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Formation control of underwater vehicles using Multi Agent System

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This paper proposes the development of a formation control algorithm of multiple acoustic underwater vehicles by employing the behaviour of autonomous mobile agents under a proposed pursuit. A robust pursuit is developed using the distributed consensus coordinated algorithm ensuring the transfer of information among the AUVs. The development of robust pursuit based on characteristics of multi-agent system is for solving the incomplete information capabilities in each agent such as asynchronous computation, decentralized data and no system global control. In unreliable and narrow banded underwater acoustic medium, the formation of AUVs based distributed coordinated consensus tracking can be accomplished under the constant or varying virtual leader's velocity. Further, the study to achieve tracking based on virtual leader AUV's velocity is extended to fixed and switching network topologies. Again for mild connectivity, an adjacency matrix is defined in such a way that an adaptive connectivity is ensured between the AUVs. The constant virtual leader vehicle velocity method based on consensus tracking is more robust to reduce inaccuracy because no accurate position and velocity measurements are required. Results were obtained using MATLAB and acquired outcomes are analysed for efficient formation control in presence of the underwater communication constraints.

Key words: AUV, Multi Agent System, formation control, switching network topology, mild connectivity

1. Introduction

Formation of multiple Acoustic Underwater Vehicles (AUVs) is a significant and mainstream control issue has received impressive consideration in case of certain system failure. Formation of AUVs in a mission provides saving the exploration time, reconfiguration ability, increase the effectiveness and robustness

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of the cooperative control [1]. Various commercial applications of formation control of AUVs are in gas and oil industries, in high resolution seabed examination, mapping and commercial assessment, post-lay pipe surveys and neutralization of undersea mines area [2]. This is only possible due to each AUV can function independently and collectively as group such as multi-agent system with the help of certain measurement, advanced control action and communication technology [3].

The present study is inspired by leader-follower formation of AUVs in presence of communication constraints to move cooperatively along a reference path. Various formation control algorithms for AUVs have been discussed in [4–30]. Nonlinear model for various marine crafts are described by Fossen [4]. Fuzzy logic based on formation control law is discussed in [5]. The leader-follower approach [6–9]; the virtual leader structure approach [10]; the artificial potential field method [11, 12]; the behavioural approach [13]; the graph-theory applications [14]; formation control based on geometrical structure [15–18]; formation induced by flocking [19–22] are proposed for the group coordination of multiple AUVs, to name just a few. Collaborative tasks based on inter AUV communication is presented in [23–25]. Cortes et al. develops the rendezvous autonomous agent using proximity graphs as described in [26]. Cyclic pursuit approach for AUVs has been proposed in [27]. The basic idea of flocking control mission is discussed in [28–30].

Formation control of multiple non-holonomic AUVs using pursuit strategy of autonomous mobile agents is proposed in this work. Virtual Leader's pathis treated as reference trajectory and those participating AUVs follow the reference path are known as followers via some controller technique [31]. Due to the constraints and imperfect channel sensing of underwater acoustic communication network, the multiple AUVs fail to keep knowledge of the state of other underwater robots with change in time. Thus a pursuit strategy is designed in such a way that the agents can sense the underwater environment and determine its next waypoint for the group coordination. Further the paper proposes distributed coordinated tracking algorithm as modelling of mobile agents using pursuit laws in shallow water environment [32]. The concept relies on consensus tracking in which each AUV obtains the information about position and velocity from its neighbours within the communication network. Analysis of the proposed algorithm provides guarantee to have global exponential tracking [33].

In this paper, the translational dynamics of AUV is being studied in detail. The contributions of the paper are as follows:

A. The consensus tracking is maintained by autonomous mobile agents using pursuit based on constant virtual leader's velocity.

B. Again the proposed pursuit algorithm based on varying velocity of virtual leaderis applied under fixed and switching network topology.

C. In hostile environment, the pursuit of autonomous mobile agents is described based on mild connectivity.

The remainder of this paper is organised as follows. The kinematics and dynamics of the AUV are discussed in Section 2. Section 3 provides problem statement based on various distributed consensus tracking. A detailed description about distributed consensus tracking using autonomous multi agent system is described in Section 4. The results are discussed in Section 5. Conclusion of the paper is provided in Section 6.

2. AUV modelling

Consider $i = 1, 2, , \ldots, N$ number of AUVs model in earth fixed frame through a cooperative motion plan, can be expressed as [4]

$$M_\eta(\eta_i)\dot{r}_i + C_\eta(\eta_i, r_i)r_i + D_\eta(\eta_i, r_i)r_i + g(\eta_i) = \bar{\tau}_i\,, \tag{1}$$

where

$$\dot{r}_i = \ddot{\eta}_i\,. \tag{2}$$

$$\dot{\eta}_i = R(\psi_i)v_i\,, \tag{3}$$

η_i describes the position of the AUV with respect to the earth fixed reference frame. v_i is the velocities of the AUV. $\bar{\tau}_i$ is the control input acting on the AUV in the earth fixed frame.

The schematic representation of an AUV in body and inertia reference frames is presented in Fig. 1 [2].

$M_\eta(\eta_i)$ is the inertia mass matrix is defined as

$$M_\eta(\eta_i) = R^T(\psi_i)MR(\psi_i). \tag{4}$$

$C_\eta(\eta_i, r_i)$ is the Coriolis and centripetal matrix given by

$$C_\eta(\eta_i, r_i) = \left[C(v) - MR^{-1}(\psi_i)\dot{R}(\psi_i)\right]R^{-1}(\psi_i). \tag{5}$$

$D_\eta(\eta_i, r_i)$ is Hydrodynamics damping matrix may be defined as

$$D_\eta(\eta_i, r_i) = D(v)R^{-1}(\psi_i). \tag{6}$$

$g(\eta_i)$ is gravitational forces and moments matrix, and the control input is given by

$$\bar{\tau}_i = R(\psi_i)\tau_i\,. \tag{7}$$

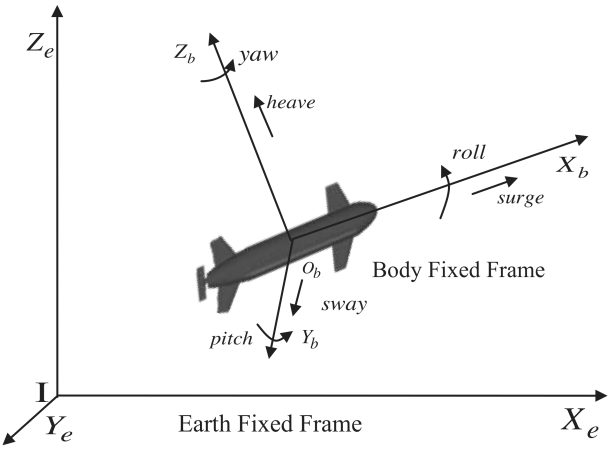

Figure 1: Schematic representation of an AUV

The velocity rotation/ transformation matrix between body and inertia frame is given by $R(\psi_i)$.

$$R(\psi_i) = \begin{bmatrix} \cos(\psi_i) & -\sin(\psi_i) & 0 \\ \sin(\psi_i) & \cos(\psi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

Assumption 1 *Referring to equation (4), (5) and (6), the subsequent assumptions have been considered [2]*

- *$M_\eta(\eta_i)$ for $i \in (1, 2, 3, \ldots, N)$ is symmetric positive stable for any number of AUVs*

$$M_\eta(\eta_i) = M_\eta^T(\eta_i) > 0, \quad \forall \eta \in R^3. \tag{9}$$

- *$C_\eta(\eta_i, r_i)$ will not be skew-symmetrical although $C_\eta(\eta)$ is a skew-symmetrical*

$$s^T \left(\dot{M}_\eta(\eta_i) - 2C_\eta(\eta_i, r_i) \right) s = 0, \quad \forall s \in R^3, \quad \eta \in R^3, \quad r \in R^3. \tag{10}$$

- *The damping matrix $D_\eta(\eta_i, r_i)$ is positive such that*

$$s^T D_\eta(\eta_i, r_i) s > 0, \qquad \forall s \neq 0, \quad \eta \in R^3, \quad r \in R^3. \tag{11}$$

- *The acceleration of gravity g will change by embedding a load results in change of mass matrix as well as mismatch of buoyancy centers and generated torques as the gravity. Here $g(\eta_i) = 0$, as we consider the motion along (X, Y) axis in three DOF.*

3. Problem statement

Each AUV acts as an autonomous mobile agent. Let the agents accomplish the formation using proposed pursuit along the reference path as presented in Fig. 2. A weighted undirected graph may be represented as $\varsigma = (\kappa, \varepsilon, A)$ the distributed consensus among the agents, where $\kappa = \{1, \ldots, n\}$ is the node set, $\varepsilon \subseteq \kappa \times \kappa$ is the edge set, and $A = \left[a_{ij}\right] \in R^{n\times n}$ is the weighted adjacency matrix. Let the Laplacian matrix $L = \left[l_{ij}\right] \in R^{n\times n}$ associated with A maybe defined as [32]

$$l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij} \qquad \text{and} \qquad l_{ij} = -a_{ij}, \quad i \neq j, \tag{12}$$

where edge (i, j) in ς may be denoted as agent i and agent j obtains information from each other. a_{ij} is a positive weight, if agent j is a neighbour of agent i if $(i, j) \in \varepsilon$, and $a_{ij} = 0$ otherwise. Note that here $a_{ij} = a_{ji}$, $\forall i \neq j$, since $(j, i) \in \varepsilon$ implies $(i, j) \in \varepsilon$. A path is a sequence of edges in an undirected graph of the form $(i_1, i_2), (i_2, i_3), \ldots,$ where $i_j \in \kappa$. L is symmetric positive semi-definite [32]. AUVs can transfer their information in the form position and velocity through undirected path exists between every pair of distinct nodes.

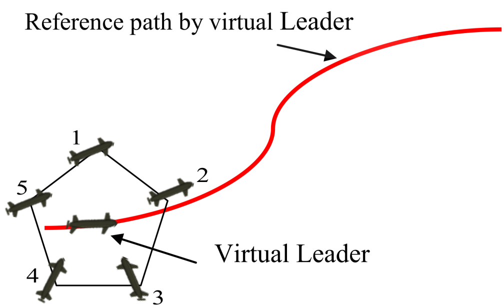

Figure 2: Formation control using proposed pursuit tracking the virtual leader's path

The formulation for the group of agents to track the reference virtual leader trajectory is given by

$$\lim_{t\to\infty} |\eta_i - \eta_r| = 0, \tag{13}$$

where η_r represents the virtual leader's path and the position of the i-th agent is given by η_i. Each agent may track the path of the agent connected through an undirected graph is given as

$$\lim_{t\to\infty} \eta_i - \eta_j = d_{ij} \quad \forall i \in (1, 2, 3, \ldots, N), \tag{14}$$

where $d_{ij} \in R^n$ is a constant vector in the global coordinate frame $\{E\}$.

4. Multi-agent System based Formation Control Design

Generally pursuit is framed considering the classical multi agent problem. Each AUV is designed as an mobile agent [34, 35]. The autonomous agents are preferred because it works without using any global information and centralized controller. A pursuit is a control law utilizes neighborhood interaction among the agents [36–38]. Suppose there is a virtual leader available by providing the reference path to the followers, having position η_o and velocity v_o.

4.1. Distributed consensus tracking with constant Leader's velocity

Based upon the distributed consensus tracking, the controller input is given as [32]

$$\bar{\tau}_i = -\sum_{j=0}^{n} a_{ij}(\eta_i - \eta_j) - \beta \mathrm{sgn}\left[\sum_{j=0}^{n} a_{ij}(v_i - v_j)\right], \tag{15}$$

where a_{ij} is a positive weight, agent j is a neighbour of agent i and β is a positive constant. The position $\eta_i \in R$ and velocity $v_i \in R$ of follower agent i and $\bar{\tau}_i \in R$ is the control input.

Assumption 2 *The constant velocity of virtual leader is v_0. For a fixed network topology, the undirected graph ς is connected and at least a_{i0} is nonzero. So*

$$v_i(t) \to v_0(t), \qquad \eta_i(t) \to \eta_0(t) \quad as \quad t \to \infty\,. \tag{16}$$

Let $\widetilde{\eta}_i = \eta_i - \eta_0$ and $\widetilde{v}_i = v_i - v_0$.,
The system can be rewritten as

$$\widetilde{r}_i = \dot{\widetilde{\eta}}_i = \widetilde{v}_i\,, \tag{17}$$

$$\dot{\widetilde{r}}_i = M_\eta^{-1}\left(\widetilde{\eta}_i\right)\left[\bar{\tau}_i - C_\eta(\eta_i, r_i)r_i - D_\eta(\eta_i, r_i)r_i - g(\eta_i)\right], \tag{18}$$

$$\dot{\widetilde{r}}_i = M_\eta^{-1}\left(\widetilde{\eta}_i\right)\left[-\sum_{j=0}^{n} a_{ij}\left(\widetilde{\eta}_i - \widetilde{\eta}_j\right) - \beta\,\mathrm{sgn}\left[\sum_{j=0}^{n} a_{ij}\left(\widetilde{v}_i - \widetilde{v}_j\right)\right]\right.$$
$$\left. -\; C_\eta(\eta_i, r_i)\widetilde{r}_i - D_\eta(\eta_i, r_i)\widetilde{r}_i - g(\eta_i)\right]. \tag{19}$$

As (9), (10) and (11) are symmetric, so Lyapunov candidate function of (19) can be given as

$$V = \frac{1}{2}\widetilde{\eta}^T M^2 \widetilde{\eta} + \frac{1}{2}\widetilde{v}^T M \widetilde{v}. \tag{20}$$

The derivative of V is given by [32]

$$\begin{aligned} \dot{V} &= \widetilde{\eta}^T M^2 \widetilde{\eta} + \widetilde{v}^T M \dot{\widetilde{v}} = \widetilde{\eta}^T M^2 \widetilde{\eta} + \widetilde{v}^T M \left[-M\widetilde{\eta} - \beta \operatorname{sgn}(M\widetilde{v}) \right] \\ &= -\beta \left\| M\widetilde{v} \right\|_1 , \end{aligned} \tag{21}$$

where $M = L + diag(a_{10}, \ldots, a_{n0})$ is symmetric positive definite [32]. It follows that $\dot{V}$ is negative semi-definite and $\dot{\widetilde{r}}_i = -M\widetilde{\eta} - \beta \operatorname{sgn}(M\widetilde{v})$ [32]. $\dot{V} \equiv 0$ implies that $\widetilde{v} \equiv 0_n$, results in $\widetilde{\eta} \equiv 0_n$ from (19). So $\widetilde{\eta}(t) \to 0_n$ and $\widetilde{v}(t) \to 0_n$ as $t \to \infty$ and $\eta_i(t) \to \eta_0(t)$ and $v_i(t) \to v_0(t)$ as $t \to \infty$.

4.2. Distributed consensus tracking for fixed network topology with varying Leader's velocity

It is assumed that the leader is having time varying velocity represented as v_0. In this method, absence of acceleration measurements may affect the follower AUVs to track the virtual leader. The proposed tracking algorithm for the agents may be given as [32]:

$$\begin{aligned} \bar{\tau}_i = &- \sum_{j=0}^{n} a_{ij} \left[(\eta_i - \eta_j) + \alpha (v_i - v_j) \right] \\ &- \beta \operatorname{sgn} \left\{ \sum_{j=0}^{n} a_{ij} \left[\gamma (\eta_i - \eta_j) + (v_i - v_j) \right] \right\}, \end{aligned} \tag{22}$$

where α, β, and γ are positive constants. It is assumed that $|\dot{v}_0| \leqslant \varphi_l$, where φ_l for the fixed network topology represents positive constant as shown in Fig. 3.

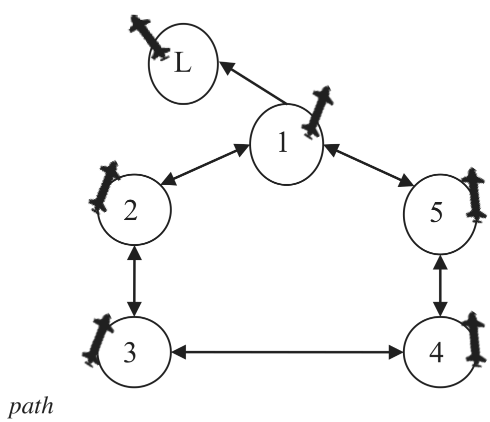

Figure 3: Proposed cyclic pursuit with a virtual leader (L) and five followers

The agents follow the virtual leader is given by $\widetilde{\eta}_i = \eta_i - \eta_0$ and $\widetilde{r}_i = v_i - v_0$, so the system of (1) may be represented as

$$\dot{\widetilde{\eta}}_i = \widetilde{v}_i\,, \tag{23}$$

$$\dot{\widetilde{r}}_i = M_\eta^{-1}\left(\widetilde{\eta}_i\right)\left[\bar{\tau}_i - C_\eta(\eta_i, r_i)\widetilde{r}_i - D_\eta(\eta_i, r_i)\widetilde{r}_i - g(\eta_i)\right] - \dot{v}_0\,. \tag{24}$$

Applying the value of $\bar{\tau}_i$ from (22) and assuming the assumption (9), (10) and (11), we have [32]

$$\dot{\widetilde{v}}_i = -M\widetilde{\eta} - \alpha M\widetilde{v} - \beta\,\mathrm{sgn}\left[M\left(\gamma\widetilde{\eta} + \widetilde{v}\right)\right] - 1\dot{v}_0\,, \tag{25}$$

where $M = L + diag(a_{10}, \ldots, a_{n0})$, γ and α are positive constants represented as $\alpha\gamma = 1$ and $\gamma < (4\lambda_{\min}(M)/4\lambda_{\min}(M) + 1)$ [32]. The Lyapunov function candidate is given as [32, 33]:

$$V = \begin{bmatrix}\widetilde{\eta}^T & \widetilde{v}^T\end{bmatrix} P \begin{bmatrix}\widetilde{\eta} \\ \widetilde{v}\end{bmatrix} = \frac{1}{2}\widetilde{\eta}^T M^2\widetilde{\eta} + \frac{1}{2}\widetilde{v}^T M\widetilde{v} + \gamma\widetilde{\eta}^T M\widetilde{v}\,, \tag{26}$$

where $P = \begin{bmatrix}(1/2)M & (\gamma/2)M \\ (\gamma/2)M & (1/2)M\end{bmatrix}$ is symmetric positive definite when γ satisfies. On differentiating V, we have

$$\begin{aligned}\dot{V} &= \widetilde{\eta}^T M^2\widetilde{\eta} + \widetilde{v}^T M\dot{\widetilde{v}} + \gamma\widetilde{v}^T M\widetilde{v} + \gamma\widetilde{\eta}^T M\dot{\widetilde{v}} \\ &= -\begin{bmatrix}\widetilde{\eta}^T & \widetilde{v}^T\end{bmatrix} Q \begin{bmatrix}\widetilde{\eta} \\ \widetilde{v}\end{bmatrix} - \left(\gamma\widetilde{\eta}^T + \widetilde{v}^T\right) M\left\{\beta\,\mathrm{sgn}\left[M\left(\gamma\widetilde{\eta} + \widetilde{v}\right)\right] + 1\dot{v}_0\right\} \\ &\leqslant -\begin{bmatrix}\widetilde{\eta}^T & \widetilde{v}^T\end{bmatrix} Q \begin{bmatrix}\widetilde{\eta} \\ \widetilde{v}\end{bmatrix} - (\beta - \varphi_l)\left\|M\left(\gamma\widetilde{\eta} + \widetilde{v}\right)\right\|_1\,,\end{aligned} \tag{27}$$

where $Q = \begin{bmatrix}\gamma M^2 & (\alpha\gamma/2)M^2 + (M^2 - \gamma M/2) \\ (\alpha\gamma/2)M^2 + (M^2 - \gamma M/2) & \alpha M^2 - \gamma M\end{bmatrix}$ is symmetric positive definite. The distributed consensus tracking may be attained at least globally exponential.

$$V \leqslant \lambda_{\max}(P)\left\|\begin{bmatrix}\widetilde{\eta}^T & \widetilde{v}^T\end{bmatrix}^T\right\|_2^2. \tag{28}$$

Equation (27) may be rewritten as

$$\dot{V} \leqslant -\begin{bmatrix}\widetilde{\eta}^T & \widetilde{v}^T\end{bmatrix} Q \begin{bmatrix}\widetilde{\eta} \\ \widetilde{v}\end{bmatrix} \leqslant -\lambda_{\min}(Q)\left\|\begin{bmatrix}\widetilde{\eta}^T & \widetilde{v}^T\end{bmatrix}^T\right\|_2^2 \leqslant -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}V\,. \tag{29}$$

Therefore, one can get that $V(t) \leqslant V(0)e^{-\lambda_{\min}(Q)/\lambda_{\max}(P)t}$ and $V \geqslant -\lambda_{\min}(P)\left\|\left[\widetilde{\eta}^T \quad \widetilde{v}^T\right]^T\right\|_2^2$ where $\lambda_{\min}(.)$ and $\lambda_{\max}(.)$ represnts the smallest and largest eigen values of the symmetric matrix respectively. So the agents tracking varying virtual leader's velocity is globally exponentially stable [33], if

$$\left\|\left[\widetilde{\eta}^T(t) \quad \widetilde{v}^T(t)\right]^T\right\|_2 \leqslant k_1 e^{-k_2 t} \tag{30}$$

where $k_1 = \sqrt{[\widetilde{\eta}^T(0) \quad \widetilde{v}^T(0)]\, P\, [\widetilde{\eta}^T(0) \quad \widetilde{v}^T(0)]^T / \lambda_{\min}(P)}$ and $k_2 = \lambda_{\min}(Q)/2\lambda_{\max}(P)$.

4.3. Distributed consensus tracking for switching network topology with varying virtual Leader's velocity

For switching network topology, the proposed algorithm for agents may be defined as [32]. $\bar{N}_i(t) \subseteq \{0, 1, \ldots, n\}$ as the neighbour set of follower AUVs i in the formation maitaining n follower AUVs and the virtual leader. Assuming $j \in \bar{N}_i(t)$, $i = 0, 1, \ldots, n$, $j = 0, 1, \ldots, n$, if $\left|\eta_i - \eta_j\right| < R$ at time t, $j \notin \bar{N}_i(t)$ otherwise, where R represents the sensing radius of the AUV.

$$\begin{aligned}\bar{\tau}_i = &- \sum_{j\in\bar{N}_i(t)} b_{ij}\left[(\eta_i - \eta_j) + \alpha(v_i - v_j)\right] \\ &- \beta \sum_{j\in\bar{N}_i(t)} b_{ij}\left(\operatorname{sgn}\left\{- \sum_{k\in\bar{N}_i(t)} b_{ik}\left[\gamma(\eta_i - \eta_j) + \alpha(v_i - v_j)\right]\right\}\right) \\ &- \operatorname{sgn}\left\{\sum_{k\in\bar{N}_j(t)} b_{jk}\left[\gamma(\eta_j - \eta_k) + \alpha(v_j - v_k)\right]\right\},\end{aligned} \tag{31}$$

where b_{ij}, $i = 1, \ldots, n$, $j = 0, \ldots, n$, are positive constants, α, β and $\gamma < \min_t 4\lambda_{\min}\left[\hat{M}(t)\right] / \left(4\lambda_{\min}\left[\hat{M}(t)\right] + 1\right)$ are positive constants and $\hat{P}(t) = \begin{bmatrix} (1/2)\hat{M}(t) & (\gamma/2)I_n \\ (\gamma/2)I_n & (1/2)I_n \end{bmatrix}$, $\hat{Q}(t) = \begin{bmatrix} \gamma\hat{M}(t) & (\alpha\gamma/2)\hat{M}(t) \\ (\alpha\gamma/2)\hat{M}(t) & \alpha\hat{M}(t) - \gamma I_n \end{bmatrix}$ are symmetric positive definite at each time instant and

$$\hat{M}(t) = \begin{cases} b_{ij} & j \in \bar{N}_i(t), \quad j \neq i, \\ 0 & j \notin \bar{N}_i(t), \quad j \neq i, \\ \sum\limits_{k\in\bar{N}_i(t)} b_{ik} & j = i. \end{cases} \tag{32}$$

Suppose the leader is having not less than one follower neighbour agent of that connected through undirected graph $\varsigma(t)$ may be defined as

$$\operatorname{sgn}\left\{\sum_{k\in\bar{N}_0(t)} b_{0\kappa}\left[\gamma(\eta_0-\eta_k)+(v_0-v_k)\right]\right\}=0. \tag{33}$$

If $\beta>\varphi_l$ and $0<\gamma<\min\limits_t\left\{\sqrt{\lambda_{\min}\left(\hat{M}(t)\right)},\ \dfrac{4\alpha\lambda_{\min}\left(\hat{M}(t)\right)}{4+\alpha^2\lambda_{\min}\left(\hat{M}(t)\right)}\right\}$ is verified, then $\eta_i(t)\to\eta_0(t)$ and $v_i(t)\to v_0(t)$ as $t\to\infty$. So the Lyapunov candidate function may be defined as [32, 33]

$$V=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}V_{ij}+\sum_{i=1}^{n}V_{i0}+\frac{1}{2}\widetilde{v}^T\widetilde{v}+\gamma\widetilde{\eta}^T\widetilde{v}\,. \tag{34}$$

4.4. Distributed consensus tracking based on mild connectivity with varying virtual Leader's velocity

If the undirected graph $\varsigma(t)$ at each time instant is linked then the leader AUV is presumed to be a neighbour of not less than one follower AUV [32]. As underwater channel model is unreliable and affected due to a number of constraints, the inter connectivity between the AUVs may not inevitably always satisfied. So when a mild connectivity is present between the virtual leader to agent 1 and between the neighbour agents by maintaining the initial pursuit. That is, if two followers are neighbours of each other at $t=0$, then the virtual leader should be a neighbour of this follower at $t>0$. So an adaptive connectivity adjacency matrix b_{ij} may be proposed in (31) and may be defined as [32, 33]

$$\begin{aligned}\bar{\tau}_i=-\alpha\sum_{j\in\bar{N}_i(t)} b_{ij}(t)(\eta_i-\eta_j)-\beta\sum_{j\in\bar{N}_i(t)} b_{ij}(t)\Bigg\{&\operatorname{sgn}\Bigg[\sum_{k\in\bar{N}_i(t)} b_{ik}(t)(\eta_i-\eta_k)\Bigg]\\ &-\operatorname{sgn}\Bigg[\sum_{k\in\bar{N}_j(t)} b_{jk}(t)(\eta_j-\eta_k)\Bigg]\Bigg\},\end{aligned} \tag{35}$$

where b_{ij}, $i=1,\ldots,n$, $j=0,\ldots,n$, are positive constants, differentiable and function of $\left\|\eta_i-\eta_j\right\|$, may be defined as

$$b_{ij}=\begin{cases}1 & \left\|\eta_i(0)-\eta_j(0)\right\|\geqslant R,\\ 0 & \left\|\eta_i(t)-\eta_j(t)\right\|<R,\\ b_{ij}(t)\to\infty & \left\|\eta_i(t)-\eta_j(t)\right\|\to R.\end{cases} \tag{36}$$

$V = (1/2)\widetilde{\eta}^T\widetilde{\eta}$ is taken as the Lyapunov function for the consensus tracking algorithm based on mild connectivity for (3) can be chosen as [32]

$$V = \begin{bmatrix} \widetilde{\eta}^T & \widetilde{v}^T \end{bmatrix} \hat{P}(t) \begin{bmatrix} \widetilde{\eta} \\ \widetilde{v} \end{bmatrix}, \tag{37}$$

$$\dot{V} = -\begin{bmatrix} \widetilde{\eta}^T & \widetilde{v}^T \end{bmatrix} \hat{Q}(t) \begin{bmatrix} \widetilde{\eta} \\ \widetilde{v} \end{bmatrix}, \tag{38}$$

where α, β, and $\gamma < \min_t 4\lambda_{\min}\left[\hat{M}(t)\right]/(4\lambda_{\min}\left[\hat{M}(t)\right] + 1)$ are positive constantsand $\hat{P}(t) = \begin{bmatrix} (1/2)\hat{M}(t) & (\gamma/2)I_n \\ (\gamma/2)I_n & (1/2)I_n \end{bmatrix}$, $\hat{Q}(t) = \begin{bmatrix} \gamma\hat{M}(t) & (\alpha\gamma/2)\hat{M}(t) \\ (\alpha\gamma/2)\hat{M}(t) & \alpha\hat{M}(t) - \gamma I_n \end{bmatrix}$ are symmetric positive definite for $0 \leqslant t_1 < t_2$,

$$\hat{M}(t) = \begin{cases} b_{ij} & j \in \bar{N}_i(t), \quad j \neq i, \\ 0 & j \notin \bar{N}_i(t), \quad j \neq i, \\ \sum\limits_{k \in \bar{N}_i(t)} b_{ik} & j = i, \end{cases} \tag{39}$$

$$x^T\left[\hat{M}(t_1) - \hat{M}(t_2)\right] \leqslant 0. \tag{40}$$

The proposed method in the mild connectivity is given by

$$\dot{V} \to -\infty \quad \text{as} \quad \left\|\eta_i(t) - \eta_j(t)\right\| \to R, \tag{41}$$

$$b_{ij}(t)\left|\eta_i(t) - \eta_j(t)\right| \to \infty \quad \text{as} \quad \left\|\eta_i(t) - \eta_j(t)\right\| \to R. \tag{42}$$

For consensus tracking the path by agents, accurate position and varying virtual Leader's velocity is required through proper connectivity.

5. Results and discussions

To verify the feasibility, a proposed control method with the AUV **hydrodynamic** parameters as given in Table 1 is simulated using MATLAB.

5.1. Simulation setup

The nominal the sampling period for inter AUV communication and virtual leader AUV velocity are $h = 0.3$ sec and $v_n = 1.8$ m/s respectively. For varying velocity of virtual leader varies from 1.5 m/s to 2.2 m/s at certain instant of time $t > 0$ and the followers uses pursuit strategy of autonomous mobile agents. The gains are taken as $k = [4, 6, 8, 10, 12]$.

Table 1: Parameters used for design of AUV [39]

Parameters	Values	Parameters	Values
AUV Mass	2234.5 kg	AUV Length	4.215 m
$X_{\dot{u}}$ (hydrodynamic derivatives for forces X)	−141.9 kg	X_{uu} (The hydrodynamic damping parameters for surge motion)	−35.4 kg/m
$Y_{\dot{v}}$ (hydrodynamic derivatives for forces Y)	−1715.4 kg	Y_{vv} (The hydrodynamic damping parameters for sway motion)	−667.5 kg/m
$N_{\dot{r}}$ (hydrodynamic derivatives for moment N)	−1349 kg.m^2/rad	N_{rr}(The hydrodynamic damping parameters for yaw motion)	−310
N_{vv} (Coriolis and centripetal parameter for yaw motion)	433.8 kg	X_{vr} (Coriolis and centripetal parameter for surge motion)	1715.4 kg/rad
Y_{ur} (Coriolis and centripetal parameter for sway motion)	103.4 kg/rad	N_{ur} (Coriolis and centripetal parameter for yaw motion)	−1427 kg.m/rad
Y_{uv} (Coriolis and centripetal parameter for sway motion)	−346.76 kg/m	N_{uv}(Coriolis and centripetal parameter for surge motion)	−686.08 kg

5.2. Results and discussion

A comparison is made for consensus tracking the reference path using autonomous mobile agents based on constant and varying Leader's velocity. The initial locations of the AUVs are given by $\eta_1(0) = [-10, 100]^T$, $\eta_2(0) = [-100, 150]^T$, $\eta_3(0) = [0, -200]^T$, $\eta_4(0) = [0, -100]^T$, $\eta_5(0) = [-50, -75]^T$ and the virtual leader path as given in (43) and (44). The desired path of the virtual leader is chosen as

$$x_0(t) = t, \tag{43}$$

$$y_0(t) = t + 200 \sin\left(\frac{2\pi}{72}t\right). \tag{44}$$

Fig. 4a displays the trajectory tracking for the desired path of virtual leader with constant velocity using mobile agents. Each AUV act as an agent and named as 1, 2, 3, 4, and 5 respectively. Fig. 4a shows taht each agent follows the reference path and track the virtual leader with constant velocity. Fig. 4b and Fig. 4c represents tracking error of X and Y respectively.

The starting locations of the AUVs are given by $\eta_1(0) = [-70, -200]^T$, $\eta_2(0) = [-25, -250]^T$, $\eta_3(0) = [-20, 270]^T$, $\eta_4(0) = [-50, 124]^T$, $\eta_5(0) =$

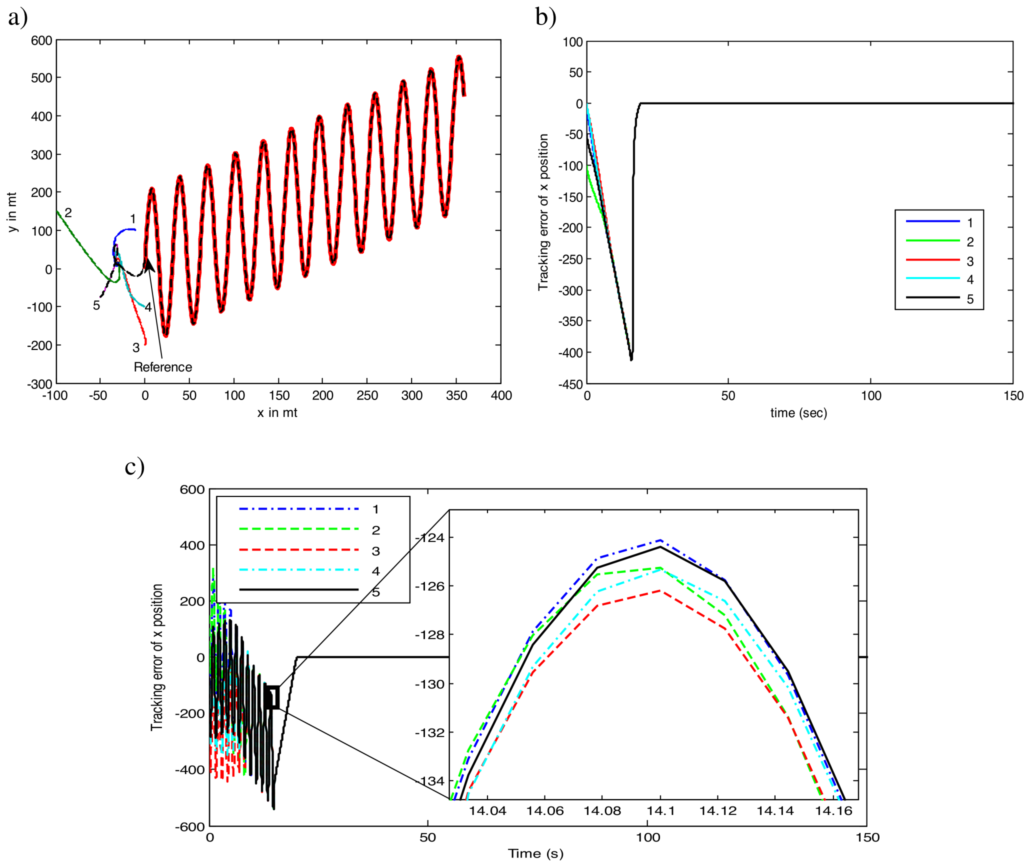

Figure 4: Trajectory path of five agents following virtual leader with constant velocity: a) Trajectories of five AUVs following virtual leader; b) X position tracking error; c) 3Y position tracking error

$[-100, 300]^T$ and the virtual leader path as given in (45) and (46). The reference path of the leader is given by

$$x_0(t) = t, \tag{45}$$

$$y_0(t) = 200 \sin\left(\frac{2\pi}{72}t\right). \tag{46}$$

Fig. 5a provides the trajectory tracking for the desired path of virtual leader with varying velocity using mobile agents. Fig. 5a shows that each agent follows the reference trajectory that track the virtual leader with varying velocity. Tracking error in X and Y direction are represented in Figs. 5b and 5c respectively. This means that first AUV 1 tracks the reference path and subsequently others with respect to time. It can also be observed that AUV 5 tracks the virtual leader with

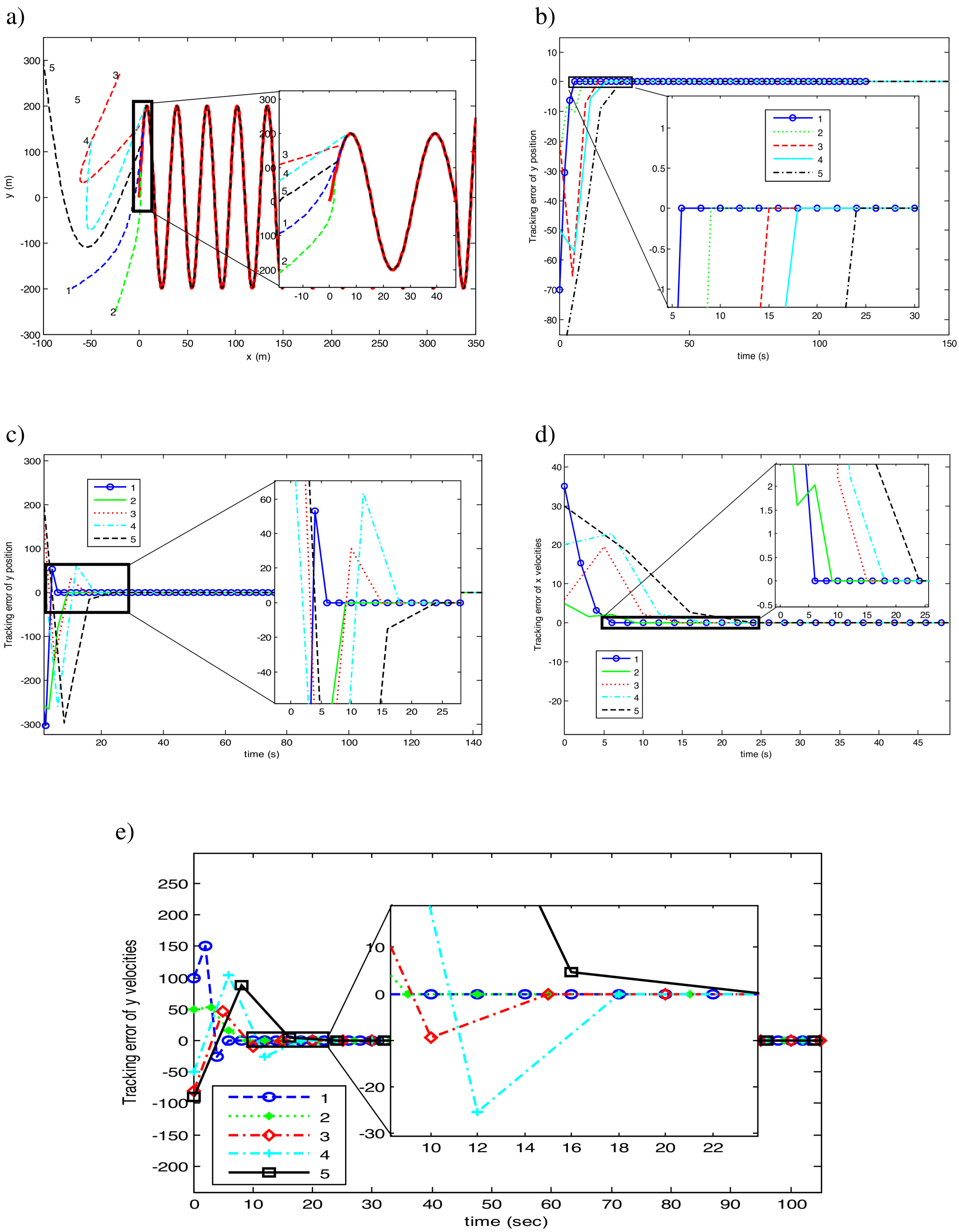

Figure 5: Trajectory path of five agents following virtual leader with varying velocity: a) Trajectories of five AUVs following virtual leader; b) X position tracking error; c) Y position tracking error; d) Tracking error of velocities in x axis; e) Tracking error of velocities in y axis

a delay than other agents. From Fig. 5d and Fig. 5e, it can be observed that the velocities of AUVs are varied with varying of virtual leader's velocity by tracking the virtual leader.

The initial positions of the agents are given by $\eta_1(0) = [45.29, 27.34]^T$, $\eta_2(0) = [45.67, 48.24]^T$, $\eta_3(0) = [6.392, 47.79]^T$, $\eta_4(0) = [5, 35]^T$, $\eta_5(0) = [31.62, 7.881]^T$ and the initial position of virtual leader is given by $\eta_0(0) = [25, 20]^T$, for a switching topology.

It can be observed from Fig. 6a that all the AUVs track the path of the virtual leader AUV using (45) and (46) at each time period by certain formation. Fig. 6b and Fig. 6c represents tracking error of X and Y respectively. For a case when a mild connectivity between the leader to agent 1 and between the neighbour AUVs is present at $t > 0$. The initial location for the follower AUVs are given as $\eta_1(0) = [-70, -200]^T$, $\eta_2(0) = [-25, -250]^T$, $\eta_3(0) = [-20, 270]^T$,

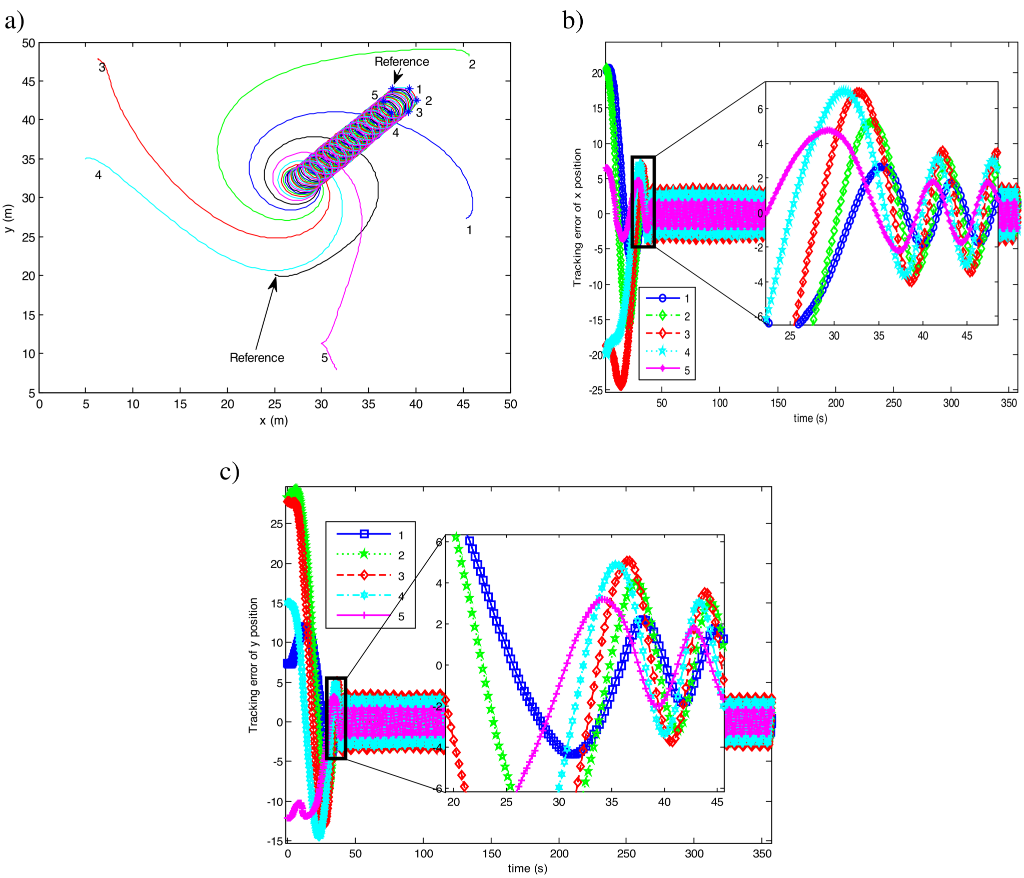

Figure 6: Trajectory path of five agents following virtual leader with switching network topology: a) Trajectories of five AUVs following virtual leader; b) X position tracking error; c) Y position tracking error

$\eta_4(0) = [-50, 124]^T$ and $\eta_5(0) = [-100, 300]^T$. Formation of five agents tracking virtual leader as given in (43) and (44) using mild connectivity is as shown in Fig. 7a. Trajectory tracking of desired virtual leader's path based on formation using five AUV followers is as shown in Fig. 7a. Due to shallow water, AUV 3 unable to track the route of the leader AUV but remain in formation. Error generated by following X and Y coordinates of cooperatively tracking AUVs are represented in Fig. 7b and Fig. 7c.

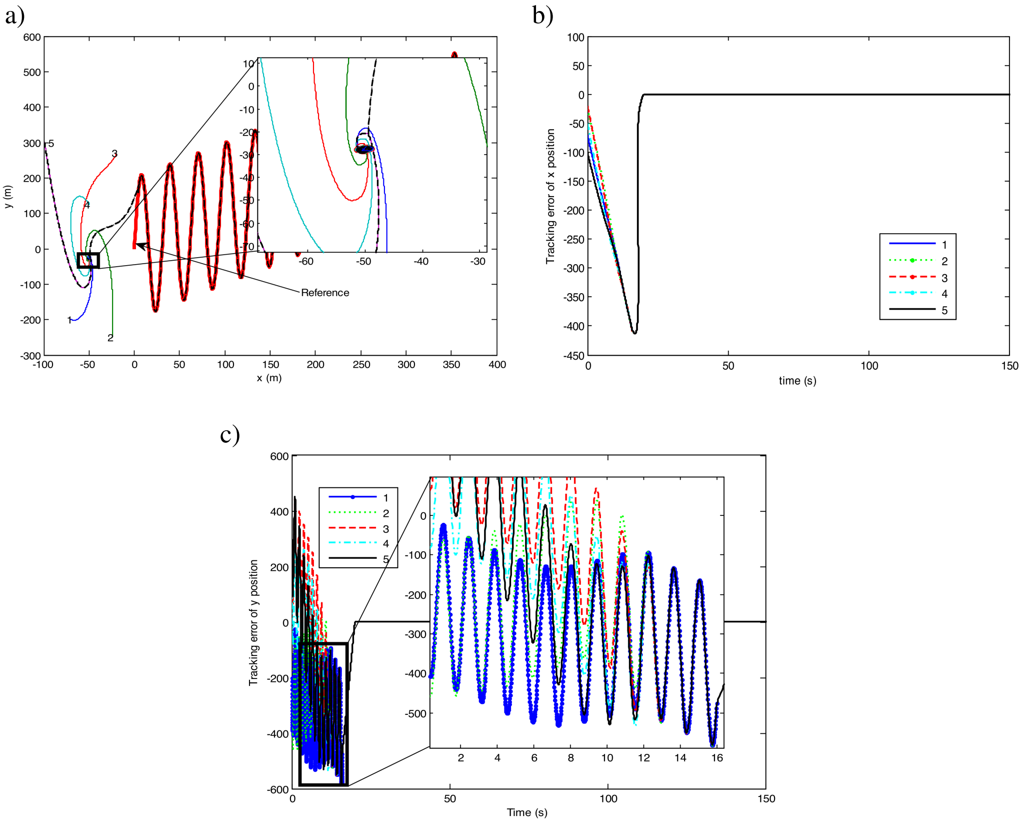

Figure 7: Trajectory path of five agents following virtual leader with mild connectivity: a) Trajectories of five AUVs following virtual leader; b) X position tracking error; c) Y position tracking error

6. Conclusions

In this paper, the development of formation control algorithms based on multi agent system using distributed consensus tracking in underwater area is presented. Distributed consensus tracking with fixed virtual Leader's velocity for fixed network topology, varying virtual Leader's velocity for fixed network topology, varying virtual Leader's velocity for switching network topology and varying virtual Leader's velocity for fixed network topology under mild connectivity are exhibited for formation of follower AUVs by tracking with the virtual leader co-operatively. Sufficient conditions on the undirected graph based on consensus tracking of AUV formations are presented such that the formation of mobile agents can be achieved asymptotically. A mild connectivity maintenance mechanism is also proposed for formation of AUVs using adjacency matrix. The adequacy and exactness of the proposed algorithms are checked through MATLAB simulations considering five AUVs as agents in formation. From the simulation, AUVs can track the reference path using the proposed formation algorithms and can be used in various underwater misions.

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