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Quantum mechanical aspects in the MEMS/NEMS technology

Abstract

According to the scaling laws for nanomechanical resonators, many of their metrological properties improve when downscaled. This fact encourages for constant miniaturization of MEMS/NEMS based sensors. It is a well known fact, that the laws of classical physics cannot be used to describe the systems which are arbitrarily small. In consequence, the classical description of nanoresonators must break down for sufficiently small and cool systems and then the quantum effects cannot be neglected. One of the fundamental question which arises is, how one may investigate quantum effects in MEMS/NEMS sensors and what is the influence of quantum effects on the performance of such systems. In this paper we would like to raise those issues by presenting the results of our work related to our estimations and calculations of MEMS/NEMS dynamics. The first and second sections are of theoretical character. In the first section (Classical modeling), we describe the classical methods for describing the resonator dynamics and the classical limit on the resolution of MEMS/NEMS based force sensors, which is set by the thermomechanical noise. In the second section (Quantum aspects), we concentrate on the quantum description of micro and nanoresonators and the influence of quantum effects, such as zero-point motion and back-action, on their performance (quantum limits). The third section is devoted to the presentation of our experimental methods of MEMS/NEMS deflection metrology, i.e. Optical Beam Deflection method (OBD) and fibre optics interferometry.

Keywords: MEMS, NEMS, OBD, FOI, quantum, force, resolution.

1. Classical modelling – classical harmonic oscillator

Practically all objects with certain inertness and elasticity are able to perform mechanical oscillations. Both, the macroscopic and nanosized systems vibrations can be characterized by their natural frequencies and eigenmode shapes, which depend on their physical properties and the geometry of the system. In the case of systems with simple geometries such as cantilevers (Fig.1.), beams (Fig.2.), and plates, their eigenfrequencies and eigenmodes can be found analytically. Often, however, the geometry of the system is not simple and then one may use the finite element method (FEM) to model the vibrations of a mechanical structure.

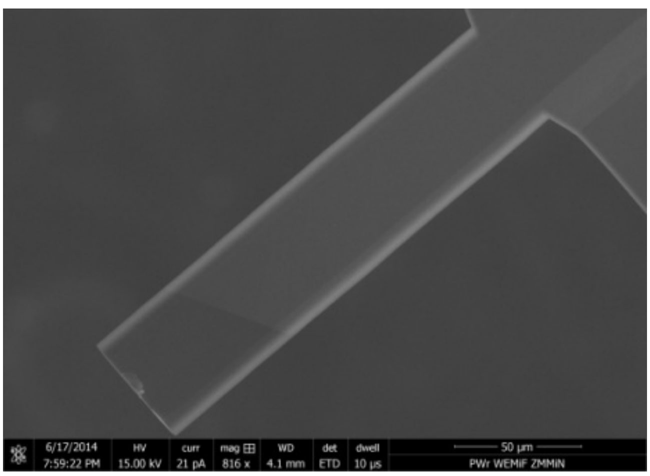

Fig. 1. Samples: MEMS/NEMS cantilever

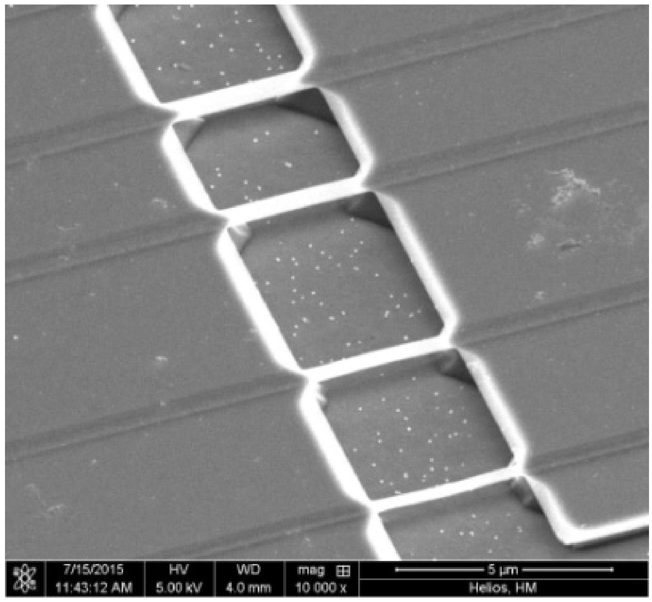

Fig. 2. Samples: An array of NEMS beams

Micro/nanobeams and cantilevers are widely used in micro/nano metrology. The vibrations of those structures are described by the Euler-Bernoulli equation with tension of the form [1]:

$$\rho A\frac{\partial^2 u}{\partial t^2}+D\frac{\partial^2 u}{\partial x^4}-T\frac{\partial^2 u}{\partial x^2}=q \tag{1}$$

where ρ is the material density, A is the cross-sectional area, D is the bending rigidity, T is the tension and q denotes the distributed load. By solving equation (1) with the appropriate boundary conditions one may obtain solutions of the form $U_n(x,t)=u_n(x)a_n(t)$, with the time dependent term being just the solution of the undamped classical harmonic oscillator problem and μ being the shape of the eigenmodes. The eigenfrequencies are described by the equation [1]:

$$\omega_n=\frac{\alpha_n^2}{l^2}\sqrt{\frac{D}{\rho A}} \tag{2}$$

with the coefficients α depending on the number of mode and boundary conditions (cantilever/beam). For instance, the eigenfrequency of the first mode for the nanobeam with the dimensions of 30 nm × 100 nm × 3.5 μm, made of silicon <100> is equal to about 19 MHz. The cantilever with the same parameters has the eigenfrequency of about 3 MHz. This kind of structure was recently made in the ZMMiN WEMiF PWr.

According to the analysis of the dynamics of driven damped nanobridges, the transfer function of such structures operating near their lowest eigenfrequency is of the form [2]:

$$H_B(\omega)=\frac{\mu_0/m}{\omega_0'^2+i\omega_o^2/Q-\omega^2} \tag{3}$$

where μ_0 is the measure of the average deflection of the fundamental mode, given by:

$$\mu_0 = \frac{1}{l^2}\int_0^l u_0(x)\,dx = 0.8309 \tag{4}$$

The transfer function (3) is very similar to the transfer function of the harmonic oscillator:

$$H_{HO}(\omega) = \frac{1/m}{\omega_0^{\,2} + i\omega_0\omega - \omega^2} \tag{5}$$

In fact, the difference between functions (3) and (5) is smaller than 1% for the $Q > 15$. This observation is consistent with the more general fact: each mode of vibrations of the MEMS/NEMS structure can be treated as a harmonic oscillator provided that displacement from the equilibrium position is small. Those oscillators are independent and according modes form a complete orthogonal base [1].

There are many noise sources in the MEMS/NEMS systems which affect their signal-to-noise ratio. Some of them may be reduced using better measurement electronics but the classical noise limit is determined by the level of thermomechanical noise. The nanoresonator is constantly exchanging energy with its environment. This interaction is modelled by the stochastic force F_n driving the resonator. The equation of motion takes the form of the Langevin equation:

$$m\ddot{x} + \Gamma x + kx = F_n \tag{6}$$

The power spectral density, (PSD) of the driving force is provided by the classical fluctuation-dissipation theorem [1]:

$$\bar{S}_{F_nF_n}(\omega) = \frac{4k_BT\omega_0 m}{Q} = 4k_BT\Gamma \tag{7}$$

The PSD of the displacement is related to the driving force PSD via the relation:

$$\bar{S}_{XX}(\omega) = \left|H_{HO}(\omega)\right|^2 S_{F_nF_n}(\omega) \tag{8}$$

which leads to the equation:

$$\bar{S}_{XX} = \frac{4k_BT\omega_0^3}{kQ\left[\left(\omega_0^2 - \omega^2\right) + \left(\omega_0\omega/Q\right)^2\right]} \tag{9}$$

Relation (9) can be used to find the spring constant of the structure via measuring its thermomechanical vibrations. Because thermomechanical noise is modelled as white noise and the transfer function of the resonator for the constant driving force is $1/k$, the variance of resonators displacement is [3]:

$$\left\langle x_{th}^2\right\rangle^{static} = \frac{1}{2\pi}\frac{1}{k^2}\int_0^B \frac{4kk_BT}{\omega_0 Q}2\pi df = \frac{4k_BTB}{k\omega_0 Q} \tag{10}$$

According to the Hooke's law, the minimal detectable static force acting along the displacement of the structure is equal to [3]:

$$F_{\min}^{static} = k\sqrt{\left\langle x_{th}^2\right\rangle^{static}} = \sqrt{\frac{4kk_BTB}{k\omega_0 Q}} \tag{11}$$

In the case of measuring a dynamic force using an Amplitude Modulation Detection Method (AMDM), the cyclic force is detected with the sensitivity improved by the Q factor. This time, the bandwidth can be assumed as very narrow and localised near the resonator eigenfrequency. As a consequence, the thermomechanical noise PSD can be assumed as constant [3]:

$$\left\langle x_{th}^2\right\rangle^{AM} = \frac{1}{2\pi}\left|H_{HO}(\omega_o)\right|^2\frac{4kk_BT}{\omega_0 Q}2\pi B = \frac{4k_BTBQ}{k\omega_0} \tag{12}$$

Thus the minimal detectable force for the AMDM-based sensors is [3].

$$F_{\min}^{AM} = k\sqrt{\left\langle x_{th}^2\right\rangle^{AM}}\Big/Q = \sqrt{\frac{4kk_BTB}{\omega_0 Q}} \tag{13}$$

2. Quantum aspects – quantum harmonic oscillator

According to the scaling laws for nanobridges/nanocantilevers [4], parameters such as mass, stiffness, dissipated mechanical energy, mechanical and thermal time constants, force noise and smallest detectable mass decrease while downscaling the structure. At the same time the resonant frequency increases. Generally, metrological parameters (e.g. sensitivity and resolution) of MEMS/NEMS sensors improve when built-in nanomechanical resonators (i.e. beams, cantilevers) become smaller. This fact explains the tendency for constant miniaturization of such systems.

When the nanoresonator is strongly cooled (e.g. to minimise the thermomechanical noise), its quantized, discrete energy structure appears [1]. Each mode of vibrations carries a quantized amount of energy, which, according to the quantum harmonic oscillator model, is equal:

$$E_n = \hbar\omega_n\left(N + \frac{1}{2}\right) \tag{14}$$

where $N = 0, 1, \ldots$ denotes the state number. The energy of the ground state (i.e. the state with the lowest possible energy) is:

$$E_0 = \frac{1}{2}\hbar\omega_n \tag{15}$$

The energy given by equation (15) is called the zero-point energy and its existence causes that even at the zero temperature, the variance of position of the quantum harmonic oscillator is not zero. Using the ground state wave function of the oscillator, one may find the RMS value of position as:

$$x_{SQL}^{rms} = \sqrt{\frac{\hbar}{2m\omega_n}} \tag{16}$$

The value (16) is called the Standard Quantum Limit (SQL). For the silicon nanobeam structure described earlier, the SQL for the first mode of vibrations is about 133 fm. The analogous structure, as a cantilever, has the corresponding SQL equal to about 337 fm.

A crucial obstacle for reaching the quantum limit of a nanomechanical mode is the thermal occupation factor N_{th} which is the measure of the level of thermomechanical noise. The average energy of the mechanical mode coupled to the thermal bath is [5]:

$$\left\langle E\right\rangle = \hbar\omega_n\left(\frac{1}{2} + \frac{1}{e^{\hbar\omega_n/k_BT} - 1}\right) = \hbar\omega_n N_{th} \tag{17}$$

Another useful parameter describing the resonator is its effective temperature T_R which is the measure of its displacement variance [1]:

$$T_R = \frac{k}{k_B}\langle x^2 \rangle \tag{18}$$

For high temperatures ($k_B T >> \hbar\omega_n$), the classical equipartition theorem holds and the effective temperature of the resonator is equal to the temperature of the reservoir (T_R=T). When the temperature become much lower ($k_B T \approx \hbar\omega_n$), the deviations from the classical behaviour cannot be neglected and $T_R \neq T$. Then, to describe all the quasi-classical thermal and quantum effects, one may use the generalized fluctuation-dissipation theorem which leads to the Callen-Welton equation for the thermomechanical force noise PSD [6]:

$$\overline{S}_{F_nF_n} = 4\hbar\omega_n\left(\frac{1}{e^{\hbar\omega_n/k_BT}-1}+\frac{1}{2}\right)\Gamma \tag{19}$$

When $k_B T << \hbar\omega_n$, N_{th} is less than 1 and the mode is "frozen out". In this state, it is likely that the resonator is in its ground state. For a 1 GHz resonator, the freeze-out of modes occurs for temperatures lower than 50 mK [5]. When the resonator is placed in the bath with zero temperature (T=0), its effective temperature is determined only by the zero-point fluctuations and is equal to $T_R = \hbar\omega_n/2k_B$ with the corresponding force PSD of the form $\overline{S}_{F_nF_n} = 2\hbar\omega_n\Gamma$. For example, the approximate temperature at which quantum effects appear, for the 19 MHz nanobridge structure described earlier, is about 6 mK.

The basic question for mechanical force detection is how precisely one may continuously measure the position of an object. To obtain the useful electric signal, a mechanical device must be coupled to a linear detector which consists of a displacement transducer (which converts position to voltage) and an amplifier. It turns out, that the performance of any linear amplifier is limited by the uncertainty principle [5]. Even for the optimal-engineered system, the measured output signal consists of an equal contribution of quantum noise from the nanomechanical resonator + transducer and from the linear amplifier forming the output signal. Even if the resonator will be placed at zero temperature environment, the quantum limit on variance of the position will be higher than the SQL by the factor of 1.35 [5]:

$$x_{QL}^{rms} = \sqrt{\frac{\hbar}{m\omega_n \ln 3}} = 1.35\, x_{SQL}^{rms} \tag{20}$$

There are two contributions to the noise which arises from the readout procedure and their nature is very different. The first one, the imprecision noise, is the consequence of all the uncertainties connected with the detector (i.e. electrical or photon shot noise) and is independent of the resonator. Because, for the fixed power of the signal, detector shot noise does not depend on the measurement of the resonator displacement, its PSD decreases when the coupling between resonator and detector increases. The second one, the back-action noise is caused by the influence of the detector on the resonator (the described process can be seen as the continuous weak quantum measurement which affects the resonator displacement and momentum via the stochastic force driving the resonator). The PSD of this noise contribution increases while the coupling between resonator and detector increases [1].

Optimally engineered coupling between the resonator and detector is achieved when those two contributions are equal [5]. Then, their total PSD is equal to the PSD of the zero-point fluctuations [6]:

$$\overline{S}_{XX}^{\min} = 2\overline{S}_{XX}^{zpf} = \frac{4\hbar}{\omega_n\Gamma} \tag{21}$$

It should be emphasized that although the total added noise is at a minimum, the uncertainty principle does not forbid the measurement sensitivity from being arbitrarily good [7].

3. Displacement measurements

There are several methods of displacement measurement in micro- and nanoscale and the optical transduction is currently one of the most prominent [8,9] compared to the other readout methods [10, 11]. The optical beam deflection and interferometry are relatively simple to implement and provide high comparable sensitivity [12, 13, 14].

3.1. Optical beam deflection

The optical beam deflection (OBD) method is one of the main techniques used to read out cantilever bending and twisting in atomic force microscopy (AFM) [12] and cantilever-based biochemical sensors applications, where the structure bending is an indication of a chemical, physical, or biological process [15, 16]. The technique was introduced by Mayer and Amer in 1988 [17].

The divergent beam from a laser diode is firstly collimated by collimator lens and then the light beam is focused on the cantilever. The light is reflected from the cantilever directed onto the position sensitive detector. The cantilever displacement is transformed into the laser spot displacement on the detector in vertical directions and horizontal direction. The position of the laser spot on the detector is determined by measuring the photocurrents from the corresponding segments of the detector.

The OBD method is highly sensitive and has excellent noise performance. In a well-designed system the resolution is limited principally by cantilever thermal noise. For a 450 µm long cantilever with 1 N/m spring constant the measured amplitude of thermomechanical noise was 4.2 pm with a thermal noise limit of 32 fm/√Hz. In this case, the resolution was limited principally by cantilever thermal noise. In the case of SQL, these resolution is sufficient for carrying out the experiment.

3.2. Fibre Optic Interferometry

Precise bulk optics systems allow non-contact investigations with no loss in the sensitivity of the sensor. Bulk optic systems like OBD require an alignment of free space elements and the mechanical construction is rather huge in a close proximity and with relation to small vibrating structures. Fibre optics based systems are simplified by using flexible waveguides that propagate the light and the only mechanical constraint occurs when coupling light to or from the fibre [18].The size of a measuring head is comparable with the small dimensions of the investigated structures and easy to use in any environment (vacuum or cryostat). This could be the main reason to use any fibre both in free standing [19,20] and integrated systems [21]. The main component is then the −3 dB fibre coupler which effectively acts as beam splitter.

The growing demand for better resolution on the one hand and the versatility on the other makes the fibre optics interferometry a remarkable technique. The simple Fabry-Perot configuration [22] can be used with the reference path reflected from the fibre-air interface at the end of the Fresnel reflection. The two ways of phase modulation read-out due to the movement of the structure can be obtained: one for displacement of the order of micrometers and one of the order of nanometers. The first type is connected with fringe pattern analysis, the second with the amplitude in comparison with the intensity of a single fringe. As far as SQL measurement for various structures are the point of interest, the achieved 359 fm/√Hz resolution has to be improved to be useful in these experiments. However, the simplicity of adjustment in any environment (measuring chamber) is so important to refine this method and make it leading for experiments connected with displacement measurements. It is foreseen to obtains a resolution of 20 fm/√Hz after the cooperation with NIST laboratory [23].

4. Conclusions

Micro- and nanoelectromechanical systems (MEMS, NEMS) form a group of transducers that can be used as sensors for very precise mass and force detection and metrology. These can be done by the resonant frequency shift or amplitude of mechanical vibration changes read-out. By scaling the MEMS/NEMS device down, the smaller resolution with better sensitivity can be obtained, however the mechanical parameters of such a system, the resonator dynamics, change significantly. On one hand the resonant frequency rises and the amplitude of vibrations decreases, on the other – for very small structures quantum effects can be observed.

We have shown the classical methods for describing MEMS/NEMS resonator dynamics and the classical limit on the resolution of MEMS/NEMS based force sensors, which is set by the thermomechanical noise (beams and cantilevers are our point of interest). We have also presented a quantum description of micro- and nanoresonators and the influence of quantum effects, such as zero-point motion and back-action, on their performance (quantum limits). We have compared the results of these considerations and calculations to present the methods of displacement metrological systems that work in the ZMMiN WEMiF PWr laboratories.

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5. References

[1] Poot M., van der Zant H. S.: Mechanical systems in the quantum regime. Physics Reports, 2012, 511(5), 273-335.

[2] Cleland A. N., Roukes M. L.: Noise processes in nanomechanical resonators. Journal of Applied Physics, 2002, 92(5), 2758-2769.

[3] Bhushan B., Kawata S.: Applied scanning probe methods VI. 2007, Springer-Verlag Berlin Heidelberg.

[4] Duraffourg L., Arcamone J.: Nanoelectromechanical Systems, 2015. John Wiley & Sons, Inc.

[5] Schwab K. C., Roukes M. L.: Putting mechanics into quantum mechanics. Physics Today, 2005, 58(7), 36-42.

[6] Montinaro M.: Coupling of nanomechanical resonators to controllable quantum systems (Doctoral dissertation, University_of_Basel), 2014.

[7] Teufel J. D., Donner T., Castellanos-Beltran M. A., Harlow J. W., Lehnert, K. W.: Nanomechanical motion measured with an imprecision below that at the standard quantum limit. Nature nanotechnology, 2009, 4(12), 820-823.

[8] Gao W.: Precision nanometrology and its applications to precision nanosystems. International Journal of Precision Engineering and Manufacturing, 2005, 6(4), 14-20.

[9] Basarir O., Bramhavar S., Basilio-Sanchez G., Morse T., Ekinci K. L.: Sensitive micromechanical displacement detection by scattering evanescent optical waves. Optics letters, 2010, 35(11), 1792-1794.

[10] Lee J. H., Yoon K. H., Kim T. S.: Characterization of resonant behavior and sensitivity using micromachined PZT cantilever. Integrated Ferroelectrics, 2002, 50(1), 43-52.

[11] Shekhawat G., Tark S. H., Dravid V. P.: MOSFET-embedded microcantilevers for measuring deflection in biomolecular sensors. Science, 2006, 311(5767), 1592-1595.

[12] Binnig G., Quate C. F., Gerber C.: Atomic force microscope. Physical review letters, 1986, 56(9), 930.

[13] Hoogenboom B. W., Frederix P. L. T., Yang J. L., Martin S., Pellmont Y., Steinacher M., Hug H. J.: A Fabry–Perot interferometer for micrometer-sized cantilevers. Applied Physics Letters, 2005, 86(7), 074101-1.

[14] Helm M., Servant J. J., Saurenbach F., Berger R.: Read-out of micromechanical cantilever sensors by phase shifting interferometry. Applied Physics Letters, 2005, 87(6).

[15] Boisen A., Dohn S., Keller S. S., Schmid S., Tenje M.: Cantilever-like micromechanical sensors. Reports on Progress in Physics, 2011, 74(3), 036101.

[16] Kopiec D., Pałetko P., Nieradka K., Majstrzyk W., Kunicki P., Sierakowski A., Gotszalk, T.: Closed-loop surface stress compensation with an electromagnetically actuated micro-cantilever. Sensors and Actuators B: Chemical, 2015, 213, 566-573.

[17] Meyer G., Amer N. M.: Novel optical approach to atomic force microscopy. Applied physics letters, 1988, 53(12), 1045-1047.

[18] Bosseboeuf A., Petitgrand S.: Application of microscopic interferometry techniques in the MEMS field. In Optical Metrology (pp. 1-16). Int. Society for Optics and Photonics, 2003, October.

[19] Rasool H. I., Wilkinson P. R., Stieg A. Z., Gimzewski J. K.: A low noise all-fiber interferometer for high resolution frequency modulated atomic force microscopy imaging in liquids. Review of Scientific Instruments, 2010, 81(2), 023703.

[20] Lawall J., Kessler E.: Michelson interferometry with 10 pm accuracy. Review of Scientific Instruments, 2000, 71(7), 2669-2676.

[21] Li Y., Mi X., Sasaki M., Hane K.: Precision optical displacement sensor based on ultra-thin film photodiode type optical interferometers. Measur. Science and Technology, 2003, 14(4), 479.

[22] Li M., Wang M., Li H.: Optical MEMS pressure sensor based on Fabry-Perot interferometry. Optics Express, 2006, 14(4), 1497-1504.

[23] Smith D. T., Pratt J. R., Howard L. P.: A fiber-optic interferometer with subpicometer resolution for dc and low-frequency displacement measurement. Review of Scientific Instruments, 2009, 80(3), 035105.

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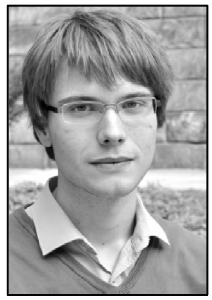

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