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SYNTHESIS OF MANIPULATION ROBOT PROGRAM TRAJECTORIES WITH CONSTRAINTS IN THE FORM OF OBSTACLES

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Abstract. *The paper presents a method of synthesis of manipulation robot motion trajectory according to mobility degrees. The configuration of the manipulator parts and obstacles are approximated by semiinfinite spaces limited by planes. The fact of robot collision with obstacles is reduced to the problem of determining the compatibility of systems of linear inequalities. The authors developed an algorithm for solving the problem based on dynamic programming method.*

Keywords: robot trajectory, the degree of mobility, the manipulator

SYNTEZA TRAJEKTORII PROGRAMOWYCH ROBOTA MANIPULACYJNEGO PRZY OGRANICZENIACH W POSTACI PRZESZKÓD

Streszczenie. *Przedstawiono metodę syntezy trajektorii ruchu robota manipulacyjnego wg stopni swobody. Konfiguracja położenia robota i przeszkód jest przybliżona półprzestrzenią ograniczoną płaszczyznami. Fakt zderzenia robota z przeszkodami ograniczono do problemu określenia zgodności systemów nierówności liniowych. Opracowano algorytm rozwiązywania zadań na podstawie metody programowania dynamicznego.*

Słowa kluczowe: trajektoria robota, stopnie swobody, manipulator

Introduction

In general, a manipulation robot is an open-loop kinematic system consisting of elements and linkage joints connecting them in series [5]. Geometric dimensions of elements, type of joints and range of their movements set the workspace at any point of which a robot gripper can be positioned. The type of joint is determined by structural features and capabilities of manipulation robot actuators. As a rule, they represent Grade 5 kinematic pairs, and can be set by logical variables p_i:

$$p_i = \begin{cases} 1, \text{ in case of translatory movement}, \\ 0, \text{ in case of rotary movement} \end{cases} \quad (1)$$

The variable determining the position and orientation of the gripper within the workspace are the values of generalized coordinates q_i, i=1,2,…,n, according to robot degrees of mobility, where n is the number of degrees of mobility.

1. Methodology for objects formal description

The problem of synthesis of program robot trajectories is to determine the values of generalized coordinates by mobility degrees providing moving the gripper along the predetermined trajectory with a given accuracy, if the conditions of mutual non-collisions of elements with each other and with barriers in technological space, i.e. in the space where they can move while the robot performs processing operations. Since a robot consists of a body and elements representing geometric objects moving in technological space containing obstacles also representing geometric objects, it is necessary to develop a technique of their formalized description. For this purpose a manipulator robot is represented as a complex geometric object consisting of a number of geometric subobjects: a fixed base and movable elements, element 2, …, element n, the position of each of which is determined by the values of the generalized coordinates. Each subobject is described as logical expressions $R_b(x,y,z)$, $R_l^1(x,y,z,q_1)$, $R_l^2(x,y,z,q_1,q_2)$, ..., $R_l^n(x,y,z,q_1,q_2,...,q_n)$, which are respectively: the basis, element 1, element 2,..., element n. Obstacles existing in the technology space are also represented as a number of fixed geometric objects that are described in the form of logical expressions: $R_p^1(x,y,z)$, $R_p^2(x,y,z)$,..., $R_p^m(x,y,z)$, respectively, where m is the number of obstacles.

The logical function describing a geometric object has the following form [4]:

$$R(x,y,z) = R_1 L R_2 L ... L R_N = 1, \quad (2)$$

R_k, (k=1,2,…N) are the logical variables defined by the following expression:

$$R_k = \begin{cases} 1, \ B_k(x,y,z) \le 0, \\ 0, \text{otherwise}. \end{cases},$$

where $B_k(x,y,z) \le 0$ is the inequality setting or approximating the *k* part of the edge of a geometrical object; N is the number of inequalities; L – the signs of logical operations of conjunction, disjunction or negation. We approximate the given trajectory of the gripper by a set of points $A_j(x_j, y_j, z_j)$, j=1,2,…,m, where m is the number of points approximating the trajectory. The distance between neighboring points $d_{j,j+1}$ is defined on the basis of condition of magnitude constancy of the trajectory curvature:

$$K = \frac{\alpha}{d_{j,j+1}},$$

Where α is the magnitude of change in the angle of the unit vector of tangents in points $A_j(x_j, y_j, z_j)$ and $A_{j+1}(x_{j+1}, y_{j+1}, z_{j+1})$; K = const, depth of camber ratio.

Let there be given a configuration of robot kinematics scheme determined by the vector $Q^j(q_1^i, q_2^i, ..., q_n^i)^T$, and the position of the gripper in points $A_j(x_j, y_j, z_j)$ in the coordinate system related to the fixed base of the robot. It is necessary to determine the vector $Q^{j+1}(q_1^{j+1}, q_2^{j+1}, ..., q_n^{j+1})^T$ providing moving the gripper to another point $A_{j+1}(x_{j+1}, y_{j+1}, z_{j+1})$ defined in the same coordinate system. Then, posing the problem of synthesis of manipulation robot trajectories by mobility can be represented as follows:

It is necessary to minimize (maximize) the kinematic quality criteria [2]:

$$J = \sum_{j=1}^{m-1} \sum_{i=1}^{n} C_i (q_i^i - q_i^{j+1})^2 \to \min(\max), \quad (3)$$

while the constraints define the condition point entries approximating the trajectory into the robot manipulation workspace of [4]:

$$\forall A_j(x_j, y_j, z_j),\ j = 1,2,...,m: D_1 L D_2 L ... L D_M = 1\cdot \quad (4)$$

where C_i is the coefficient characterizing the dynamic indicators of the drive of the i-th degree of mobility according to a predetermined parameter (energy consumption, speed, accuracy, etc.); q_i^j, q_i^{j+1}- elements of vectors Q^j and Q^{j+1} correspondingly, D_k, (k=1,2,…,M) – logical variables defined by the following expression:

$$D_k = \begin{cases} 1, & B_k(x,y,z) \le 0 \\ 0, & otherwise \end{cases},$$

where $B_k(x,y,z) \le 0$ – the inequality defining or approximating the k[th] part of the manipulation robot workspace, M - the number of inequalities, as well as additional constraints, taking into account possible mutual collision of robot elements (5) and robot elements with obstacles (6):

$$\mathrm{R_b} \wedge \mathrm{R_l^1} \wedge \mathrm{R_l^2} \wedge \cdots \wedge \mathrm{R_l^n} = 0, \quad (5)$$

$$(R_b \vee R_l^1 \vee R_l^2 \cdots \vee R_l^n) \wedge (R_p^1 \vee R_p^2 \vee \cdots \vee R_p^N) = 0, \quad (6)$$

where n is the number of manipulator elements, N is the number of obstacles in the technology space. For practical implementation of synthesis algorithm based on possible mutual collisions of elements and robot elements with the existing in the working area obstacles it is necessary to determine the fact of collision. For this purpose the obstacles located in the working area, and the robot elements are approximated by polyhedra described by the systems of linear inequalities (7):

$$\begin{cases} a_{1,1}^k x + a_{1,2}^k y + a_{1,3}^k z + a_{1,4}^k \ge 0, \\ a_{2,1}^k x + a_{2,2}^k y + a_{2,3}^k z + a_{2,4}^k \ge 0, \\ \dots\dots\dots\dots \\ a_{m,1}^k x + \mathrm{a}_{\mathrm{m},2}^{\mathrm{k}} y + \mathrm{a}_{\mathrm{m},3}^{\mathrm{k}} z + \mathrm{a}_{\mathrm{m},4}^{\mathrm{k}} \ge 0. \end{cases} \quad (7)$$

where $a_{i,j}^k$, j=1,2,3,4, i=1,2,...,m, are the coefficients defining the faces of the polyhedron in space OXYZ, approximating the kth obstacle (element), m is the number of faces of a kth polyhedron. Then the mutual collision of the lth obstacle (element) of the manipulation robot with the kth obstacle (element) is determined by the condition of the existence of solution of the following system of linear inequalities:

$$\begin{cases} a_{1,1}^k x + a_{1,2}^k y + a_{1,3}^k z + a_{1,4}^k \ge 0, \\ a_{2,1}^k x + a_{2,2}^k y + a_{2,3}^k z + a_{2,4}^k \ge 0, \\ \dots\dots\dots\dots \\ a_{m,1}^k x + a_{\mathrm{m},2}^{\mathrm{k}} y + a_{\mathrm{m},3}^{\mathrm{k}} z + a_{\mathrm{m},4}^{\mathrm{k}} \ge 0, \\ a_{1,1}^l x + a_{1,2}^l y + a_{1,3}^l z + a_{1,4}^l \ge 0, \\ a_{2,1}^1 x + a_{2,2}^l y + a_{2,3}^l z + a_{2,4}^l \ge 0, \\ \dots\dots\dots\dots \\ a_{n,1}^l \mathrm{x} + a_{\mathrm{n},2}^{1} \mathrm{y} + a_{\mathrm{n},3}^{1} \mathrm{z} + a_{\mathrm{n},4}^{1} \ge 0. \end{cases} \quad (8)$$

Using known methods of the theory of linear algebra we can determine the fact of the collision of manipulation robot elements with existing obstacles and the robot elements among themselves that arises from the condition of compatibility of the system of linear inequalities (8) [3]. One way of solving the problem is to compute all possible minors 3 of the system of linear inequalities (8). If there is a minor different from zero, then the system (8) is not compatible, i.e. there is no fact of collision of elements among themselves and with the obstacles existing in the working area. This problem can also be solved using the theory of matrices, based on the Kronecker-Capelli theorem [3]. The initial system of inequalities (8) by adding free variables is reduced to systems of linear equations. The coefficient matrix rank and the rank of the augmented matrix are calculated. When there is a coincidence, the system has at least one solution; otherwise there is no solution.

The algorithm of synthesis of program trajectories by robot manipulation mobility based on mutual collisions of robot elements and robot elements with the obstacles in the working area consists of two stages. At the first stage of the algorithm there is built a weighted graph whose vertices are the possible values of q_i, and the edges correspond to the kinematic pairs. At the second stage, basing on Bellman dynamic programming method we find the set $Q(q_{i,j})$ that minimizes the criterion of quality (3). Identification of possible values of q_i with sampling interval Δq_i depends on the type of the robot kinematic pair, i.e. on the values of the parameter p_i, p_{i+1}.

Let us describe the sections of the working space formed by the movement of the elements, starting from the rth degree of mobility of the manipulator robot, in the form of a logical expression:

$$L_r = D_1 L D_2 L ... L D_m = 1, \quad (9)$$

where D_k, (k=1,2,…,m) are the logical variables specifying or approximating a section of manipulation robot workspace.

Expression (9) is obtained as follows: using lengths of elements, i = 1,2, ..., n; parameters of joints $p_1, p_2, ..., p_n$, as well as considering the design constraints imposed on the changes of values of the generalized coordinates:

$$\mathrm{q_i^н} \le \mathrm{q_i} \le \mathrm{q_i^в}, \quad (10)$$

where $q_i^н$, $q_i^в$ - lower and upper values of the generalized coordinate of the ith degree of mobility of the manipulator. It is necessary to show a section of the robot workspace graphically. The resulting graphical representation is used to determine the elementary surfaces limiting the workspace section described by logical variables D_k. Linking the variables D_k, in accordance with the obtained graphical configuration of a part of the workspace, we obtain an expression of the form (9). Suppose that we are given the initial configurati on of the kinematic scheme which ensures the condition of non-collision of robot links among themselves and with barriers in technological space.

2. Algorithm of programmed trajectories synthesis

The algorithm is based on the analysis of the ways in the weighted graph containing all possible solutions and has the following form:

Step 1. Entering initial values: the coordinates of points approximating the trajectory of the gripper $A_j(x_j, y_j, z_j)$, $j = 1,2,...,\ m;$ logical expressions $L_1, L_2, .., L_{n-2}$ describing the working space and the subspaces; logical expressions:

$R_p^1(x,y,z)$, $R_p^2(x,y,z)$, ..., $R_p^m(x,y,z)$,

$R_b(x,y,z)$,

$R_l^1(x,y,z,q_1)$, $R_l^1(x,y,z,q_1,q_2)$, ..., $R_l^1(x,y,z,q_1,q_2,...,q_n)$

describing the obstacles, the robot base and links as geometric objects; $\Delta q_1, \Delta q_2, ..., \Delta q_n$ magnitude steps of finding a solution by mobility; $Q^{\text{н}}(q_1^{\text{н}}, q_2^{\text{н}}, ..., q_n^{\text{н}})^T$, $Q^{\text{в}}(q_1^{\text{в}}, q_2^{\text{в}}, ..., q_n^{\text{в}})^T$ - the vectors defining lower and upper values of change of the generalized coordinates values; the initial position of the manipulation robot configuration $Q^0(q_1^0, q_2^0, ..., q_n^0)^T$.

Step 2. If the condition

$$\forall A_j(x_j, y_j, z_j), \quad j = 1,2,...,m: \quad L_1 = 1$$

is satisfied, then go to Step 3, otherwise the problem has no sense and the solution is over.

Step 3. $j = 1$

Step 4. By each mobility degree the interval of change of the generalized coordinates is divided into n equal intervals. As a result of this procedure we receive the digraph of possible configurations. We define this graph by a triangular adjacency matrix of dimension (nm+2×nm+2), where n is the number of robot mobility degrees, m is the number of generalized values satisfying the condition (12), the adjacency matrix elements are determined based on the following expression:

$$a_{i,j}^{i+1,j} = \begin{cases} 1, \text{if vertex } q_{i,j} \text{ is adjacent to vertex } q_{i+1,j}, \\ 0, \text{otherwise.} \end{cases}$$

Step 5. For each possible configuration of the manipulation robot we verify the condition $R_b \wedge R_l^1 \wedge R_l^2 \wedge \cdots \wedge R_l^n = 0$, then the value $a_{i,j}^{i+1,j} = 1$, otherwise $a_{i,j}^{i+1,j} = 0$.

Step 6. For each possible configuration of the manipulation robot we verify the condition:

$$(R_b \vee R_l^1 \vee R_l^2 \cdots \vee R_l^n) \wedge (R_p^1 \vee R_p^2 \vee \cdots \vee R_p^m) = 0,$$

then the value $a_{i,j}^{i+1,j} = 1$, otherwise $a_{i,j}^{i+1,j} = 0$.

Step 7. Further we again define the values $q_{i,j}$ based on the following expression:

$$q_{i,j} = \min \sum_{j=1}^{m} C_i (q_i^0 - q_{i,j})^2.$$

Step 8. For the final vertex of the graph we have:

$$q^{\kappa} = \min \sum_{j=1}^{m} C_n (q_n^0 - q_{n,j})^2$$

Step 9. $j = j + 1$.

Step 10. If $j \le n$, then go to Step 4, otherwise to Step 11.

Step 11. We derive the values of the generalized coordinates by robot mobility of the vector $Q(q_{i,j})$.

Further the problem is solved according to a well-known scheme [2, 5]. On the basis of the solution of the inverse problem by statute using the method of spline functions in the space of generalized coordinates we can obtain the trajectory that ensures coincidence of the grip with program movement at the points approximating the motion trajectory. Then given the constraints on the velocity and acceleration, we obtain the manipulation robot control program.

The analysis of the obtained solution can be made as follows: we describe the robot links and obstacles with a reserve [2], i.e. these geometric objects are placed in the spaces completely covering them and also described by systems (7), (8). Then the robot is moved along a predetermined path based on control programs; and at each sampling interval it is analyzed for consistency with newly acquired, with respect to the entered reserve, systems of equations (7), (8). In this case, if no collisions are detected during the robot movements, the control program is admissible and optimal according to the selected criterion (3).

3. Conclusions

The proposed approaches for solving the problem of synthesis of manipulation robot movements allow avoiding some of the disadvantages (solvability condition, redundancy of solutions, the possibility of practical implementation, etc.) of the known approaches [2, 5].

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