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Joanna KWIECIEŃ
Bogusław FILIPOWICZ

OPTIMIZATION OF COMPLEX SYSTEMS RELIABILITY BY FIREFLY ALGORITHM

OPTYMALIZACJA NIEZAWODNOŚCI ZŁOŻONYCH SYSTEMÓW ZA POMOCĄ ALGORYTMU ŚWIETLIKA*

Algorithms based on swarm intelligence are more and more frequently applied to problems of systems reliability. The article presents the application of a firefly algorithm to the reliability optimization of two systems: bridge and 10-unit, with minimal paths set, minimal cuts set and decomposition methods. The obtained results are presented and compared with the available literature data.

Keywords: *system reliability optimization problems, reliability optimization methods, RRAP system, firefly algorithm.*

Algorytmy bazujące na inteligencji stadnej są coraz częściej stosowane w problemach niezawodności systemów. Artykuł prezentuje zastosowanie algorytmu świetlika do optymalizacji niezawodności dwóch systemów: mostkowego i 10-elementowego, z wykorzystaniem metod zbioru minimalnych ścieżek, minimalnych cięć oraz metody dekompozycji. Uzyskane rezultaty zostały przedstawione i porównane z dostępnymi danymi literaturowymi.

Słowa kluczowe: *problemy optymalizacji niezawodności systemów, metody optymalizacji niezawodności, system RRAP, algorytm świetlika.*

1. Introduction

The problem of testing the reliability of devices which influence the quality of technical object operation can be seen as an analysis of a system, i.e. intentionally separated collection of subsystems associated with dependencies or interactions. A system model can be represented as an ordered pair $<N, f>$, where N is the set of natural numbers assigned to the elements, and f is the function called the system structure, which expresses the state of the system depending upon the state of its components. If the object has two states {operating, failed}, this function takes binary values, where “1” implies operating state and “0” is assigned to failed state. In order to achieve the required reliability of the whole system, there must be adequate reliabilities of its individual components. It is important to take into account the specific limits being imposed, such as the total cost of the various components of the equipment as well as the sum weight and volume.

There are many different approaches that allow us to solve the problem of optimizing the reliability of complex systems. Many papers relate to the application of algorithms belonging to the group of swarm algorithms, i.e. those based on the behavior of social insects or animal herds. Analyzing the current literature, it can be observed that such research mainly concern the effectiveness of ant algorithms [1], particle swarm optimization [4, 10, 15], bees algorithms [18] and cuckoo search algorithm [5, 6, 13, 14], which simultaneously indicates the advantage of the cuckoo search over other swarm algorithms. This study focuses on examining the usefulness of applying the firefly algorithm to systems consisting of 5 and 10 elements, taking into consideration several methods of determining the reliability of these systems.

2. Reliability of complex systems

When designing a highly reliable system, it is very important to achieve a balance between reliability, and other resources, such as cost, volume or weight. The problem of optimizing reliability with respect to redundancy (RRAP, *reliability redundancy allocation problem*) is treated as a nonlinear programming problem which has one or more resources constraints. Among these known systems, two cases were considered: a bridge system and a system consisting of 10 elements.

2.1. Bridge system

The bridge system shown in Figure 1 can be formulated as follows [12, 14]:

$$\begin{aligned} Max\ f(r,n) = {} & R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 + \\ & - R_1R_2R_4R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \end{aligned} \tag{1}$$

with constraints taking into account the upper limit of the total volume and weight (V), cost (C) and system weight (W):

$$g_1(r,n) = \sum_{i=1}^{m} w_i v_i^2 n_i^2 - V \le 0$$

$$g_2(r,n) = \sum_{i=1}^{m} \alpha_i \left(-\frac{1000}{\ln(r_i)} \right)^{\beta_i} \left[n_i + e^{0.25 n_i} \right] - C \le 0$$

$$g_3(r,n) = \sum_{i=1}^{m} w_i n_i e^{0.25 n_i} - W \le 0$$

$$0 \le i \le m, \quad 0 \le r_i \le 1, \quad n_i \in Z^+$$

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

where:
m – the number of subsystems in the system,
n_i – the number of components in subsystem i,
r_i – the reliability of each component in subsystem i,
R_i – the reliability of subsystem i,
α_i, β_i – physical features of components,
w_i, v_i, c_i – the weight, volume, cost of element in subsystem i.

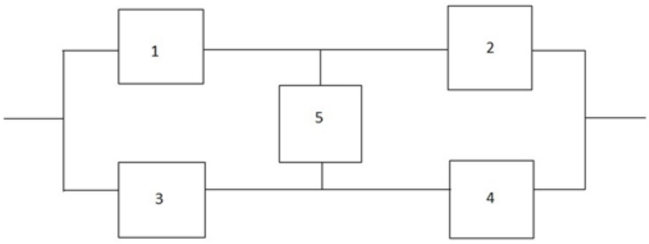

Fig. 1. The scheme of bridge system

The parameter settings of the bridge system can be found in the literature. According to [14] the following values shown in Table 1 were selected.

Table 1. Data used in the bridge system

i	$10^5\alpha_i$	β_i	$w_iv_i^2$	w_i	V	C	W
1	2.330	1.5	1	7	110	175	200
2	1.450	1.5	2	8			
3	0.541	1.5	3	8			
4	8.050	1.5	4	6			
5	1.950	1.5	2	9			

2.2. The system consists of 10 elements

The reliability structure of a 10-unit system is shown in Figure 2 [14].

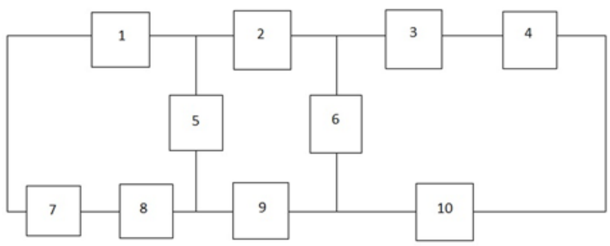

Fig. 2. The diagram of 10-unit system

Assuming by $R_i(x_i)$ reliability of the subsystem i equals $1-(1-r_i)^{x_i}$ and $Q_i = 1 - R_i$ this problem can be formulated as follows [1]:

$$\begin{aligned} Max\ f(x) = {} & R_1R_2R_3R_4 + R_1R_2R_6R_{10}(Q_3 + R_3Q_4) + R_1R_5R_9R_{10}(Q_2 + R_2Q_3Q_6 + \\ & + R_2R_3Q_4Q_6) + R_7R_8R_9R_{10}(Q_1 + R_1Q_2Q_5 + R_1R_2Q_3Q_5Q_6 + R_1R_3Q_4Q_5Q_6) + \\ & + R_2R_3R_4R_5R_7R_8Q_1(Q_9 + R_9Q_{10}) + Q_1R_3R_4R_6R_7R_8R_9Q_{10}(Q_2 + R_2Q_5) + \\ & + R_1Q_2R_3R_4R_6R_7R_8R_9Q_{10} + R_1Q_2R_3R_4R_5R_6R_9Q_{10}(Q_7 + R_7Q_8) + \\ & + Q_1R_2R_5R_6R_7R_8Q_9R_{10}(Q_3 + R_3Q_4) \end{aligned} \tag{2}$$

subject to m constraints:

Table 2. Data used for 10-unit system

i	r	c_1	c_2	c_3	c_4	c_5
1	0.6796	33.2468	35.6054	13.7848	44.1345	10.9891
2	0.7329	27.5668	44.9520	96.7365	25.9855	68.0713
3	0.6688	13.3800	28.6889	85.8783	19.2621	1.0164
4	0.6102	0.4710	0.4922	63.0815	12.1687	29.4809
5	0.7911	51.2555	39.6833	78.5364	23.9668	59.5441
6	0.8140	82.9415	59.2294	11.8123	28.9889	46.5904
7	0.8088	51.8804	78.4996	97.1872	47.8387	49.6226
8	0.7142	77.9446	86.6633	45.0850	25.0545	59.2594
9	0.8487	26.8835	7.8195	3.6722	76.9923	87.4070
10	0.7901	85.8722	27.7460	55.3950	53.3007	55.3175

$$g_y(r,n) = \sum_{i=1}^{10} c_{yi}x_i \le b_y, \quad y = 1,2,...,m, \quad x_i \in Z^+$$

For this system the coefficients c_{yi} represent random numbers in the range [0, 100], r_i are generated in the range [0.65, 0.85], whereas the parameter $b_y = rand(1.5, 3.5)\cdot\sum_{i=1}^{10} c_{yi}$. The values of setting parameters of the model are summarized in Table 2, based on the data available in the literature [14].

3. Methods of determining the reliability of systems

In systems defined as having redundant reliability structure, the case of incompatibility of some features of the system with the specified requirements does not lead to system's failure. The two minimal subsets of elements can be distinguished, giving the possibility of estimating upper and lower bounds of the system reliability [2, 3, 8, 11]:
- minimal path - a set of components whose proper functioning (all) ensure the successful operation of the whole system, however the failure of even one of these elements will cause a failure state for the system as a whole; components of a minimal path are connected in series and the actual reliability structure of the system can be mapped to the structure of a parallel-series, in which the minimal paths are connected in parallel [3]; denoting by P_1, ..., P_r the minimal paths set of the system, the structure function of the system is given as [2]:

$$f(x) = 1 - \prod_{j\in\{1,...,r\}}\left(1 - \prod_{i\in P_j} R_i\right) \tag{3}$$

- minimal cut - is a set of components, which being in a failure state cause the system malfunction, however damage to any subset of this elements set does not damage the system; elements of the minimal cut are connected in a parallel combination and the real reliability structure of system can be converted to an equivalent series-parallel structure, wherein the minimal cuts are connected in series [3]. If C_1, ..., C_s denote the set of minimal cuts, therefore we have:

$$f(x) = \prod_{j\in\{1,...,s\}}\left(1 - \prod_{i\in C_j}(1 - R_i)\right) \tag{4}$$

One of the known methods for determining the reliability of complex systems is called the decomposition method which consists of performing consecutive structural operations converting a n-elements object of any structure to a certain number of simple objects with the series-parallel structures for which the reliability can be determined using known methods [3, 11]. In each operation this decomposition is always performed with respect to one chosen ith element with the reliability R_i. The two structures consisting of n-1 elements are considered. In one of them the chosen element is replaced by an absolutely reliable one – "short circuit" ($R_i = 1$), and in the second structure by an absolutely failed component ($R_i = 0$) called "break". The reliability of the whole n-elements system $R^{(n)}$ can be computed by using the following recursive formula:

$$R^{(n)} = R_i R_i^{(n-1)} + \left(1 - R_i\right) R_{\underline{i}}^{(n-1)} \tag{5}$$

where $R_i^{(n-1)}$, $R_{\underline{i}}^{(n-1)}$ denote the reliability of decomposed structure with "short circuit" ($R_i = 1$) and with "break" ($R_i = 0$), respectively.

3.1. Case: bridge system

There are four minimal paths in the bridge system shown in Figure 1, namely: $P_1 = \{1, 2\}$, $P_2 = \{3, 4\}$, $P_3 = \{1, 4, 5\}$ and $P_4 = \{2, 3, 5\}$. For these minimal paths the function which describes the reliability of the system in terms of its elements has following form:

$$f = 1 - (1 - R_1R_2)(1 - R_3R_4)(1 - R_1R_4R_5)(1 - R_2R_3R_5) \tag{6}$$

Analyzing minimal cuts set method, the bridge structure is characterized by the following cuts: $C_1 = \{1, 3\}$, $C_2 = \{2, 4\}$, $C_3 = \{2, 3, 5\}$, $C_4 = \{1, 4, 5\}$, and on the basis of Eq.(4) the function is given as:

$$f = [1 - (1 - R_1)(1 - R_3)][1 - (1 - R_2)(1 - R_4)][1 - (1 - R_2)(1 - R_3)(1 - R_5)][1 - (1 - R_1)(1 + - R_4)(1 - R_5)] \tag{7}$$

As can be seen the bridge system contains sets $\{2, 3, 5\}$, $\{1, 4, 5\}$, which are both the minimal path and minimal cut.

Applying the decomposition method, the reliability of the bridge system can be calculated in terms of decomposed structures reliabilities with respect to the chosen component 5 which are:

$$R_5^{(4)} = \left[1 - (1 - R_1)(1 - R_3)\right]\left[1 - (1 - R_2)(1 - R_4)\right]$$
$$R_{\underline{5}}^{(4)} = R_1R_2 + R_3R_4 - R_1R_2R_3R_4$$

For these specific reliabilities, the total reliability of the bridge system is given by the following formula:

$$R^{(5)} = R_5R_5^{(4)} + \left(1 - R_5\right)R_{\underline{5}}^{(4)} = R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 + - R_1R_2R_4R_5 - R_1R_2R_3R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \tag{8}$$

In the case of homogeneous system ($R_i = r$), the total reliability reduces to form:

$$R^{(5)} = 2r^2 + 2r^3 - 5r^4 + 2r^5$$

3.2. Case: structure of 10 elements

In order to evaluate the total reliability of structure shown in Figure 2, methods of minimal cuts set and minimal paths set were used. Applying minimal paths set method one can find eight paths. Therefore we have four minimal paths of order 4: $P_1 = \{1, 2, 3, 4\}$, $P_2 = \{7, 8, 9, 10\}$, $P_3 = \{1, 5, 9, 10\}$, $P_4 = \{1, 2, 6, 10\}$ and their appropriate reliabilities: $\pi(P_1) = R_1R_2R_3R_4$, $\pi(P_2) = R_7R_8R_9R_{10}$, $\pi(P_3) = R_1R_5R_9R_{10}$, $\pi(P_4) = R_1R_2R_6R_{10}$. Then, for four paths of order 6: $P_5 = \{7, 8, 5, 2, 3, 4\}$, $P_6 = \{1, 5, 9, 6, 3, 4\}$, $P_7 = \{7, 8, 9, 6, 3, 4\}$, $P_8 = \{7, 8, 5, 2, 6, 10\}$, we have $\pi(P_5) = R_7R_8R_5R_2R_3R_4$, $\pi(P_6) = R_1R_5R_9R_6R_3R_4$, $\pi(P_7) = R_7R_8R_9R_6R_3R_4$, $\pi(P_8) = R_7R_8R_5R_2R_6R_{10}$.
The reliability of the whole system can be described as follows:

$$f = 1 - \prod_{i=1}^{n}(1 - \pi(P_i)) = 1 - (1 - R_1R_2R_3R_4)(1 - R_7R_8R_9R_{10})(1 - R_1R_5R_9R_{10})(1 - R_1R_2R_6R_{10}) (1 - R_7R_8R_5R_2R_3R_4)(1 - R_1R_5R_9R_6R_3R_4)(1 - R_7R_8R_9R_6R_3R_4)(1 - R_7R_8R_5R_2R_6R_{10}) \tag{9}$$

Structure of 10 elements is characterized by 16 minimal cuts, including:

- five of second order: $C_1 = \{1, 7\}$, $C_2 = \{1, 8\}$, $C_3 = \{2, 9\}$, $C_4 = \{3, 10\}$, $C_5 = \{4, 10\}$,
- six of third order: $C_6 = \{1, 5, 9\}$, $C_7 = \{2, 6, 10\}$, $C_8 = \{2, 5, 8\}$, $C_9 = \{3, 6, 9\}$, $C_{10} = \{2, 5, 7\}$, $C_{11} = \{4, 6, 9\}$,
- five of fourth order: $C_{12} = \{7, 6, 5, 3\}$, $C_{13} = \{8, 6, 5, 3\}$, $C_{14} = \{7, 5, 6, 4\}$, $C_{15} = \{8, 5, 6, 4\}$, $C_{16} = \{1, 5, 6, 10\}$.

Using Eq.(4), the reliability of the whole structure is determined from the following formula:

$$f = [1 - (1 - R_1)(1 - R_7)][1 - (1 - R_1)(1 - R_8)][1 - (1 - R_2)(1 - R_9)][1 - (1 - R_3)(1 - R_{10})][1 + - (1 - R_4)(1 - R_{10})][1 - (1 - R_1)(1 - R_5)(1 - R_9)][1 - (1 - R_2)(1 - R_6)(1 - R_9)][1 - (1 - R_2) (1 - R_5)(1 - R_8)][1 - (1 - R_3)(1 - R_6)(1 - R_9)][1 - (1 - R_2)(1 - R_5)(1 - R_7)][1 - (1 - R_4)(1 + - R_6)(1 - R_9)][1 - (1 - R_7)(1 - R_6)(1 - R_5)(1 - R_3)][1 - (1 - R_8)(1 - R_6)(1 - R_5)(1 - R_3)][1 + - (1 - R_7)(1 - R_6)(1 - R_5)(1 - R_4)][1 - (1 - R_8)(1 - R_5)(1 - R_6)(1 - R_4)][1 - (1 - R_1)(1 - R_5) (1 - R_6)(1 - R_{10})] \tag{10}$$

In the case of the decomposition method, performed analysis on the new elements is repeated until structures resulting from replacement of elements are sufficiently simple for the calculation. Hence the initial structure $R^{(10)}$ can be decomposed with respect to its element 5, which is replaced by "short circuit" and "break". Therefore, the structure reliability is given as:

$$R^{(10)} = R_5R_5^{(9)} + (1 - R_5)R_{\underline{5}}^{(9)} \tag{11}$$

In the next step, structures $R_5^{(9)}$ and $R_{\underline{5}}^{(9)}$ are decomposed with respect to the chosen component 6, for which:

$$\begin{aligned} R_5^{(9)} &= R_6R_{5,6}^{(8)} + (1 - R_6)R_{5,\underline{6}}^{(8)} \\ R_{\underline{5}}^{(9)} &= R_6R_{\underline{5},6}^{(8)} + (1 - R_6)R_{\underline{5},\underline{6}}^{(8)} \end{aligned} \tag{12}$$

The structures $R_{5,6}^{(8)}$, $R_{5,\underline{6}}^{(8)}$, $R_{\underline{5},6}^{(8)}$, $R_{\underline{5},\underline{6}}^{(8)}$ obtained in this way are simple structures, therefore their reliability can be easily computed.

4. Firefly algorithm

The firefly algorithm (FA), based on the behavior of fireflies flying towards a light source and their interaction with bioluminescent signals, is one of the algorithms belonging to the group of swarm algorithms. The phenomenon of a firefly moving towards the brighter individual is the basis of the algorithm. One of the rules used in the firefly algorithm is that all fireflies are unisex. Moreover, attractiveness of fireflies is proportional to the intensity of their emitted light, wherein the light intensity determined by the value of the objective function (it is proportional to this value for maximization problems) decreases with increasing distance between the fireflies. If there is no more attractive individual, a firefly moves randomly [16, 17]. Each firefly has a certain light intensity I, which varies according to the distance r between two individuals, and attractiveness β, which is proportional to the light intensity seen by the neighboring fireflies. Therefore, attractiveness (β) is dependent on distance and the light absorption coefficient γ [17]:

$$\beta(r) = \beta_0 e^{-\gamma r^m}, \quad m \geq 1 \tag{13}$$

where β_0 denotes the attractiveness at $r = 0$.

The movement, during which the firefly i being in the position x_i tries to get closer to the more attractive individual j in the position x_j is determined by the following formula [17]:

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_j - x_i) + \alpha(rand - \frac{1}{2}) \tag{14}$$

where x_i is the current position of a firefly i, the second term denotes attractiveness and the third term is due to random movement (*rand* is a random number generator uniformly distributed in the range [0, 1], and $\alpha \in [0, 1]$).

The general structure of the FA is as follows [7, 16, 17]:

1. Initialize algorithm's parameters (β_0, γ, stopping criterion) and randomly generate initial population of n fireflies; define the objective function $f(x)$.
2. Compute the light intensity of each individual, whereby the light intensity of ith firefly I_i is determined by the value of the objective function $f(x_i)$.
3. While the stopping criterion has not been met, do the following:
   - compare all pairs of fireflies in terms of light intensity: if ($I_j > I_i$) then move firefly i towards another firefly j,
   - determine new values of the objective function $f(x_i)$, evaluate new solutions, update the light intensity.
4. If the stopping criterion has been met, determine the best solution.

The firefly algorithm was originally developed for the continuous optimization problems. Applying it to reliability optimization of selected structures with continuous and discrete decision variables requires certain additional operations. Correctly determining the distance and ways of movement of individuals, in order to ensure the validity of the solutions are the main elements of the algorithm, which should be adapted. We assume that the distance between the two fireflies is determined as the norm of the difference between values of the decision variables assigned to the two individuals. The movement of each firefly in the direction of the brighter individual consists in performing the specified number of steps, in which the length of the step does not exceed the predefined maximum changes of values for the continuous variables (STEP_MAX_CV) and for the discrete variables (STEP_MAX_DV). If after the performed step the firefly finds itself outside the acceptable area, the maximum length of the step is reduced (multiplied by a random number from the range [0.5, 0.99]). If after a specified number of trials (MAX_P) the solution does not find itself in the acceptable area the firefly will not move.

5. Results of experiments

Using the set values of various parameters in the selected two systems, listed in Tables 1 and 2, many experiments have been performed to investigate the suitability of the firefly algorithm in solving selected reliability problems. As we know, the reliability for minimal cuts (i.e. lower bound) is less than for minimal paths (i.e. upper bound), which represents the basis for seeking out optimal values. Within the confines of the testing, for the chosen set parameters, the efficiency of the firefly algorithm was checked and the obtained results were compared to the best previously known solutions. The presented results of the applied algorithm to solve the problem of the reliability of system with 10 elements were limited to discussing the results where $m = 5$. The firefly algorithm was implemented in the Matlab 2015a environment. During the experiments, to verify the quality of the results of the algorithm, the following values of its parameters were established:

Table 3. Results for the bridge system after 50 runs

Method	Number of fireflies	Best value	Worst value	Mean value
decomposition	30	0.999889027392830	0.999692113944072	0.999867770075797
	10	0.999882704854672	0.999535345770864	0.999789510970645
cuts	30	0.999887373640587	0.999709686550381	0.999839910069411
	10	0.999881295104186	0.999561725825725	0.999795740150406
paths	30	**0.999998825015460**	0.999995999869590	**0.999997854290027**
	10	0.99999874719315	0.999992668983783	0.999997639615957

Table 4. Results of 10-unit system (m = 5)

Method	Number of fireflies	Best value	Worst value	Mean value
decomposition	30	0.999124934817144	0.998712767969089	0.999029684217294
	10	0.999124934817144	0.997639045897561	0.998706554859805
cuts	30	0.999123179843347	0.998518087543003	0.998951987420550
	10	0.999123179843347	0.997349605284400	0.998697533072659
paths	30	**0.999999983601514**	0.999999961168195	**0.999999979369618**
	10	**0.999999983601514**	0.999999967299759	0.999999978215736

Table 5. Compilation of best obtained results from the literature data

Structure: bridge system		
Algorithm	Best result	Mean value
PSO [4]	0.99988957	0.99988594
PSO [15]	0.99988963	-
MPSO [10]	0.9998896376	0.9998891423
ABC [18]	0.99988962	0.99988362
CS-GA [6]	0.99988964	0.9998854
CS [13, 14]	0.99988964	0.99987998
BAT [9]	0.9998896376	0.9998894767
Structure: 10-unit system		
Algorithm	Best result	Mean value
ACO [1]	0.999991	0.9980477
CS [13, 14]	0.67189992	0.67189992

the stopping criterion of a single run – 1000 iterations, population size – 10 or 30 individuals, MAX_P = 100, STEP_MAX_CV = 0.5, STEP_MAX_DV = 2, $\gamma = 0.1$. For each instance there were 50 independent repetitions of the algorithm.

The results of the experiments are presented in Tables 3 and 4, which list the best and worst obtained results as well as the mean value of 50 runs. The results of the research suggest an advantage of the presented algorithm using minimal paths versus other methods.

Analysis of the literature data regarding the best solutions obtained by various methods inspired by swarm behavior, including ant colony optimization (ACO), particle swarm optimization (PSO) and modified PSO (MPSO), artificial bee colony (ABC), cuckoo search (CS) and bat algorithm (BAT), allowed for their collective summary (Table 5).

As is evident from the calculations, in the case of the bridge system the firefly algorithm with minimal paths set was the one method, which enabled the obtainment of results (0.999998825015460), exceeding the results of PSO, MPSO, ABC, CS, CS-GA and BAT. Unfortunately, such a conclusion cannot be drawn when comparing the FA using other methods. In the case of the 10-unit system, the firefly algorithm can clearly be seen to have an advantage over the cuckoo search. It should be noted, however, that for the considered examples, the results, obtained during maximization with the minimal paths method, differ significantly from the results of both the decomposition method and minimal cuts set method. Therefore, we can conclude that in the design of a variety of real systems, the safest approach is to adopt the lower estimated value for reliability.

6. Conclusions

The paper presents research results obtained using the firefly algorithm in reliability-redundancy allocation problems. In order to examine the effectiveness of the algorithm, two systems and three methods of determining the reliability were chosen, i.e. the minimal paths set, the minimal cuts set and the method of decomposition. Analyzing the results, it can be concluded that for the considered systems, significantly better results for the firefly algorithm were obtained in conjunction with the use of the minimal paths set method. It is worth noting that the results concerning the use of swarm algorithms presented in the literature turned out to be worse than those that managed to obtain by proposed implementation of the firefly algorithm.

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Joanna KWIECIEŃ
AGH University of Science and Technology
Faculty of Electrical Engineering, Automatics, Computer Science and Biomedical Engineering
Al. Mickiewicza 30, 30-059 Krakow, Poland

Bogusław FILIPOWICZ
State Higher Vocational School in Tarnow
Polytechnic Institute
Ul. Mickiewicza 8, 33-100 Tarnow, Poland

E-mail: kwiecien@agh.edu.pl, fil@agh.edu.pl