}=0. \quad (29)$$

It has the following form:

$$\lambda_0=\lambda_1=\lambda_2=-\frac{R}{2L}. \quad (30)$$

Therefore, we anticipate the particular integral of the equation (9) in the form:

$$i_1(t)=e^{\lambda_0 t}, \quad (31)$$

$$i_2(t)=t\cdot e^{\lambda_0 t}. \quad (32)$$

The general integral of the equation (9) is given by the formula:

$$i(t)\equiv A_1 i_1(t)+A_2 i_2(t)=A_1 e^{\lambda_0 t}+A_2 t\cdot e^{\lambda_0 t}, \quad (33)$$

where $A_1, A_2 \in \Re$. By differentiating the equation (33) with respect to variable t, we obtain:

$$\frac{di(t)}{dt}=A_1\lambda_0 e^{\lambda_0 t}+A_2 e^{\lambda_0 t}+A_2\lambda_0 t e^{\lambda_0 t}= \\ =A_1\lambda_0 e^{\lambda_0 t}+(1+\lambda_0 t)A_2 e^{\lambda_0 t}. \quad (34)$$

[2] Hyperbolic sine is defined as follows: $\sinh(t)\equiv 0{,}5[\exp(t)-\exp(-t)]$.

Constants A_1 and A_2, taking into account the conditions (6)-(7) in equations (33)-(34), we get from the system:

$$\begin{cases} i(0) = A_1 = 0 \\ \left.\dfrac{di}{dt}\right|_{(t=0)} = A_1\lambda_0 + A_2 = \dfrac{E}{L}. \end{cases} \tag{35}$$

Thus, the system (35) has solutions:

$$\begin{cases} A_1 = 0 \\ A_2 = \dfrac{E}{L}. \end{cases} \tag{36}$$

By inserting the integration constants A_1 and A_2, defined by formulas (36), to the general solution (33), the particular integral of the equation (9) under the initial conditions (6) and (7) receives the following form:

$$i(t) = \frac{E}{L} t \cdot \exp(\lambda_0 t). \tag{37}$$

After introducing formula (30) to (37), the particular integral of the equation (9) has the following form:

$$i(t) \equiv i2(t) = \frac{E}{L} \cdot t \cdot \exp\left[\left(-\frac{R}{2L}\right)t\right], \tag{38}$$

for $R^2 - 4\dfrac{L}{C} = 0$.

• <u>Case 3</u>. *Oscillatory case − (Sinusoidal vibrations suppressed exponentially).*

Two different complex conjugated roots of the equation (13).

The characteristic equation (13) has two different complex conjugated roots when:

$$\frac{R^2}{L^2} - 4\frac{1}{LC} < 0 \quad \Leftrightarrow \quad (-4)\left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right] < 0 \quad \Leftrightarrow \quad \frac{1}{LC} - \left(\frac{R}{2L}\right)^2 > 0. \tag{39}$$

They have the following form:

$$\lambda_1 = -\frac{R}{2L} + \frac{\sqrt{-4}}{2} \cdot \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = -\frac{R}{2L} + j \cdot \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}, \tag{40}$$

$$\lambda_2 = -\frac{R}{2L} - \frac{\sqrt{-4}}{2} \cdot \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = -\frac{R}{2L} - j \cdot \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}. \tag{41}$$

Considering the formula (40) the particular integrals of the equation (9) we anticipate in the following form:

$$i_1(t) = \left[\exp\left(-\frac{R}{2L} \cdot t\right)\right] \cdot \cos\left(t \cdot \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\right), \tag{42}$$

$$i_2(t) = \left[\exp\left(-\frac{R}{2L}\cdot t\right)\right]\cdot \sin\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right). \tag{43}$$

The general integral of equation (9) has the following form:

$$i(t) \equiv i3(t) = A_1 i_1(t) + A_2 i_2(t) =$$
$$= A_1\left[\exp\left(-\frac{R}{2L}\cdot t\right)\right]\cdot \cos\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right) + A_2\left[\exp\left(-\frac{R}{2L}\cdot t\right)\right]\cdot \sin\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right), \tag{44}$$

$$i(t) \equiv i3(t) = \left[\exp\left(-\frac{R}{2L}\cdot t\right)\right]\cdot\left[A_1\cos\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right) + A_2\sin\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)\right]. \tag{45}$$

where $A_1, A_2 \in \Re$.

In turn, by differentiating the equation (45) with respect to variable t, we obtain:

$$\frac{di(t)}{dt} = \left(-\frac{R}{2L}\right)\left[\exp\left(-\frac{R}{2L}\cdot t\right)\right]\cdot\left[A_1\cos\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right) + A_2\sin\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)\right] +$$
$$+\left[\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right]\left[\exp\left(-\frac{R}{2L}\cdot t\right)\right]\cdot\left[A_2\cos\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right) - A_1\sin\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)\right]. \tag{46}$$

Constants A_1 and A_2, taking into account the conditions (6) and (7) in equations (45) and (46), we determine from the system:

$$\begin{cases} i(0) = A_1 = 0 \\ \left.\frac{di}{dt}\right|_{(t=0)} = A_1\left(-\frac{R}{2L}\right) + A_2\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2} = \frac{E}{L}. \end{cases} \tag{47}$$

The system (47) has the solution:

$$\begin{cases} A_1 = 0 \\ A_2 = \dfrac{1}{\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}}\dfrac{E}{L}. \end{cases} \tag{48}$$

By inserting the integration constants A_1 and A_2, defined by formulas (48), to the general solution (45), the particular integral of the equation (9) under the initial conditions (6) and (7) has the following form:

$$i(t) \equiv i3(t) = \frac{E\cdot\exp\left(-\frac{R}{2L}\cdot t\right)}{L\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}}\cdot \sin\left(t\cdot\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right), \tag{49}$$

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} < 0 \quad \Leftrightarrow \quad \frac{1}{LC} - \left(\frac{R}{2L}\right)^2 > 0. \tag{50}$$

3. Laplace transformation

3.1. Determination of Laplace transformation

Lapalace transformation

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt = F(s) \tag{51}$$

assigns the function f(t) of the real variable t, called the original, to some other function F(s) of the complex variable s, called its image [2]-[7].

It is assumed that the original f (t) on its domain $t \geq 0$, it is a function of smooth pieces (i.e. regular), which for $t \to +\infty$ increasing no faster than $e^{\alpha t}$, where $\alpha > 0$ [3]. Recall that the function is regular in a certain area if it is differentiable at each point of the domain.

Laplace transformation is a transformation of a set of functions for which Laplace integral is convergent in a set of complex functions of a complex variable. In contrast, Laplace transform is only an image of a certain function f (t) by Laplace transform.

3.2. Selected properties of the Laplace transform

Property 1. *(Homogeneity)*

$$\mathcal{L}\{\lambda \cdot f(t)\} = \lambda \cdot \mathcal{L}\{f(t)\}, \quad \lambda \in R. \tag{52}$$

Property 2. *(Additivity)*

$$\mathcal{L}\{f_1(t) + f_2(t)\} = \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\}. \tag{53}$$

Property 3. *(Linearity)*

$$\mathcal{L}\{\lambda_1 f_1(t) + \lambda_2 f_2(t)\} = \lambda_1 \cdot \mathcal{L}\{f_1(t)\} + \lambda_2 \cdot \mathcal{L}\{f_2(t)\}. \tag{54}$$

3.3. Reverse Laplace transform

Knowing the transform of the function (its image), you can use the formula [2]-[7]:

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} e^{st} F(s)\, ds = \begin{cases} f(t) \text{ dla } t > 0 \\ 0 \text{ dla } t > 0 \end{cases} \tag{55}$$

and set the function itself (original).

The integration path in this case is a straight line parallel to the imaginary axis with the equation:

$$\mathrm{Re}(s) = c \tag{56}$$

where $\mathrm{Re}(s) = c > \alpha$.

If t = 0, then the function has a jump.

However, when $\lim_{t \to +\infty} f(t) \neq 0$, then integral takes the average value $\frac{1}{2} f(0^+)$.

3.4. Laplace transform and differential equations

Equation (5) is transformed into an equation in the Laplace s domain [3]:

$$L \cdot \mathcal{L}\left\{\frac{di}{dt}\right\} + R \cdot \mathcal{L}\{i(t)\} + \frac{1}{C} \cdot \mathcal{L}\left\{\int_0^t i(\tau)\, d\tau\right\} = E \cdot \mathcal{L}\{1\}. \tag{57}$$

In the equation (57), the following properties are introduced for the Laplace transform [2]-[7]:

$$L \cdot \mathcal{L}\left\{\frac{di}{dt}\right\} = L \cdot s \cdot I(s), \tag{58}$$

$$R \cdot \mathcal{L}\{i(t)\} = R \cdot I(s), \tag{59}$$

$$\frac{1}{C} \cdot \mathcal{L}\left\{\int_0^t i(\tau)\, d\tau\right\} = \frac{1}{C} \cdot \frac{1}{s} \cdot I(s), \tag{60}$$

$$E \cdot \mathcal{L}\{1\} = E \cdot \frac{1}{s} \cdot I(s). \tag{61}$$

After entering the relationship (58)-(61) in the equation (57) in the domain of Laplace we have:

$$s \cdot L \cdot I(s) + R \cdot I(s) + \frac{1}{s \cdot C} \cdot I(s) = \frac{E}{s} \cdot I(s). \tag{62}$$

From here we get:

$$I(s) = \frac{E}{s \cdot \left(s \cdot L + R + \frac{1}{s \cdot C}\right)}. \tag{63}$$

Current in the time domain i(t) is equal to Laplace inverse transform I(s) [2]-[7]:

$$i(t) = \mathcal{L}^{-1}\left[\frac{E}{s \cdot \left(s \cdot L + R + \frac{1}{s \cdot C}\right)}\right]. \tag{64}$$

As a result of applying the Laplace transformations, we move from the differential-integral equation (5) to the algebraic equation (62).

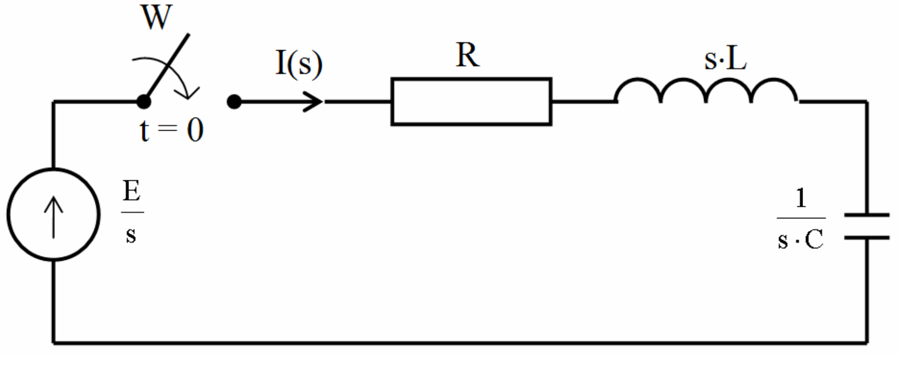

Fig. 2. Substitute circuit of the *RLC* serial circuit in Laplace field
Source: Elaboration of the Authors

[3] In this equation we accept $u_C(t) = 0$.

4. Numerical analysis in the *Mathematica* program of the serial switching RLC of the circuit for constant voltage

4.1. Data for numerical analysis of the intensity function

Name:	Designation:	SI unit:	Conversions:
Voltage	$E = 100$	[E] = [V]	$\left[\frac{1}{LC}\right]=\left[\frac{1}{VsA^{-1}AsV^{-1}}\right]=\left[\frac{1}{s^2}\right]$
Inductance	$L = 2\cdot10^{-3}$	[L] = [H] [L] =[VsA^{-1}]	$\left[\frac{R}{2L}\right]=\left[\frac{\Omega}{H}\right]=\left[\frac{VA^{-1}}{VsA^{-1}}\right]=\left[\frac{1}{s}\right]$
Resistance	$\Delta > 0$ $R_1=85$ $\Delta = 0$ $R_2=20\sqrt{10}$ $\Delta < 0$ $R_3=4$	[R] = [Ω] [R] = [VA^{-1}]	$\left[\left(\frac{R}{2L}\right)^2\right]=\left[\frac{1}{s^2}\right]$
Capacity	$C = 2\cdot10^{-6}$	[C] = [F] [C] = [AsV^{-1}]	$-\frac{R}{L}\pm\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}=\left[\frac{1}{s}\right]$
Intensity	i	[i] = [A]	$\left[\frac{E}{2L}\left(\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)^{-1}\right]=\left[\frac{Vs}{VsA^{-1}}\right]=[A]$
Time	t	[t] = [s]	$t\cdot\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}=\left[s\cdot\frac{1}{s}\right]=[-]$

Source: Elaboration of the Authors

Note that the accepted values: E=100 [V], L = $2\cdot10^{-3}$ [H], C = $2\cdot10^{-6}$ [F] and for:

resistance $0 < R < 20\sqrt{10}$ current intensity is defined by the formula (28),
resistance $R = 20\sqrt{10}$ current intensity is defined by the formula (38),
resistance $20\sqrt{10} < R < +\infty$ current intensity is defined by the formula (49).

Checking a discriminant of the characteristic equation (13) for given values:

E = 100 [V], L = $2\cdot10^{-3}$ [H], C = $2\cdot10^{-6}$ [F]

Case 1:

$$R_1^2-4\frac{L}{C}>0 \quad\Leftrightarrow\quad 85^2-4\frac{2\cdot10^{-3}}{2\cdot10^{-6}}>0,\quad 7225-4000>0,\quad 3225>0 \tag{65}$$

Case 2:

$$R_2^2-4\frac{L}{C}=0 \quad\Leftrightarrow\quad (20\sqrt{10})^2-4\frac{2\cdot10^{-3}}{2\cdot10^{-6}}=0,\quad 4000-4000=0,\quad 0=0 \tag{66}$$

Case 3:

$$R_3^2-4\frac{L}{C}<0 \quad\Leftrightarrow\quad 4^2-4\frac{2\cdot10^{-3}}{2\cdot10^{-6}}<0,\quad 16-4000<0,\quad -3986<0 \tag{67}$$

4.2. Mathematica algorithms for intensity formulae and graphs

Mathematica - Program 1 Graph of the function (28), (Fig. 3) [1], [8]

```
E1:=100; R1:=85; L1:=2*10^(-3); C1:=2*10^(-6)
Plot[(2*E1*(Exp[(-R1*t)/(2*L1)])*
Sinh[0.5*t*Sqrt[(R1/L1)^2-4/(L1*C1)]])/(L1*Sqrt[(R1/L1)^2-
4/(L1*C1)]),{t,0,0.0005},PlotRange→{0,1},GridLines→Automatic,
PlotStyle→Thickness[0.005],AxesLabel→{"t","i1(t)"}]
```

Mathematica - Program 2 Formula and graph of the function (28) according to Laplace's reverse transformation (64), (Fig. 4) [1]

```
I1=InverseLaplaceTransform[100/(s*(s*L1+R1+(1/(s*C1)))),s,t]
Plot[i1,{t,0,0.0005},PlotRange→{0,1.5},GridLines→Automatic,
PlotStyle→{Thickness[0.005],Red},AxesLabel→{"t","i1(t)"}]
```

$$\mathrm{i1(t)} = (-20)\cdot\frac{\exp[(-21250-12250\sqrt{129})t]-\exp[(-21250+12250\sqrt{129})t]}{\sqrt{129}} \tag{68}$$

Mathematica - Program 3 Graph of the function (38), (Fig. 5) [1], [8]

```
E2:=100; R2:=20*Sqrt[10]; L2:=2*10^(-3); C2:=2*10^(-6)
Plot[(E2*t/L2)*Exp[((-1)*R2)/(2*L2)*t],{t,0,0.0005}, PlotRan-
ge→{0,1.4},GridLines→Automatic, Plot-
Style→Thickness[0.005],AxesLabel→{"t","i2(t)"}]
```

Mathematica - Program 4 Formula and graph of the function (38) according to Laplace's reverse transformation (64), (Fig. 6) [1]

```
i2=InverseLaplaceTransform[100/(s*(s*L2+R2+(1/(s*C2)))),s,t]
Plot[i2,{t,0,0.0005},PlotRange→{0,1.5},GridLines→Automatic,
PlotStyle→{Thickness[0.005],Red},AxesLabel→{"t","i2(t)"}]
```

$$\mathrm{i2(t)} = 50000\cdot t\cdot \exp(-1000\sqrt{10}\cdot t) \tag{69}$$

Mathematica - Program 5 Graph of the function (49), (Fig. 7) [1], [8]

```
E3:=100; R3:=4; L3:=2*10^(-3); C3:=2*10^(-6)
Plot[(((E3*Exp[(-R3/(2*L3))*t])/(L3*Sqrt[(1/(L3*C3)-
(R3/(2*L3))^2)]))*
(Sin[t*Sqrt[(1/(L3*C3)(R3/(2*L3))^2)]])),{t,0,0.004},
PlotRange→{3,3},GridLines→Automatic,PlotStyle→Thickness[0.005]
,AxesLabel→{"t","i3(t)"}]
```

Mathematica - Program 6 Formula and graph of the function (49) according to Laplace's reverse transformation (64), (Fig. 8)

```
i3=InverseLaplaceTransform[100/(s*(s*L3+R3+(1/(s*C3)))),s,t]
Plot[i3,{t,0,0.004},PlotRange→{-3.5,3.5},
GridLines→Automatic,PlotStyle→{Thickness[0.005],Red},
AxesLabel→{"t","i3(t)"}]
```

$$\mathrm{i3(t)} = \frac{50\cdot\exp(-1000\cdot t)}{\sqrt{249}}\cdot\sin(1000\sqrt{249}\cdot t) \tag{70}$$

Source: Programs in Mathematica were elaborated by the Authors

4.3. Graphs in the Mathematica program of the intensity function

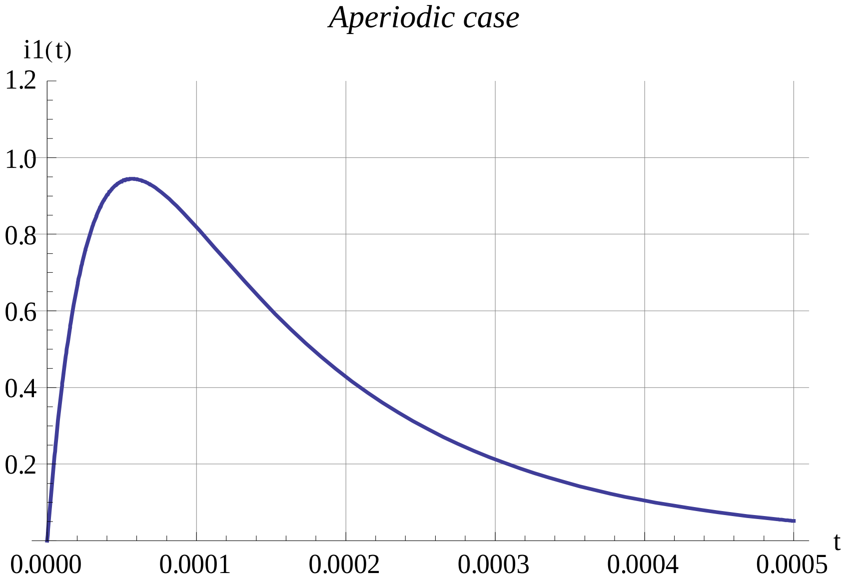

Fig. 3. Graph of the function i1(t) [A] obtained from the formula (28) in program 1 and data:
E = 100 [V]
R = 85 [Ω]
L = $2 \cdot 10^{-3}$ [H]
C = $2 \cdot 10^{-6}$ [F]

Source: Elaboration of the Authors

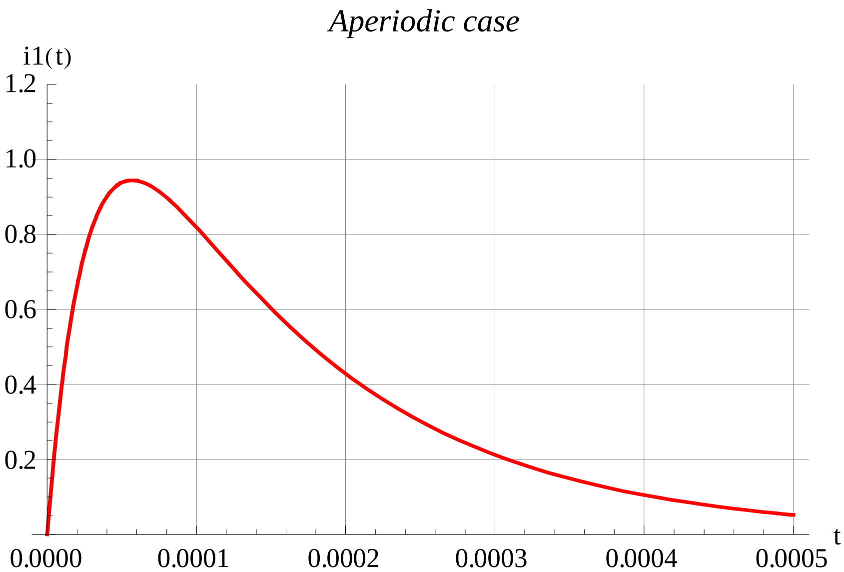

Fig. 4. Graph (68) of the function (28) obtained according to the inverse Laplace transform (64) and in program 2 with data:
E = 100 [V]
R = 85 [Ω]
L = $2 \cdot 10^{-3}$ [H]
C = $2 \cdot 10^{-6}$ [F]

Source: Elaboration of the Authors

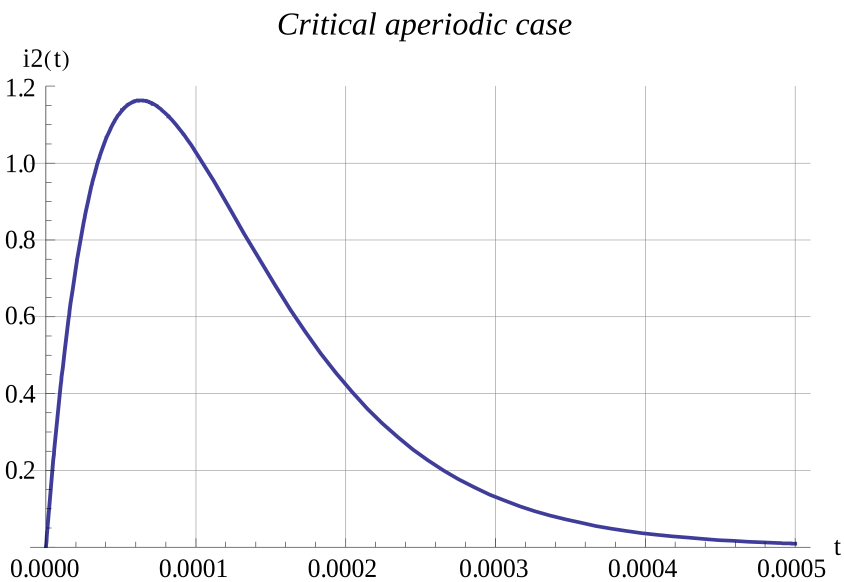

Fig. 5. Graph of the function i2(t) [A] obtained from the formula (38) in program 3 and data:
E = 100 [V],
R = $20\sqrt{10}$ [Ω],
L = $2 \cdot 10^{-3}$ [H],
C = $2 \cdot 10^{-6}$ [F]

Source: Elaboration of the Authors

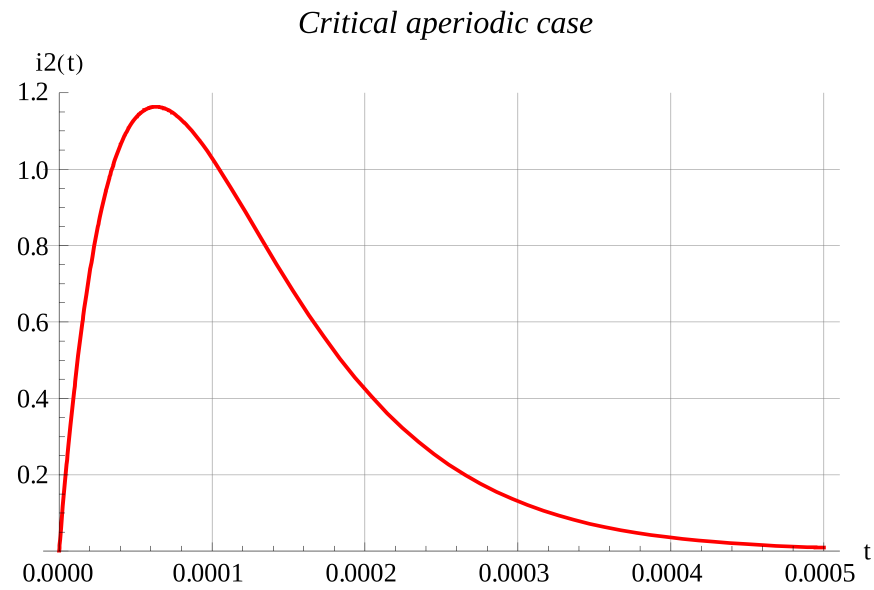

Fig 6. Graph (69) of the function (38) obtained according to the inverse Laplace transform (64) and in program 4 with data:
E = 100 [V]
R = $20\sqrt{10}$ [Ω]
L = $2\cdot10^{-3}$ [H]
C = $2\cdot10^{-6}$ [F]

Source:
Elaboration of the Authors

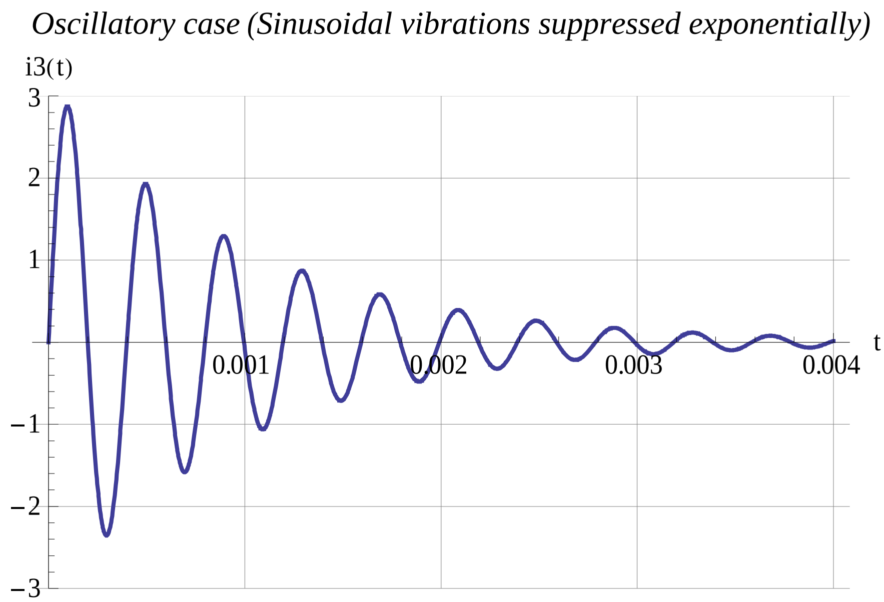

Fig. 7. Graph of the function i3(t) [A] obtained from the formula (49) in program 5 and data:
E = 100 [V]
R = 4 [Ω]
L = $2\cdot10^{-3}$ [H]
C = $2\cdot10^{-6}$ [F]

Source:
Elaboration of the Authors

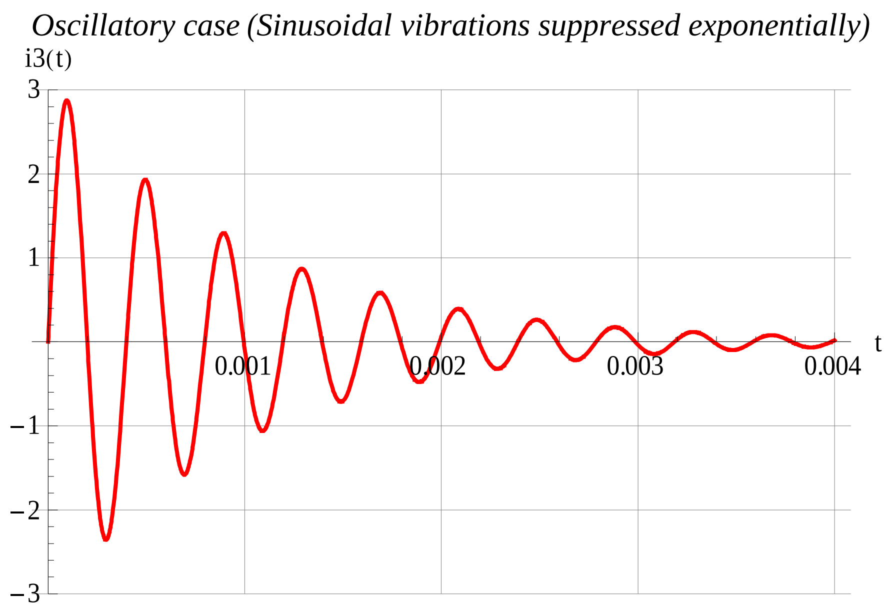

Fig 8. Graph (70) of the function (49) obtained according to the inverse Laplace transform (64) in program 6 with data:
E = 100 [V]
R = 4 [Ω]
L = $2\cdot10^{-3}$ [H]
C = $2\cdot10^{-6}$ [F]

Source:
Elaboration of the Authors

5. Scheme of Laplace transformation method

The following diagram is presented to illustrate the used methods.

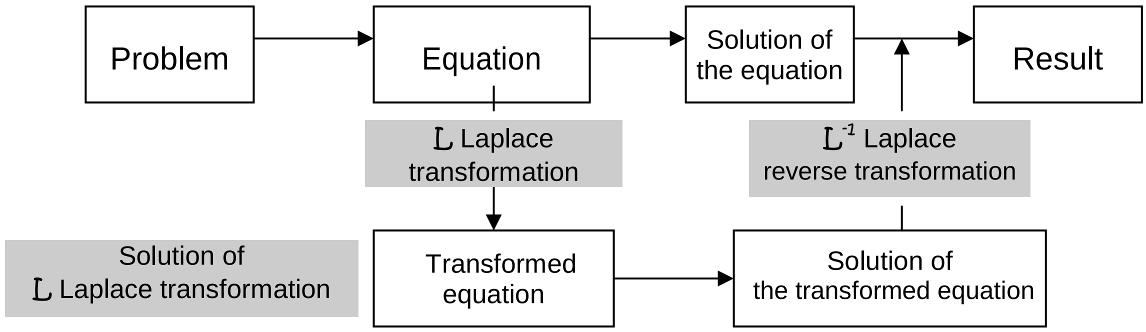

Fig. 9. Scheme of Laplace operator method
Source: Elaboration of the Authors

6. Conclusions

- The transient state of the RLC serial circuit when constant voltage is applied is described by a differential-integral equation, from which is obtained a second order differential equation with constant coefficients.
- The differential-integral equation is solved by means of Laplace transformations. However, a second order differential equation with constant coefficients is calculated using the analytical method.
- Using both the Laplace transform method and the analytical method, the same transient currents are obtained as a function of time.
- Using both the *MathCAD* numerical program and the *Mathematica* numerical program to create graphs of the intensity functions using Laplace transformations, the same transient currents are obtained as a function of time.
- Laplace's transformation and inverse Laplace transformations are applicable to the solution of certain classes of integral or differential-integral equations.

Literature

[1] Abel M.L., Braselton J.P.: *Mathematica by example. Revised edition.* AP Professional, A Division of Hartcourt Brace & Company, Boston San Diego New York London Sydney Tokyo Toronto, 1994.

[2] Hayt W.H. Jr., Durbin S.M., Kemmerly J.E.: *Engineering circuit analysis.* 1971.

[3] Lathi B.P.: *Signal processing and linear systems.* Oxford University Press, 2000, USA.

[4] Nillson J.W., Riedel S.A.: *Electric circuits. Sixth edition.* Prentice Hall, 2001.

[5] Siebert W.M.: *Circuits, singals and systems.* Mit Press, 1985

[6] Theraja B.L.: *Fundamentals of electrical engineering and electronics.* Chand (S.) & Co Ltd, 1997, India.

[7] Theraja B.L., Theraja A.K.: *ABC of electrical engineering.* Chand & Co Ltd, 2012.

[8] Trott M.: *The Mathematica guide book for graphics.* Springer Science+Business Media Inc., 2004