

## FURTHER RESULTS ON ROBUST FUZZY DYNAMIC SYSTEMS WITH LMI $\mathcal{D}$ -STABILITY CONSTRAINTS

WUDHICHAI ASSAWINCHAICHOTE

Department of Electronic and Telecommunication Engineering  
King Mongkut's University of Technology Thonburi, 126 Prachautits Rd., Bangkok 10140, Thailand  
e-mail: wudhichai.asa@kmutt.ac.th

This paper examines the problem of designing a robust  $\mathcal{H}_\infty$  fuzzy controller with  $\mathcal{D}$ -stability constraints for a class of nonlinear dynamic systems which is described by a Takagi–Sugeno (TS) fuzzy model. Fuzzy modelling is a multi-model approach in which simple sub-models are combined to determine the global behavior of the system. Based on a linear matrix inequality (LMI) approach, we develop a robust  $\mathcal{H}_\infty$  fuzzy controller that guarantees (i) the  $\mathcal{L}_2$ -gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value, and (ii) the closed-loop poles of each local system to be within a specified stability region. Sufficient conditions for the controller are given in terms of LMIs. Finally, to show the effectiveness of the designed approach, an example is provided to illustrate the use of the proposed methodology.

**Keywords:** fuzzy controller, robust  $\mathcal{H}_\infty$  control, LMI approach,  $\mathcal{D}$ -stability, Takagi–Sugeno fuzzy model.

### 1. Introduction

In the last few decades, nonlinear  $\mathcal{H}_\infty$  theories have been extensively developed and well applied by many researchers (see Fu *et al.*, 1992; Isidori and Astolfi, 1992; van der Schaft, 1992; Ball *et al.*, 1993; 1994; Mansouri *et al.*, 2009; Rezac and Hurak, 2013).  $\mathcal{H}_\infty$  control problems basically involve MIMO systems as well as disturbance and model error problems. The nonlinear  $\mathcal{H}_\infty$  control problem can be stated as follows: Given a dynamic system with exogenous input noise and a measured output, find a controller such that the  $\mathcal{L}_2$  gain of the mapping from the exogenous input noise to the regulated output is less than or equal to a prescribed value.

Currently, there are two commonly practical methods for solving solutions to nonlinear  $\mathcal{H}_\infty$  control problems. The first one is based on the nonlinear version of the classical bounded real lemma (see Isidori and Astolfi, 1992; van der Schaft, 1992; Ball *et al.*, 1994). The other is based on dissipativity theory and the theory of differential games (see Hill and Moylan, 1980; Willems, 1972; Wonham, 1970; Basar and Olsder, 1982). Both methods show that the solution of the nonlinear  $\mathcal{H}_\infty$  control problem is in fact related to the solvability of Hamilton–Jacobi inequalities (HJIs). To the best of our knowledge, there has been no easy computation technique

for solving those inequalities yet.

Recently, many problems in  $\mathcal{H}_\infty$  control theories have been extensively investigated (see Chen *et al.*, 2000; Chilali and Gahinet, 1996; Chilali *et al.*, 1999; Vesely *et al.*, 2011), with the desired controllers designed in terms of the solution to linear matrix inequalities (LMIs). So far, it has been proven that the LMI technique is one of the best alternatives for the basic analytical method and can be supported by efficient interior-point optimization (see Yakubovich, 1976a, 1976b; Boyd *et al.*, 1994; Gahinet *et al.*, 1995; Scherer *et al.*, 1997). A prominent advantage of the LMI approach is the feasibility to combine various design multi-objectives in a numerically tractable manner. However, most of the existing results are restricted to linear dynamic systems.

So far, there have been numerous research advances devoted to the design of an  $\mathcal{H}_\infty$  fuzzy controller for a class of nonlinear systems which can be represented by a Takagi–Sugeno (TS) fuzzy model (see Yakubovich, 1967a; Han and Feng, 1998; Han *et al.*, 2000; Tanaka *et al.*, 1996; Assawinchaichote and Nguang, 2004a; 2004b; 2006; Assawinchaichote, 2012; Yeh *et al.*, 2012). Fuzzy system theory utilizes qualitative, linguistic information for a complex nonlinear system to construct a mathematical model for it. Recent studies (Zadeh, 1965; Tanaka and Sugeno, 1992; Tanaka and Sugeno, 1995;

Teixeira and Zak, 1999; Wang *et al.*, 1996; Yoneyama *et al.*, 2000; Zhang *et al.*, 2001; Joh *et al.*, 1998; Ma *et al.*, 1998; Park *et al.*, 2001; Bouarar *et al.*, 2013) show that fuzzy submodels can be used to approximate global behaviors of a uncertain nonlinear system.

Since fuzzy sub-models in different state space regions are represented by local linear systems, the global model of the system is obtained by combining these linear models through nonlinear fuzzy membership functions. It is a fact that fuzzy modelling is a multi-model approach in which simple submodels are combined to determine the global system behavior while conventional modelling uses a single model to describe the global system behavior. Recent contributions (Chayaopas and Assawinchaichote, 2013; Assawinchaichote and Chayaopas, 2013) have considered an  $\mathcal{H}_\infty$  fuzzy controller based on an LMI approach and a robust  $\mathcal{H}_\infty$  fuzzy control design. However, these works did not address satisfactorily the system dynamic characteristics which might change on the transient response.

Although the robustness and/or the stability of the closed-loop system are basically the first issue needed to be considered, the system dynamic characteristic sometimes does not meet the desired objectives such as the rise time, the settling time, and transient oscillations in many applications or real physical systems due to poor transient responses. A satisfactory transient response can be obtained by enforcing the closed-loop pole to lie within a suitable region. The problem of assigning all poles of a system in a specified region is the so-called  $\mathcal{D}$ -stable pole placement problem. Recently, Han *et al.* (2000) have studied  $\mathcal{H}_\infty$  controller design of fuzzy dynamic systems with pole placement constraints. However, their methods require the system to be in a state subspace for a period of time and also require switching controllers. Therefore, with this motivation, we examine the problem of designing a robust  $\mathcal{H}_\infty$  fuzzy controller for a class of fuzzy dynamic systems. First, we approximate this class of uncertain nonlinear systems by a Takagi–Sugeno fuzzy model. Then, based on an LMI approach, we develop a technique for designing robust  $\mathcal{H}_\infty$  fuzzy controllers such that the  $\mathcal{L}_2$ -gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value and the closed-loop system is  $\mathcal{D}$ -stable, i.e., we enforce eigenvalue clustering in a specified region. It is necessary to note that the requirement of the system to be in a state subspace for a period of time is not mandatory, and also our proposed robust  $\mathcal{H}_\infty$  fuzzy controller is not a switching controller.

This paper is organized as follows. In Section 2, preliminaries and definitions are presented. In Section 3, based on an LMI approach, we develop a technique for designing robust  $\mathcal{H}_\infty$  fuzzy controllers such that the  $\mathcal{L}_2$ -gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value

and the closed-loop poles of each local system are stable within a pre-specified region for the system described in Section 2. The validity of this approach is demonstrated by an example from the literature in Section 4. Finally, conclusions are given in Section 5.

## 2. Preliminaries and definitions

In this paper, we first examine the following standard TS fuzzy system with parametric uncertainties:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \mu_i(\nu(t)) \left[ [A_i + \Delta A_i]x(t) \right. \\ &\quad \left. + B_w w(t) + [B_i + \Delta B_i]u(t) \right], \\ z(t) &= \sum_{i=1}^r \mu_i(\nu(t)) \left[ [C_i + \Delta C_i]x(t) \right. \\ &\quad \left. + [D_i + \Delta D_i]u(t) \right], \end{aligned} \tag{1}$$

where  $x(0) = 0$ ,  $\nu(t) = [\nu_1(t) \ \dots \ \nu_\vartheta(t)]$  is the premise variable vector that may depend on states in many cases,  $\mu_i(\nu(t))$  denotes the normalized time-varying fuzzy weighting functions for each rule (i.e.,  $\mu_i(\nu(t)) \geq 0$  and  $\sum_{i=1}^r \mu_i(\nu(t)) = 1$ ),  $\vartheta$  is the number of fuzzy sets,  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^m$  is the control input,  $w(t) \in \mathbb{R}^p$  is the disturbance which belongs to  $\mathcal{L}_2[0, \infty)$ ,  $z(t) \in \mathbb{R}^s$  is the controlled output, the matrices  $A_i$ ,  $B_w$ ,  $B_i$ ,  $C_i$ , and  $D_i$  are of appropriate dimensions, and  $r$  is the number of IF-THEN rules. The matrices  $\Delta A_i$ ,  $\Delta B_i$ ,  $\Delta C_i$ , and  $\Delta D_i$  represent the system uncertainties and satisfy the following assumption.

### Assumption 1.

$$\begin{aligned} \Delta A_i &= E_{1_i} F(x(t), t) H_{1_i}, \\ \Delta B_i &= E_{2_i} F(x(t), t) H_{2_i}, \\ \Delta C_i &= E_{3_i} F(x(t), t) H_{3_i}, \\ \Delta D_i &= E_{4_i} F(x(t), t) H_{4_i}, \end{aligned}$$

where  $E_{j_i}$  and  $H_{j_i}$ ,  $j = 1, \dots, 4$  are known matrix functions which characterize the structure of the uncertainties. Furthermore,

$$\|F(x(t), t)\| \leq \rho \tag{2}$$

for some known positive constant  $\rho$ .

Throughout this paper, we assume that the fuzzy model satisfies the following assumption.

**Assumption 2.** The pairs  $(A_i, B_i)$  are locally controllable for every  $i \in \{1, 2, \dots, r\}$ .

Next, let us recall the following definition.

**Definition 1.** Let  $\gamma$  be a given positive number. The system (1) is said to have an  $\mathcal{L}_2$ -gain less than or equal to  $\gamma$  if

$$\int_0^{T_f} z^T(t)z(t) dt \leq \gamma^2 \int_0^{T_f} w^T(t)w(t) dt \quad (3)$$

for all  $T_f \geq 0$ ,  $x(0) = 0$ , and  $w(t) \in \mathcal{L}_2 [0, T_f]$ .

Note that, for the symmetric block matrices, we use the asterisk (\*) as a placeholder for a term that is induced by symmetry.

### 3. Main results

In this section, we first consider the problem of designing a robust  $\mathcal{H}_\infty$  fuzzy controller based on an LMI approach so that the inequality (3) holds. Then, LMI-based sufficient conditions for each local system (1) to have all its closed-loop poles within a prescribed LMI region are presented. Finally, the problem of designing an  $\mathcal{H}_\infty$  fuzzy controller with  $\mathcal{D}$ -stability constraints is examined.

**3.1. Robust  $\mathcal{H}_\infty$  fuzzy control design.** A robust  $\mathcal{H}_\infty$  fuzzy state-feedback controller is readily established in the form

$$u(t) = \sum_{j=1}^r \mu_j K_j x(t), \quad (4)$$

where  $K_j$  is the controller gain such that (3) holds. The state space form of the fuzzy system model (1) with the controller (4) is given by

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left[ (A_i + B_i K_j) + (\Delta A_i + \Delta B_i K_j) \right] x(t) + B_w w(t). \quad (5)$$

The following theorem provides sufficient conditions for the existence of a robust  $\mathcal{H}_\infty$  fuzzy state-feedback controller. These sufficient conditions can be derived by the Lyapunov approach.

**Theorem 1.** Consider the system (1). Given a prescribed  $\mathcal{H}_\infty$  performance  $\gamma > 0$ , if there exist a matrix  $P = P^T$  and matrices  $Y_j$ ,  $j = 1, 2, \dots, r$ , satisfying the following linear matrix inequalities:

$$P > 0, \quad (6)$$

$$\Xi_{ii} < 0, \quad i = 1, 2, \dots, r, \quad (7)$$

$$\Xi_{ij} + \Xi_{ji} < 0, \quad i < j \leq r, \quad (8)$$

where

$$\Xi_{ij} = \begin{pmatrix} \Psi_{1ij} & (*)^T & (*)^T & (*)^T \\ \Psi_{2ij} & -\Gamma + \tilde{E}_i^T \tilde{E}_i & (*)^T & (*)^T \\ \Psi_{3ij} & 0 & -I & (*)^T \\ \Psi_{4ij} & 0 & 0 & -I \end{pmatrix}, \quad (9)$$

$$\Psi_{1ij} = A_i P + P A_i^T + B_i Y_j + Y_j^T B_i^T,$$

$$\Psi_{2ij} = \tilde{B}_{w_i}^T + \tilde{E}_i^T C_i P + \tilde{E}_i^T D_i Y_j,$$

$$\Psi_{3ij} = \tilde{C}_i P + \tilde{D}_i Y_j,$$

$$\Psi_{4ij} = C_i P + D_i Y_j,$$

with

$$\tilde{B}_{w_i} = [ E_{1_i} \quad E_{2_i} \quad B_w \quad 0 \quad 0 ],$$

$$\tilde{C}_i = [ \rho H_{1_i}^T \quad \rho H_{3_i}^T \quad 0 \quad 0 ]^T,$$

$$\tilde{D}_i = [ 0 \quad 0 \quad \rho H_{2_i}^T \quad \rho H_{4_i}^T ]^T,$$

$$\tilde{E}_i = [ 0 \quad 0 \quad 0 \quad E_{3_i} \quad E_{4_i} ],$$

$$\Gamma = \text{diag}\{I, I, \gamma^2 I, I, I\},$$

then the inequality (3) holds. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^r \mu_j K_j x(t), \quad (10)$$

where

$$K_j = Y_j P^{-1}. \quad (11)$$

*Proof.* According to Assumption 1, the closed-loop fuzzy system (5) can be expressed as follows:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left( [A_i + B_i K_j] x(t) + \tilde{B}_{w_i} \tilde{w}(t) \right), \quad (12)$$

where

$$\tilde{B}_{w_i} = [ E_{1_i} \quad E_{2_i} \quad B_w \quad 0 \quad 0 ],$$

and the disturbance  $\tilde{w}(t)$  is

$$\tilde{w}(t) = \begin{bmatrix} F(x(t), t) H_{1_i} x(t) \\ F(x(t), t) H_{2_i} K_j x(t) \\ w(t) \\ F(x(t), t) H_{3_i} x(t) \\ F(x(t), t) H_{4_i} K_j x(t) \end{bmatrix}. \quad (13)$$

Consider the Lyapunov function

$$V(x(t)) = x^T(t) Q x(t),$$

where  $Q = P^{-1}$ . Differentiating  $V(x(t))$  along the trajectories of the closed-loop system (12) yields

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}^T(t) Q x(t) + x^T(t) Q \dot{x}(t) \\ &= \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left( x^T(t) (A_i + B_i K_j)^T Q x(t) \right. \\ &\quad \left. + x^T(t) Q (A_i + B_i K_j) x(t) \right. \\ &\quad \left. + \tilde{w}^T(t) \tilde{B}_{w_i}^T Q x(t) + x^T(t) Q \tilde{B}_{w_i} \tilde{w}(t) \right). \end{aligned} \quad (14)$$

Adding and subtracting

$$-z^T(t)z(t) + \sum_{i=1}^r \sum_{j=1}^r \sum_{m=1}^r \sum_{n=1}^r \mu_i \mu_j \mu_m \mu_n [\tilde{w}^T(t)\Gamma\tilde{w}(t)]$$

to and from (14), combined with the fact that  $\|F(x(t), t)\| \leq \rho$ , we get

$$\begin{aligned} \dot{V}(x(t)) &= \sum_{i=1}^r \sum_{j=1}^r \sum_{m=1}^r \sum_{n=1}^r \mu_i \mu_j \mu_m \mu_n \times \left( \begin{bmatrix} x^T(t) & \tilde{w}^T(t) \end{bmatrix} \right. \\ &\quad \times \left( \begin{array}{c} (A_i + B_i K_j)^T Q \\ + Q(A_i + B_i K_j) \\ + (\tilde{C}_i + \tilde{D}_i K_j)^T \\ \times (\tilde{C}_m + \tilde{D}_m K_n) \\ + (C_i + D_i K_j)^T \times \\ (C_m + D_m K_n) \\ \tilde{B}_{w_i}^T Q + \\ \tilde{E}_i^T (C_i + D_i K_j) \end{array} \right) \left. \begin{array}{c} (*)^T \\ -\Gamma + \tilde{E}_i^T \tilde{E}_i \end{array} \right) \\ &\quad \times \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix} \Big) - z^T(t)z(t) + \gamma^2 w^T(t)w(t), \end{aligned} \tag{15}$$

where

$$\begin{aligned} \tilde{C}_i &= [\rho H_{1_i}^T \quad \rho H_{3_i}^T \quad 0 \quad 0]^T, \\ \tilde{D}_i &= [0 \quad 0 \quad \rho H_{2_i}^T \quad \rho H_{4_i}^T]^T, \\ \tilde{E}_i &= [0 \quad 0 \quad 0 \quad E_{3_i} \quad E_{4_i}], \\ \Gamma &= \text{diag}\{I, I, \gamma^2 I, I, I\}. \end{aligned}$$

Note that

$$\begin{aligned} z^T(t)z(t) &= \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left( x^T(t) [C_i + E_{3_i} F(x(t), t)H_{3_i} \right. \\ &\quad \left. + D_i K_j + E_{4_i} F(x(t), t)H_{4_i} K_j]^T \right. \\ &\quad \times [C_i + E_{3_i} F(x(t), t)H_{3_i} + D_i K_j \\ &\quad \left. + E_{4_i} F(x(t), t)H_{4_i} K_j] x(t) \right) \\ &= \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} (C_i + D_i K_j)^T \times \\ (C_i + D_i K_j) \\ \tilde{E}_i^T (C_i + D_i K_j) \end{array} \begin{array}{c} (*)^T \\ \tilde{E}_i^T \tilde{E}_i \end{array} \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix} \end{aligned}$$

and

$$\tilde{w}^T(t)\Gamma\tilde{w}(t) = \begin{bmatrix} F(x(t), t)H_{1_i}x(t) \\ F(x(t), t)H_{2_i}K_jx(t) \\ w(t) \\ F(x(t), t)H_{3_i}x(t) \\ F(x(t), t)H_{4_i}K_jx(t) \end{bmatrix}^T \Gamma \begin{bmatrix} F(x(t), t)H_{1_i}x(t) \\ F(x(t), t)H_{2_i}K_jx(t) \\ w(t) \\ F(x(t), t)H_{3_i}x(t) \\ F(x(t), t)H_{4_i}K_jx(t) \end{bmatrix}$$

$$\leq \gamma^2 w^T(t)w(t) + \rho^2 x^T \{H_{1_i}^T H_{1_i} + K_j^T H_{2_i}^T H_{2_i} K_j + H_{3_i}^T H_{3_i} + K_j^T H_{4_i}^T H_{4_i} K_j\} x(t).$$

Note that (9) can be rewritten as follows:

$$\begin{pmatrix} (A_i P + B_i Y_j)^T & (*)^T & (*)^T & (*)^T \\ + (A_i P + B_i Y_j) & & & \\ \begin{pmatrix} \tilde{B}_{w_i}^T \\ + \tilde{E}_i^T C_i P \\ + \tilde{E}_i^T D_i Y_j \end{pmatrix} & -\Gamma + \tilde{E}_i^T \tilde{E}_i & (*)^T & (*)^T \\ \tilde{C}_i P + \tilde{D}_i Y_j & 0 & -I & (*)^T \\ C_i P + D_i Y_j & 0 & 0 & -I \end{pmatrix} < 0.$$

Thus, pre- and post-multiplying (7) and (8) by

$$\begin{pmatrix} Q & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$$

yields

$$\begin{pmatrix} (A_i + B_i K_j)^T Q & (*)^T \\ + Q(A_i + B_i K_j) & \\ \begin{pmatrix} \tilde{B}_{w_i}^T Q \\ + \tilde{E}_i^T C_i \\ + \tilde{E}_i^T D_i K_j \end{pmatrix} & -\Gamma + \tilde{E}_i^T \tilde{E}_i \\ \tilde{C}_i + \tilde{D}_i K_j & 0 \\ C_i + D_i K_j & 0 \\ (*)^T & (*)^T \\ (*)^T & (*)^T \\ -I & (*)^T \\ 0 & -I \end{pmatrix} < 0, \tag{16}$$

$i, j = 1, 2, \dots, r$ . Applying the Schur complement to (16) and rearranging them, we have

$$\begin{pmatrix} \begin{pmatrix} (A_i + B_i K_j)^T Q \\ + Q(A_i + B_i K_j) \\ + (\tilde{C}_i + \tilde{D}_i K_j)^T \times \\ (\tilde{C}_m + \tilde{D}_m K_n) \\ + (C_i + D_i K_j)^T \times \\ (C_m + D_m K_n) \\ \tilde{B}_{w_i}^T Q + \\ \tilde{E}_i^T (C_i + D_i K_j) \end{pmatrix} & (*)^T \\ & -\Gamma + \tilde{E}_i^T \tilde{E}_i \end{pmatrix} < 0,$$

$i, j, m, n = 1, 2, \dots, r$ . Using (17) and the fact that

$$\begin{aligned} &\sum_{i=1}^r \sum_{j=1}^r \sum_{m=1}^r \sum_{n=1}^r \mu_i \mu_j \mu_m \mu_n M_{ij}^T N_{mn} \\ &\leq \frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j [M_{ij}^T M_{ij} + N_{ij} N_{ij}^T], \end{aligned} \tag{17}$$

$\mu_i \geq 0$  and  $\sum_{i=1}^r \mu_i = 1$ , (15) becomes

$$\dot{V}(x(t)) \leq -z^T(t)z(t) + \gamma^2 w^T(t)w(t). \tag{18}$$

Integrating both the sides of (18) yields

$$\begin{aligned} & \int_0^{T_f} \dot{V}(x(t)) dt \\ & \leq \int_0^{T_f} \left[ -z^T(t)z(t) + \gamma^2 w^T(t)w(t) \right] dt, \end{aligned}$$

which is

$$\begin{aligned} & V(x(T_f)) - V(x(0)) \\ & \leq \int_0^{T_f} \left[ -z^T(t)z(t) + \gamma^2 w^T(t)w(t) \right] dt. \end{aligned}$$

Using the fact that  $x(0) = 0$  and  $V(x(T_f)) \geq 0$  for all  $T_f \neq 0$ , we get

$$\int_0^{T_f} z^T(t)z(t) dt \leq \gamma^2 \int_0^{T_f} w^T(t)w(t) dt$$

Hence, the inequality (3) holds.  $\blacksquare$

**3.2.  $\mathcal{D}$ -stability constraints.** To begin this subsection, we recall the following definition.

**Definition 2.** (Chilali and Gahinet, 1996) A subset  $\mathcal{D}$  of the complex plane is called an *LMI region* if there exist a symmetric matrix  $L = [L_{kl}] = [L_{lk}] \in \mathbb{R}^{g \times g}$  and a matrix  $M = [M_{kl}] \in \mathbb{R}^{g \times g}$  such that

$$\mathcal{D} = \{z = x + jy \in \mathcal{C} : f_{\mathcal{D}}(z) < 0\}, \quad (19)$$

with the characteristic function

$$\begin{aligned} f_{\mathcal{D}}(z) &= L + Mz + M^T \bar{z} \\ &= [L_{kl} + M_{kl}z + M_{lk}\bar{z}]_{1 \leq k, l \leq g}. \end{aligned} \quad (20)$$

The following lemma will be needed to derive the main results in this subsection.

**Lemma 1.** (Chilali and Gahinet, 1996) Given a dynamic system  $\dot{x}(t) = Ax(t)$ , for an LMI region, a matrix  $A \in \mathbb{R}^{n \times n}$  is  $\mathcal{D}$ -stable in an LMI region, i.e.,  $\Lambda(I, A) \subset \mathcal{D}$  if there exists a matrix  $P \in \mathbb{R}^{n \times n}$  such that

$$\begin{aligned} & L \otimes P + M \otimes (AP) + M^T \otimes (AP)^T \\ &= [L_{kl}P + M_{kl}AP + M_{lk}PA^T]_{1 \leq k, l \leq n} < 0, \\ & P > 0, \end{aligned}$$

where  $\Lambda(I, A)$  is the set of generalized eigenvalues of the  $(I, A)$  pair, i.e.,  $\det(sI - A) = 0$ , and  $\otimes$  denotes the Kronecker product of the matrices.

Using Lemma 1, we have the following result.

**Theorem 2.** Given any LMI region, if there exist a matrix  $P_D$  and matrices  $Y_j$  for  $j = 1, 2, \dots, r$ , satisfying the following linear matrix constraints:

$$\Phi_{ii} < 0, \quad i = 1, 2, \dots, r, \quad (21)$$

$$\Phi_{ij} + \Phi_{ji} < 0, \quad i < j \leq r, \quad (22)$$

where

$$\begin{aligned} \Phi_{ij} &= L \otimes P_D + M \otimes A_i P_D + M \otimes B_i Y_j \\ &+ M^T \otimes P_D A_i^T + M^T \otimes Y_j^T B_i^T, \end{aligned} \quad (23)$$

then the closed-loop poles of each local system of (5) are  $\mathcal{D}$ -stable in the given LMI region. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^r \mu_j K_j x(t), \quad (24)$$

where  $K_j = Y_j P_D^{-1}$ .

*Proof.* Using Assumptions 1 and 2, the closed-loop fuzzy system (5) can be expressed as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left( [A_i + B_i K_j] x(t) \right. \\ &\quad \left. + \tilde{B}_w \tilde{w}(t) \right) \end{aligned} \quad (25)$$

where

$$\tilde{B}_w = \begin{bmatrix} E_{1_i} & E_{2_i} & B_w & 0 & 0 \end{bmatrix},$$

and the disturbance is

$$\tilde{w}(t) = \begin{bmatrix} F(x(t), t) H_{1_i} x(t) \\ F(x(t), t) H_{2_i} K_j x(t) \\ w(t) \\ F(x(t), t) H_{3_i} x(t) \\ F(x(t), t) H_{4_i} K_j x(t) \end{bmatrix}. \quad (26)$$

According to Lemma 1, the system (25) is  $\mathcal{D}$ -stable if there exists a  $Q_D$  such that

$$\begin{aligned} F_{\mathcal{D}} &\triangleq M_{kl} \left[ \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j (A_i + B_i K_j) \right] Q_D \\ &+ M_{lk} Q_D \left[ \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j (A_i + B_i K_j)^T \right] \\ &+ L_{kl} Q_D < 0. \end{aligned} \quad (27)$$

Now, we have to show that there exists a  $P_D$  such that  $F_{\mathcal{D}} < 0$ . Letting  $Q_D = P_D^{-1}$  and substituting it into (27), we get

$$\begin{aligned} F_{\mathcal{D}} &= L_{kl} P_D^{-1} \\ &+ M_{kl} \left[ \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j (A_i + B_i K_j) \right] P_D^{-1} \\ &+ M_{lk} P_D^{-1} \left[ \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j (A_i + B_i K_j)^T \right]. \end{aligned} \quad (28)$$

Pre-and post-multiplying both the sides of (28) by  $P_D$ , we have

$$\begin{aligned}
 & P_D F_D P_D \\
 &= L_{kl} P_D \\
 &+ M_{kl} \left[ \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j P_D (A_i + B_i K_j) \right] \\
 &+ M_{lk} \left[ \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j (A_i + B_i K_j)^T P_D \right].
 \end{aligned}$$

Using (21),(22) and the fact that  $\mu_i \geq 0$  and  $\sum_{i=1}^r \mu_i = 1$ , we deduce that there exists  $F_D < 0$ . Hence, we show that the closed-loop poles of each local system of (5) are  $\mathcal{D}$ -stable. ■

**3.3.  $\mathcal{H}_\infty$  fuzzy controller with  $\mathcal{D}$ -stability constraints.** In this section, we consider a multi-objective robust  $\mathcal{H}_\infty$  fuzzy controller such that the closed-loop poles of each local system of (5) are  $\mathcal{D}$ -stable in an LMI region and the inequality (3) is satisfied. In order to obtain solutions, we seek a common  $P$ , i.e., by enforcing  $P = P_D$ . The last result in this paper is given by the following theorem.

**Theorem 3.** Consider the system (1). Given a prescribed  $\mathcal{H}_\infty$  performance  $\gamma > 0$ , if there exist a matrix  $P = P^T$ , matrices  $Y_j$ ,  $j = 1, 2, \dots, r$ , a symmetric matrix  $L$  and  $M$  satisfying the following linear matrix inequalities:

$$\begin{aligned}
 & P > 0, \\
 & \Phi_{ii} < 0, \quad i = 1, 2, \dots, r, \\
 & \Phi_{ij} + \Phi_{ji} < 0, \quad i < j \leq r, \\
 & \Xi_{ii} < 0, \quad i = 1, 2, \dots, r, \\
 & \Xi_{ij} + \Xi_{ji} < 0, \quad i < j \leq r,
 \end{aligned}$$

where

$$\begin{aligned}
 \Phi_{ij} &= L \otimes P + M \otimes A_i P + M \otimes B_i Y_j \\
 &+ M^T \otimes P A_i^T + M^T \otimes Y_j^T B_i^T, \\
 \Xi_{ij} &= \begin{pmatrix} \Psi_{1ij} & (*)^T & (*)^T & (*)^T \\ \Psi_{2ij} & -\Gamma + \tilde{E}_i^T \tilde{E}_i & (*)^T & (*)^T \\ \Psi_{3ij} & 0 & -I & (*)^T \\ \Psi_{4ij} & 0 & 0 & -I \end{pmatrix},
 \end{aligned}$$

$$\begin{aligned}
 \Psi_{1ij} &= A_i P + P A_i^T + B_i Y_j + Y_j^T B_i^T, \\
 \Psi_{2ij} &= \tilde{B}_w^T + \tilde{E}_i^T C_i P + \tilde{E}_i^T D_i Y_j, \\
 \Psi_{3ij} &= \tilde{C}_i P + \tilde{D}_i Y_j, \\
 \Psi_{4ij} &= C_i P + D_i Y_j,
 \end{aligned}$$

with

$$\tilde{B}_w = [ E_{1i} \quad E_{2i} \quad B_w \quad 0 \quad 0 ],$$

$$\tilde{C}_i = [ \rho H_{1i}^T \quad \rho H_{3i}^T \quad 0 \quad 0 ]^T,$$

$$\tilde{D}_i = [ 0 \quad 0 \quad \rho H_{2i}^T \quad \rho H_{4i}^T ]^T,$$

$$\tilde{E}_i = [ 0 \quad 0 \quad 0 \quad E_{3i} \quad E_{4i} ],$$

$$\Gamma = \text{diag}\{I, I, \gamma^2 I, I, I\},$$

the inequality (3) holds and the closed-loop poles of each local system of (5) are  $\mathcal{D}$ -stable in the given LMI region. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^r \mu_j K_j x(t),$$

where

$$K_j = Y_j P^{-1}.$$

*Proof.* The desired result can be obtained by using Theorems 1 and 2, together with enforcing  $P = P_D$ . ■

### 4. Illustrative example

Consider a tunnel diode circuit shown in Fig. 1, where the tunnel diode is characterized by (Assawinchaichote and Nguang, 2006)

$$i_D(t) = -0.2v_D(t) - 0.01v_D^3(t).$$

Let  $x_1(t) = v_C(t)$  be the capacitor voltage and  $x_2(t) =$

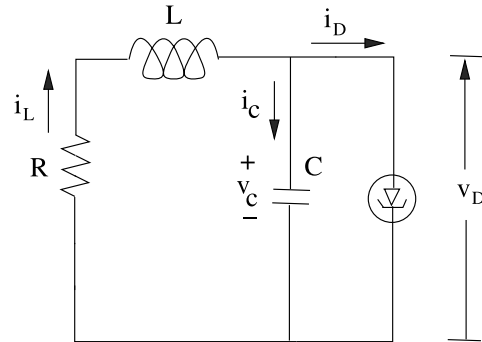


Fig. 1. Tunnel diode circuit (Assawinchaichote and Nguang, 2006).

$i_L(t)$  be the inductor current. Then, the circuit shown in Figure 1 can be modelled by the following state equations:

$$\begin{aligned}
 C\dot{x}_1(t) &= 0.2x_1(t) + 0.01x_1^3(t) + x_2(t) \\
 &+ 0.01w_1(t), \\
 L\dot{x}_2(t) &= -x_1(t) - Rx_2(t) + u(t) \\
 &+ 0.1w_2(t), \\
 z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},
 \end{aligned} \tag{29}$$

where  $u(t)$  is the control input,  $w_1(t)$  and  $w_2(t)$  are the process disturbances which may represent unmodelled dynamics,  $z(t)$  is the controlled output,  $x(t) = [x_1^T(t) \ x_2^T(t)]^T$  and  $w(t) = [w_1^T(t) \ w_2^T(t)]^T$ . Note that the variables  $x_1(t)$  and  $x_2(t)$  are treated as the deviation variables (variables deviate from its desired trajectories). The parameters in the circuit are given by  $C = 100$  mF,  $L = 1000$  mH and  $R = 1 \pm 0.3\% \ \Omega$ . With these, (29) can be rewritten as

$$\begin{aligned} \dot{x}_1(t) &= 2x_1(t) + (0.1x_1^2(t)) \cdot x_1(t) + 10x_2(t) \\ &\quad + 0.1w_1(t), \\ \dot{x}_2(t) &= -x_1(t) - (1 \pm \Delta R)x_2(t) + u(t) \\ &\quad + 0.1w_2(t), \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \end{aligned} \quad (30)$$

For simplicity, we will use as few rules as possible. Assuming that  $|x_1(t)| \leq 3$ , the nonlinear network system (30) can be approximated by the following TS fuzzy model:

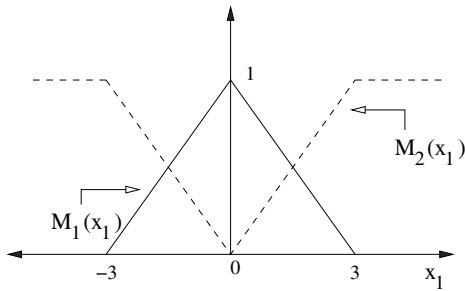


Fig. 2. Membership functions for the two fuzzy sets considered (Assawinchaichote and Nguang, 2006).

**Plant Rule 1:** IF  $x_1(t)$  is  $M_1(x_1(t))$  THEN

$$\begin{aligned} \dot{x}(t) &= [A_1 + \Delta A_1]x(t) + B_w w(t) + B_1 u(t), \\ z(t) &= C_1 x(t). \end{aligned}$$

**Plant Rule 2:** IF  $x_1(t)$  is  $M_2(x_1(t))$  THEN

$$\begin{aligned} \dot{x}(t) &= [A_2 + \Delta A_2]x(t) + B_w w(t) + B_2 u(t), \\ z(t) &= C_2 x(t), \end{aligned}$$

where  $x(0) = 0$ ,  $x(t) = [x_1^T(t) \ x_2^T(t)]^T$ ,  $w(t) = [w_1^T(t) \ w_2^T(t)]^T$ ,

$$\begin{aligned} A_1 &= \begin{bmatrix} 2 & 10 \\ -1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2.9 & 10 \\ -1 & -1 \end{bmatrix}, \\ B_w &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \end{aligned}$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_1 = E_{1_1} F(x(t), t) H_{1_1},$$

$$\Delta A_2 = E_{1_2} F(x(t), t) H_{1_2}.$$

Now, by assuming that, in (2),  $\|F(x(t), t)\| \leq \rho = 1$  and since the values of  $R$  are uncertain but bounded within 30% of their nominal values given in (29), we have

$$E_{1_1} = E_{1_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$H_{1_1} = H_{1_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

**Robust  $\mathcal{H}_\infty$  fuzzy controller design with  $\mathcal{D}$ -stability constraints.** Let us place the closed-loop poles of each local system within an LMI disk region with center  $q = -20$  and radius  $r = 19$ .

Note that the LMI disk region has the following characteristic function:

$$f_{\mathcal{D}}(z) = \begin{pmatrix} -r & q+z \\ q+\bar{z} & -r \end{pmatrix},$$

and

$$L = \begin{bmatrix} -r & q \\ q & -r \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Using Theorem 3 with  $\gamma = 1$ , we obtain

$$P = \begin{bmatrix} 0.5602 & -0.4132 \\ -0.4132 & 0.6602 \end{bmatrix},$$

$$Y_1 = [ -9.2411 \quad -8.0988 ],$$

$$Y_2 = [ -8.6991 \quad -8.0365 ],$$

$$K_1 = [ -47.4436 \quad -41.9590 ],$$

$$K_2 = [ -45.5172 \quad -40.6590 ].$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^2 \mu_j K_j x(t), \quad (31)$$

where

$$\mu_1 = M_1(x_1(t)) \text{ and } \mu_2 = M_2(x_1(t)).$$

The proposed approach yields a robust  $\mathcal{H}_\infty$  fuzzy controller which guarantees that (i) the inequality (3) holds and (ii) the closed-loop poles of each local system are within the given LMI stability region. The responses

of the state variables  $x_1(t)$  and  $x_2(t)$  are shown in Fig. 3 while the disturbance input signal,  $w(t)$ , which was used during simulation is given in Fig. 4. It is necessary to note that the disturbance cannot always be modelled as white noise, while measurement noise can be quite well described by a random process. The ratio of the regulated output energy to the disturbance input noise energy obtained by using the  $\mathcal{H}_\infty$  fuzzy controller (31) is depicted in Fig. 5. After 2 seconds, the ratio of the regulated output energy to the disturbance input noise energy tends to a constant value, which is about 0.145. Accordingly,  $\gamma = \sqrt{0.145} = 0.381$ , which is less than the prescribed values 1.

Finally, Table 1 shows a comparison of the location of closed-loop poles of each local system of the proposed method and the previous works. It is shown that the closed-loop poles of the proposed method are only located within the pre-specified region, but this is not valid for the other approaches. However, note that the proposed algorithm turns out to be efficient to apply for low-order problems; the computational time might not be suitable for high-order problems since the convergence time depends on the ‘size’ of the feasible solution set. In addition, due to the increasing size of LMI results produced using the proposed algorithm, the feasibility issue might jeopardize the existence of a solution.

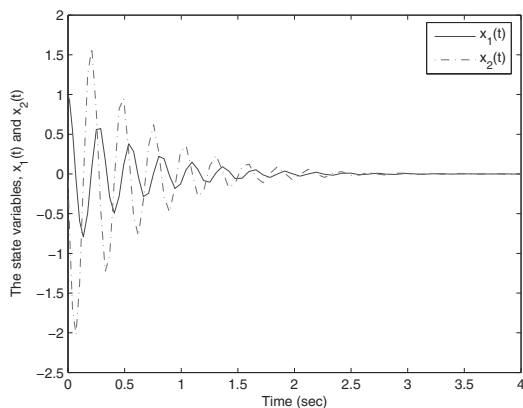


Fig. 3. State variables,  $x_1(t)$  and  $x_2(t)$ .

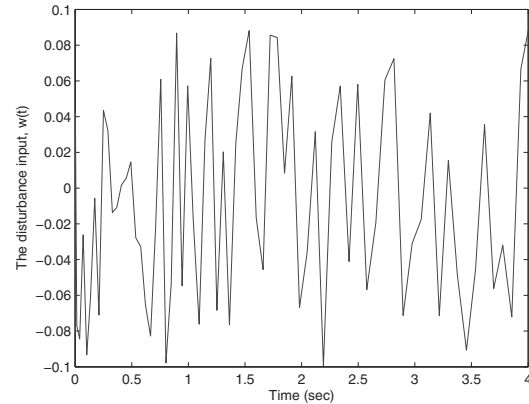


Fig. 4. Disturbance input noise,  $w(t)$ , used during simulation.

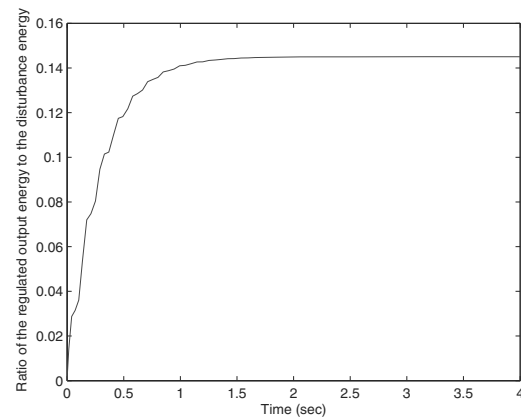


Fig. 5. Ratio of the regulated output energy to the disturbance noise energy,  $\left( \int_0^{T_f} z^T(t)z(t) dt / \int_0^{T_f} w^T(t)w(t) dt \right)$ .

### 5. Conclusion

This paper has presented a robust  $\mathcal{H}_\infty$  fuzzy controller design procedure for a class of fuzzy dynamic systems with  $\mathcal{D}$ -stability constraints described by a TS fuzzy model. Based on an LMI approach, we developed a technique for designing a robust  $\mathcal{H}_\infty$  fuzzy controller which guarantees the  $\mathcal{L}_2$ -gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value and the poles of each local system to be within a pre-specified region such that a satisfactory transient response can be obtained by enforcing the closed-loop pole to lie within a suitable region. Finally, a numerical example was given to show the effectiveness of the synthesis procedure developed in this paper. However, since in the designed approach the convergence time depends on the ‘size’ of the feasible solution set, the proposed method might not be suitable for large-order control problems. Therefore, the designing of a high performance multi-objectives controller can be

Table 1. Closed-loop poles of each local system.

Method	Plant Rule 1	Plant Rule 2
Proposed theorem	-15.9088	-13.7934
	-25.0502	-24.9656
Chayaopas <i>et al.</i> (2013)	-0.1201	-0.8964
	-13.6601	-18.1151
Assawinchaichote <i>et al.</i> (2013)	-20.9088	-18.7934
	-39.1702	-34.3356

\*\*Disk region with center  $q = -20$  and radius  $r = 19$ \*\*



considered in our possible future research work.

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### References

- Assawinchaichote, W. (2012). A non-fragile  $\mathcal{H}_\infty$  output feedback controller for uncertain fuzzy dynamical systems with multiple time-scales, *International Journal Computers, Communications & Control* **7**(1): 8–16.
- Assawinchaichote, W. and Chayaopas, N. (2013). Robust  $\mathcal{H}_\infty$  fuzzy speed control design for brushless DC motor, *International Conference on Computer, Electrical, and Systems Sciences, and Engineering, Tokyo, Japan*, pp. 1592–1598.
- Assawinchaichote, W. and Nguang, S.K. (2004a).  $\mathcal{H}_\infty$  filtering for fuzzy singularly perturbed systems with pole placement constraints: An LMI approach, *IEEE Transactions on Signal Processing* **52**(6): 1659–1667.
- Assawinchaichote, W. and Nguang, S.K. (2004b).  $\mathcal{H}_\infty$  fuzzy control design for nonlinear singularly perturbed systems with pole placement constraints: An LMI approach, *IEEE Transactions on Systems, Man, and Cybernetics: Part B* **34**(1): 579–588.
- Assawinchaichote, W. and Nguang, S.K. (2006). Fuzzy  $\mathcal{H}_\infty$  output feedback control design for singularly perturbed systems with pole placement constraints: An LMI approach, *IEEE Transactions Fuzzy Systems* **14**(3): 361–371.
- Ball, J.A., Helton, W.J. and Walker, M.L. (1993).  $\mathcal{H}_\infty$  control for nonlinear systems with output feedback, *IEEE Transactions on Automatic Control* **38**(4): 546–559.
- Ball, J.A., Helton, W.J. and Walker, M.L. (1994).  $\mathcal{H}_\infty$  control of systems under norm bounded uncertainties in all systems matrices, *IEEE Transactions on Automatic Control* **39**(6): 1320–1322.
- Basar, T. and Olsder, G.J. (1982). *Dynamic Noncooperative Game Theory*, Academic Press, New York, NY.
- Bouarar, T., Guelton, K. and Manamanni, N. (2013). Robust non-quadratic static output feedback controller design for Takagi–Sugeno systems using descriptor redundancy, *Engineering Applications of Artificial Intelligence* **26**(42): 739–756.
- Boyd, S., Ghaoui, L.E., Feron, E. and Balakrishnan, V. (1994). *Linear Matrix Inequalities in Systems and Control Theory*, SIAM Books, Philadelphia, PA.
- Chayaopas, N. and Assawinchaichote, W. (2013). Speed control of brushless DC motor with  $\mathcal{H}_\infty$  fuzzy controller based on LMI approach, *International Conference Modelling, Identification and Control, Phuket, Thailand*, pp. 21–26.
- Chen, B.-S., Tseng, C.-S. and Uang, H.-J. (2000). Mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  fuzzy output feedback control design for nonlinear dynamic systems: An LMI approach, *IEEE Transactions on Fuzzy Systems* **8**(3): 249–265.
- Chilali, M. and Gahinet, P. (1996).  $\mathcal{H}_\infty$  design with pole placement constraints: An LMI approach, *IEEE Transactions on Automatic Control* **41**(3): 358–367.
- Chilali, M., Gahinet, P. and Apkarian, P. (1999). Robust pole placement in LMI regions, *IEEE Transactions on Automatic Control* **44**(12): 2257–2270.
- Fu, M., de Souza, C.E. and Xie, L. (1992).  $\mathcal{H}_\infty$  estimation for uncertain systems, *International Journal of Robust and Nonlinear Control* **2**(1): 87–105.
- Gahinet, P., Nemirovski, A., Laub, A.J. and Chilali, M. (1995). *LMI Control Toolbox—For Use with MATLAB*, The MathWorks, Inc., Natick, MA.
- Han, Z.X. and Feng, G. (1998). State-feedback  $\mathcal{H}_\infty$  controllers design for fuzzy dynamic system using LMI technique, *IEEE World Congress on Computational Intelligence, Anchorage, AL, USA*, pp. 538–544.
- Han, Z.X., Feng, G., Walcott, B.L. and Zhang, Y.M. (2000).  $\mathcal{H}_\infty$  controller design of fuzzy dynamic systems with pole placement constraints, *American Control Conference, Chicago, IL, USA*, pp. 1939–1943.
- Hill, D.J. and Moylan, P.J. (1980). Dissipative dynamical systems: Basic input-output and state properties, *Journal of the Franklin Institute* **309**(5): 327–357.
- Isidori, A. and Astolfi, A. (1992). Disturbance attenuation and  $\mathcal{H}_\infty$ -control via measurement feedback in nonlinear systems, *IEEE Transactions on Automatic Control* **37**(9): 1283–1293.
- Joh, J., Chen, Y.H. and Langari, R. (1998). On the stability issues of linear Takagi–Sugeno fuzzy models, *IEEE Transactions on Fuzzy Systems* **6**(3): 402–410.
- Ma, X.J., Qi Sun, Z. and He, Y.Y. (1998). Analysis and design of fuzzy controller and fuzzy observer, *IEEE Transactions on Fuzzy Systems* **6**(1): 41–51.
- Mansouri, B., Manamanni, N., Guelton, K., Kruszewski, A. and Guerra, T. (2009). Output feedback LMI tracking control conditions with  $\mathcal{H}_\infty$  criterion for uncertain and disturbed T–S models, *Journal of the Franklin Institute* **179**(4): 446–457.
- Park, J., Kim, J. and Park, D. (2001). LMI-based design of stabilizing fuzzy controller for nonlinear system described by Takagi–Sugeno fuzzy model, *Fuzzy Sets and Systems* **122**(1): 73–82.
- Rezac, M. and Hurak, Z. (2013). Structured MIMO design for dual-stage inertial stabilization: Case study for HIFOO and Hinfstruct solvers, *Physics Procedia* **23**(8): 1084–1093.
- Scherer, C., Gahinet, P. and Chilali, M. (1997). Multiobjective output-feedback control via LMI optimization, *IEEE Transactions on Automatic Control* **42**(7): 896–911.

- Tanaka, K., Ikeda, T. and Wang, H.O. (1996). Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability,  $\mathcal{H}_\infty$  control theory, and linear matrix inequality, *IEEE Transactions on Fuzzy Systems* **4**(1): 1–13.
- Tanaka, K. and Sugeno, M. (1992). Stability analysis and design of fuzzy control systems, *Fuzzy Sets and Systems* **45**(2): 135–156.
- Tanaka, K. and Sugeno, M. (1995). Stability and stabilizability of fuzzy neural linear control systems, *IEEE Transactions on Fuzzy Systems* **3**(4): 438–447.
- Teixeira, M. and Zak, S.H. (1999). Stabilizing controller design for uncertain nonlinear systems using fuzzy models, *IEEE Transactions on Fuzzy Systems* **7**(2): 133–142.
- van der Schaft, A.J. (1992).  $\mathcal{L}_2$ -gain analysis of nonlinear systems and nonlinear state feedback  $\mathcal{H}_\infty$  control, *IEEE Transactions on Automatic Control* **37**(6): 770–784.
- Vesely, V., Rosinova, D. and Kucera, V. (2011). Robust static output feedback controller LMI based design via elimination, *Journal of the Franklin Institute* **348**(9): 2468–2479.
- Wang, H.O., Tanaka, K. and Griffin, M.F. (1996). An approach to fuzzy control of nonlinear systems: Stability and design issues, *IEEE Transactions on Fuzzy Systems* **4**(1): 14–23.
- Willems, J.C. (1972). Dissipative dynamical systems, Part I: General theory, *Archive for Rational Mechanics and Analysis* **45**(5): 321–351.
- Wonham, W.M. (1970). Random differential equations in control theory, *Probabilistic Methods in Applied Mathematics* **2**(3): 131–212.
- Yakubovich, V.A. (1967a). The method of matrix inequalities in the stability theory of nonlinear control system I, *Automation and Remote Control* **25**(4): 905–917.
- Yakubovich, V.A. (1967b). The method of matrix inequalities in the stability theory of nonlinear control system II, *Automation and Remote Control* **26**(4): 577–592.
- Yeh, K., Chen, C., Chen, C., Lo, D. and Chung, P. (2012). A fuzzy Lyapunov LMI criterion to a chaotic system, *Physics Procedia* **25**(1): 262–269.
- Yoneyama, J., Nishikawa, M., Katayama, H. and Ichikawa, A. (2000). Output stabilization of Takagi–Sugeno fuzzy system, *Fuzzy Sets and Systems* **111**(2): 253–266.
- Zadeh, L.A. (1965). Fuzzy set, *Information and Control* **8**(3): 338–353.
- Zhang, J.M., Li, R.H. and Zhang, P.A. (2001). Stability analysis and systematic design of fuzzy control system, *Fuzzy Sets and Systems* **120**(1): 65–72.



**Wudhichai Assawinchaichote** received the B.Eng. (Hons.) degree in electronic engineering from Assumption University, Bangkok, Thailand, in 1994, the M.Sc. degree in electrical engineering from the Pennsylvania State University (Main Campus), USA, in 1997, and the Ph.D. degree from the Department of Electrical and Computer Engineering of the University of Auckland, New Zealand (2001–2004). He is currently working as a lecturer in the Department of Electronic and Telecommunication Engineering at King Mongkut's University of Technology Thonburi, Bangkok. His research interests include fuzzy control, robust control and filtering, Markovian jump systems and singularly perturbed systems.

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