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BEAM PATTERN PROBABILITY DENSITY FUNCTION FOR MULTIPLE ECHO STATISTICS

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When fish target strength is estimated indirectly from single beam echo peak values the inverse techniques of solving the "single-beam integral equation" are recently quite satisfactorily used. This approach needs prior knowledge of beam pattern PDF, as it represents the kernel of the integral equation and is usually estimated under assumption of uniform spatial distribution of fish.

However, it may be shown that in some cases this assumption not necessarily must be valid. For instance, when the density of fish is increased one receives multiple echoes from one fish observed in successive transmissions. Moreover, in some situations uniform assumption works properly only for the cases of large number of samples. Taking into account this phenomenon the accuracy of solution may be improved by including fish counting statistics in calculating the beam pattern PDF.

1. INTRODUCTION

Target strength estimation when using single beam echosounder systems leads into inverse problem in which probability density function (PDF) of target strength is estimated from fish echoes. Due to hydroacoustic system characteristics the reconstruction is based on incomplete data. This kind of problems is an example of statistical linear inverse problem, which is typically ill-conditioned and can be solved using direct inverse techniques based on regularization or iterative ones in which additional constraints are specified. In most cases the observed data are restricted to the certain range limited by side-lobe level. This approach allows omitting the problem of ambiguity of beam pattern function [1]. However, to calculate this function additional assumption on distribution of fish behavior need to be made.

2. BEAM PATTERN PDF BASICS

For beam pattern PDF calculation let us consider first the ideal circular piston in a infinite baffle. Its one-way beam pattern function b is:

$$b(\theta) = \frac{2J_1(x)}{x} \tag{1}$$

where is x defined by $x=x(\theta)=ka\ sin\theta$ (k – wave number, a – transducer radius) and $J_n()$ – Bessel function of first kind order n. The logarithmic version of two-way pattern is derived by transform $B(\theta)=10\ log\ b(\theta)^2=20\ log\ b(\theta)$.

The kernel function of the inverse problem $p_B()$ for two-way system is obtained from its absolute version $p_b()$, which may be expressed as a parametric function $p_b(b)=(b^2(\theta) , p_b(\theta))$ with angle θ as a parameter:

$$p_b(b) = \left(\left(\frac{2J_1(x)}{x}\right)^2 , \frac{p_\theta(\theta)tan\theta}{\left|\frac{8J_1(x)J_2(x)}{x}\right|} \right) \tag{2}$$

where $p_\theta()$ is a probablity density function of random angular position of fish, $J_n(\ )$ represents Bessel function of first kind of order n and $x=ka\ sin\theta$ (k-wave number, a-transducer radius). Then using logarithmic transform of variables $B(b)=20\ log\ b$ its PDF relation may be written as:

$$p_B(B) = \frac{\ln 10}{20}\left|10^{\frac{B}{20}}\right| p_b\left(10^{\frac{B}{20}}\right) \tag{3}$$

Typical approach in PDF calculation of angular fish position $p_\theta()$ is based on the assumption of uniform distribution of fish in water column (called further non-multiple echoes statistics), which gives *sine*-law distribution of angular position θ [1]:

$$p_\theta(\theta) = \frac{1}{1-\cos\theta_{\max}}\sin\theta \tag{4}$$

where θ_{max} is maximum angle of beam pattern involved in calculation. However as will be shown in next section, for datasets obtained during survey where several echoes from one fish are present in consecutive pings, more accurate asumption can be made.

3. STATISTICS OF ANGULAR POSITION OF THE FISH FOR MULTIPLY ECHO TRACES

Let us consider now the distribution of angular position of fish $\underline{\theta}$ that is necessary for calculation of beam pattern PDF (Fig.1):

$$\underline{\theta} = arc\cos\frac{\underline{z}}{\underline{R}} = arc\cos\frac{1}{\sqrt{1+(\underline{\rho}/\underline{z})^2}} \tag{5}$$

where random variable $\underline{z}$ represents fish depth and random variables $\underline{R}$ and $\underline{\rho}$ represents coordinates of fish position related by equation: $\underline{R}^2=\underline{\rho}^2+\underline{z}^2$. Let us also consider the random variable $\underline{t}$ called trace distance, which represents distance of the fish from the crossing point of circular slice. Assuming that fish swims on the chord and is "sampled" uniformly in the consecutive pings, we may treat its distribution as uniform in a range (0, $2\ \underline{r}\ sin\ \underline{\alpha}$). Thus, the trace distance random variable may be expressed as $\underline{t} = 2\ \underline{r}\ sin\ \underline{\alpha}\ \underline{u}$, where $\underline{u}$ has again normalised uniform distribution. Taking into account the cosine law in the non-right angled triangle (Fig.1) we obtain:

$$\underline{\rho}^2 = \underline{r}^2 + \underline{t}^2 - 2\underline{r}\,\underline{t}\sin\underline{\alpha} = \underline{r}^2\left(1-(2\sin\underline{\alpha})^2(\underline{u}-\underline{u}^2)\right) \tag{6}$$

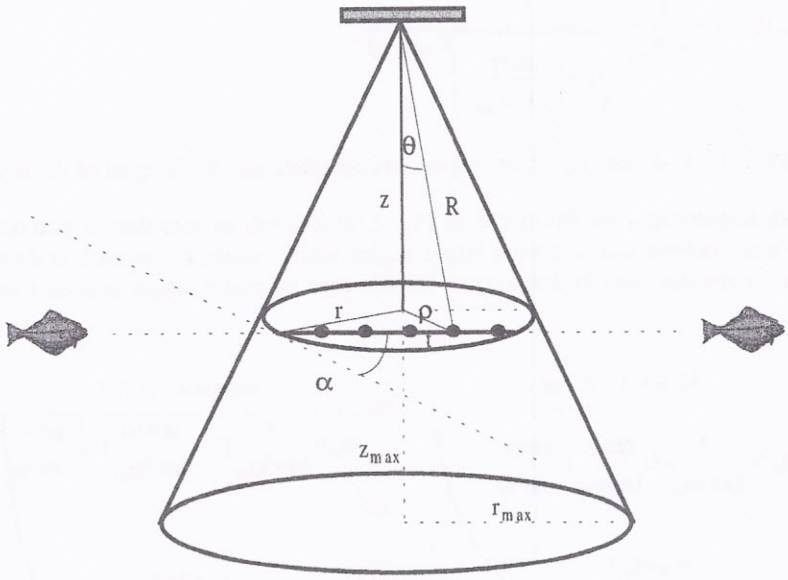

Fig 1. Geometry of multiple echo traces of the fish

Substituting $\underline{z} = \underline{r}\ tan\theta_{max}$ which removes $\underline{z}$ dependence from Eq.(5) and $\underline{r}$ dependence in Eq.(6), we receive the PDF of angular position as:

$$\underline{\theta} = arc\cos\frac{1}{\sqrt{2-(2\sin\underline{\alpha})^2(\underline{u}-\underline{u}^2)}} \tag{7}$$

Eq. (7) shows that distribution of angular position $\underline{\theta}$ depends only on distribution of crossing angle $\underline{\alpha}$ as random variable $\underline{u}$ represents uniform distribution resulting in PDF of variable $\underline{u}$-$\underline{u}^2$ expressed as $p_{u-u^2}(x) = (1/4-x)^{-1/2}$. The distribution of variable $\underline{\alpha}$ depends on angular relations between fish and vessel movement [2] and can change from *sine*-law when stationary fish model is used to uniform distribution when stationary vessel model is used. Both models give distribution of variable $4sin^2\underline{\alpha}$ as $p_{4\sin^2\alpha}(x) = (4x-x^2)^{-1/2}/\pi$ for *sine*-law model or $p_{4\sin^2\alpha}(x) = (4-x)^{-1/2}/4$ for uniform distribution one. Finally using the formulae for PDF of the product of random variables and transforming according to Eq.(7) we receive for the first and the second model respectively:

$$p_{\theta 1}(\theta) = \frac{1}{tan^2\theta_{max}} K\left(\frac{tan\theta}{tan\theta_{max}}\right)\frac{\sin\theta}{\cos^3\theta} \tag{8}$$

$$p_{\theta 2}(\theta) = \frac{1}{tan^2\theta_{max}} \frac{1}{\sqrt{1-\left(\frac{tan\theta}{tan\theta_{max}}\right)^2}} \frac{\sin\theta}{\cos^3\theta} \tag{9}$$

where $K(k) = \int_0^{\pi/2} (1\text{-}k^2 \sin^2 \varphi)^{-1/2} d\varphi$ represents complete elliptic integral of the first kind.

Both distributions are illustrated in Fig. 2. It is worth to note that as one could expect there are more echoes received from larger angles which results in increase in distribution as compared to *sin*-like distribution known for the case of non-multiple echoes from a single fish.

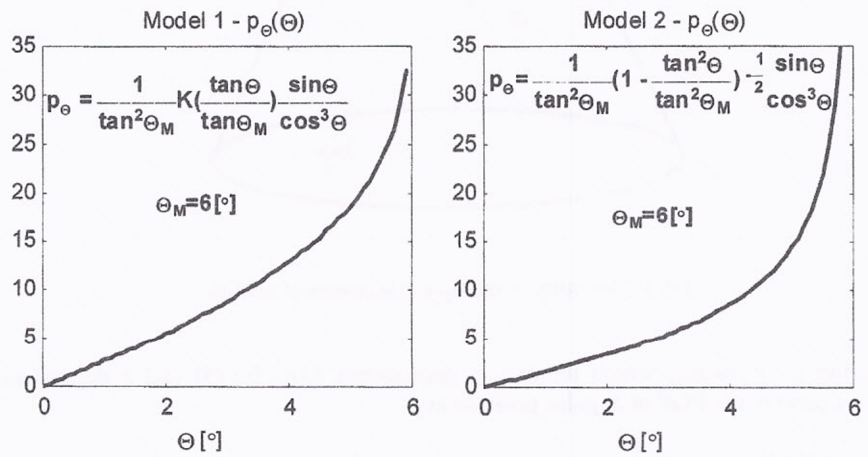

Fig.2. Theoretical distribution of angular position θ for two analysed models

4. BEAM PATTERN PDF FOR ACTUAL SYSTEM

The way of calculation of beam pattern PDF for Biosonics's dual-beam ESP (Echosounder Signal Processor) system is presented. The ESP system uses dual-beam (6°/15°) digital echosounder of 420 kHz operating frequency and 0.4 ms pulse length. The calculation of beam pattern is performed only for narrow beam channel as echo from this channel may be used for inverting taget strength of the fish. The beam pattern was fitted using following approximation proposed in [1]:

$$b(\theta) = \left(1 - (1 - 2^{-\gamma}) \frac{(1-\cos\theta)}{1-\cos\theta_{3dB}}\right)^{\frac{1}{\gamma}} \tag{10}$$

where exponential coefficient γ=–0.1 was fitted numerically to actual pattern. The logarithmic transform and inclusion of non-multiple echo statistics Eq.(4) leads to following equation:

$$p_B(B) = \frac{\ln 10}{20}\frac{\gamma}{1-2^{-\gamma}}\frac{1-\cos\theta_{3dB}}{1-\cos\theta_{max}}10^{\frac{\gamma B}{20}} \tag{11}$$

and inclusion of multiple echo statistics Eq. (8) leads to equation :

$$p_B(B) = \frac{\ln 10}{20}\frac{\gamma}{1-2^{-\gamma}}\frac{1-\cos\theta_{3dB}}{tan^2\theta_{max}}K\left(\frac{tan\theta}{tan\theta_{max}}\right)\frac{1}{\cos^3\theta}10^{\frac{\gamma B}{20}} \tag{12}$$

where θ can be calculated as inverse of (10): $\theta = arc\cos(1-(1-10^{0.05\gamma B})(1-\cos\theta_{3dB})/(1-2^{-\gamma}))$

Fig. 3 illustrates approximation of actual beam pattern of narrow beam channel and two PDF functions one with non-multiple-echo assumption and the other with multiple echo assumption.

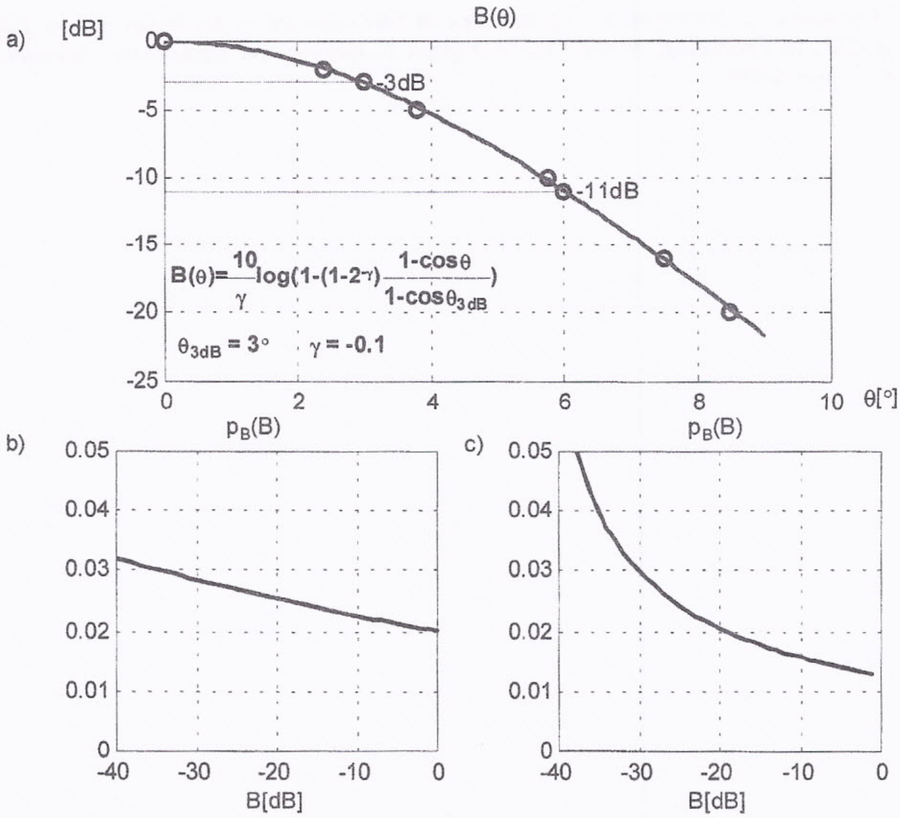

Fig. 3. a) Beam pattern approximation b) beam pattern PDF for non-multiple echoes assumptions c) beam pattern PDF for multiple echoes assumption

5. CONCLUSION

The results of using two different assumptions on statistics of fish echo traces in the process of calculation of beam pattern PDF is presented in the paper. When the data acquired during measurements contain multiple echoes from one fish more adequate approach than typically used is suggested. The presence of multiple number of fish echo traces can be verified numerically during data post-processing resulting in distribution of number of multiple echoes in fish traces. The results presented in the paper allow using multiple echo statistics for beam pattern calculation.

REFERENCE

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