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Mechanical vibrations: recent trends and engineering applications

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Abstract. Although the study of oscillatory motion has a long history, going back four centuries, it is still an active subject of scientific research. In this review paper prospective research directions in the field of mechanical vibrations were pointed out. Four groups of important issues in which advanced research is conducted were discussed. The first are energy harvester devices, thanks to which we can obtain or save significant amounts of energy, and thus reduce the amount of greenhouse gases. The next discussed issue helps in the design of structures using vibrations and describes the algorithms that allow to identify and search for optimal parameters for the devices being developed. The next section describes vibration in multi-body systems and modal analysis, which are key to understanding the phenomena in vibrating machines. The last part describes the properties of granulated materials from which modern, intelligent vacuum-packed particles are made. They are used, for example, as intelligent vibration damping devices.

Key words: mechanical vibrations; energy harvesting; modal analysis; granular materials.

1. INTRODUCTION

Although the development of the theory of mechanical vibrations has a long history, extending back four centuries, it is still an active subject of scientific research. One can observe this activity looking for example at the number of papers devoted to theory of mechanical vibrations and its applications (Table 1), which has been published in prestigious international Journals within last five years. The data is taken from the Web of Science Core Collection database and indicated 14,890 papers on mechanical vibrations related to wide variety of scientific categories, at the top of which are obviously Mechanical Engineering and Mechanics, but we can find here also Material Science, Electrical and Civil Engineering (Fig. 1). Moreover, in Fig. 2, we can find similar comparison ordered according to countries, where we can recognize significant contribution from China, but also USA and Iran. Poland is in this ranking on the 13th position, which proves that large group of Polish researchers is working in that field as well. Therefore, the Authors of this overview decided to provide in this paper a synthetic description subjectively selected topics related to more general theory of mechanical vibrations.

The article presents four selected issues from the broad field of mechanical vibration and their engineering applications.

Table 1
Number of articles devoted to mechanical vibrations
(*data from last 5 years according to Web of Sciences Core Collection*)

topic	no. of papers
mechanical vibrations in general:	14 890
– energy harvesting	974
– parametric identification	38
– modal analysis	731
– granular materials	45

In Section 2 discusses the latest achievements in the use of vibrations to obtain energy. This is one of the key challenges facing science. The constantly increasing demand for energy and the challenges related to environmental protection encourage the search for new forms of energy harvesting. Some solutions related to energy harvesting using the energy of mechanical waves allow at the same time to reduce noise, which has a detrimental effect on the lives of people of that time.

The multitude of solutions in the field of designing devices using vibration means that the search for the optimal solution could take a considerable amount of time. Modern solutions in the field of information methods allow the design of devices for a specific application. Section 3 discusses, inter alia, works that use solutions such as heuristic optimization algorithms, including genetic algorithms (GA), particle swarm optimization (PSO) and stochastic subspace identification, a system equiva-

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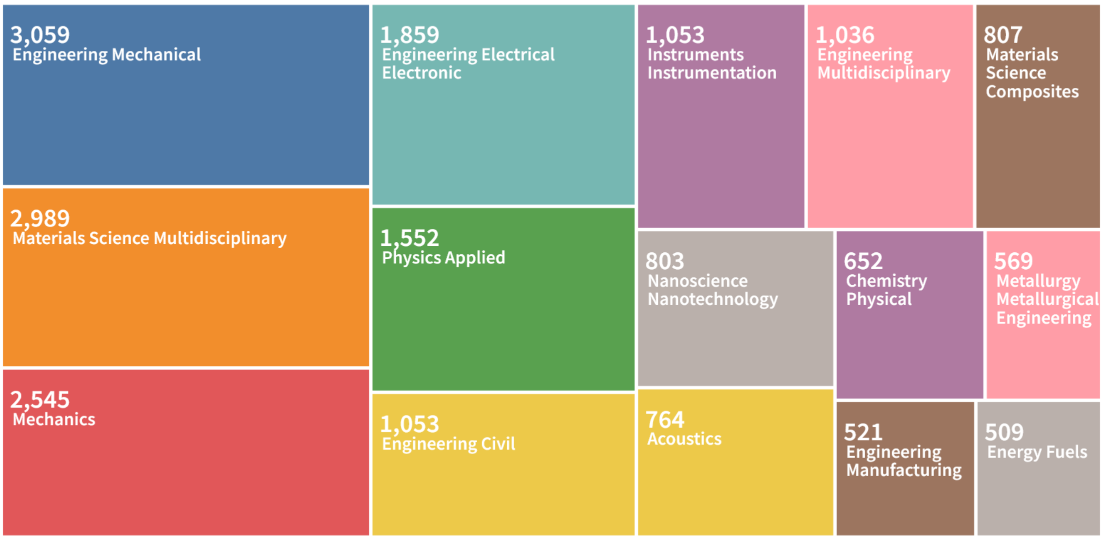

Fig. 1. Treemap of Web of Science categories with corresponding number of papers devoted to various aspects of mechanical vibrations

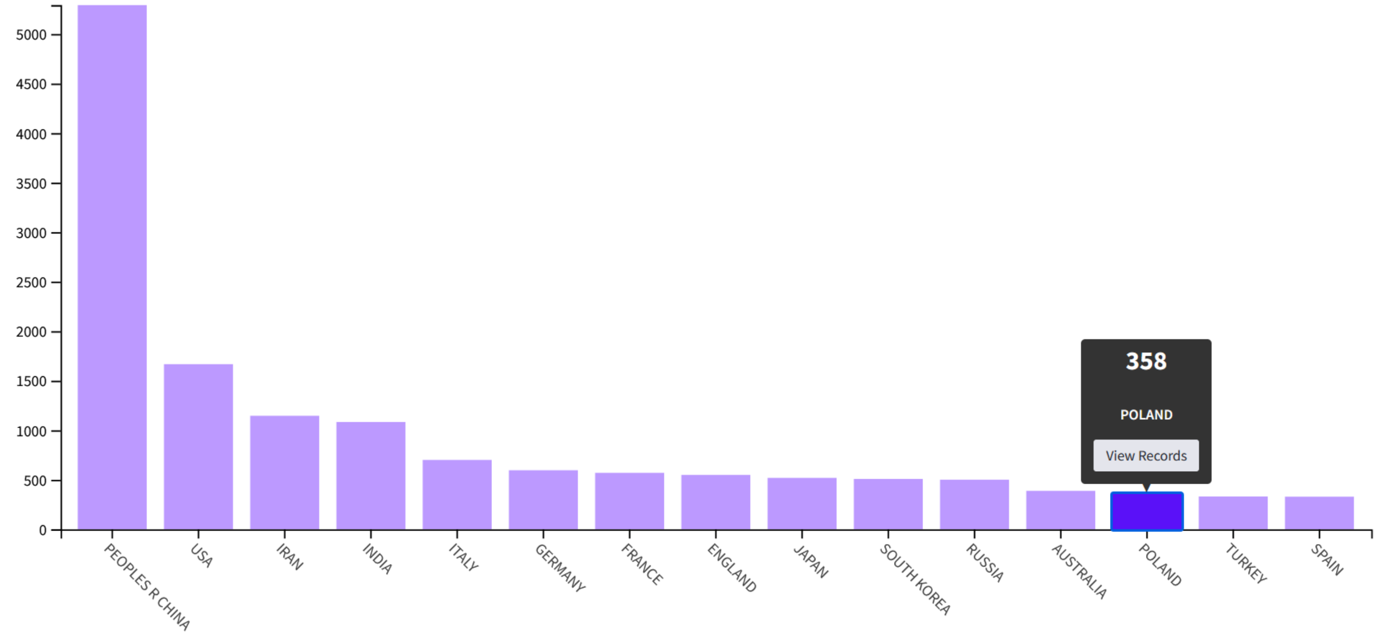

Fig. 2. Bar chart of countries with corresponding number of papers devoted to various aspects of mechanical vibrations

lent reduction process used to identify parameters of vibrating systems. Modern techniques of finding faults, e.g. using artificial neural networks, were also described.

The third goal of this overview is to present recent developments in theory of modal analysis and comprehensive overview of various aspects of that field has been presented in Section 4. The section starts with short information about linear modal analysis, the most widely applied type of modal analysis and the moves forward more complicated topics like application of component mode synthesis for linearized equation of motion or nonlinear modes theory.

The last section of the article focuses on the construction of granular materials that serve as the main element of vacuum-packed particles (VPP) devices. Enclosed in a flexible housing, the granules with the aid of an adjustable negative pressure can react as damping devices with non-linear characteristics adapting to the application. Smart materials and devices will make up a significant part of Industry 4.0.

2. VIBRATION PROBLEMS IN ELECTROMECHANICAL ENERGY HARVESTING

The rapid technological change on development visible in recent years has resulted in another industrial revolution. The concept of Industry 4.0 is closely related to the issues of the Internet of Things, artificial intelligence and cyber-physical systems. At the stage of designing or constructing new facilities, methods are increasingly used to automate it, optimize the structure or simplify its operation process. One of such methods is the energy harvesting technology, which enables the elimination of an external power source in the newly created devices and the use of environmental energy (electromagnetic radiation, sunlight, wind, heat or vibrations) [1]. Nowadays, due to high availability and development of MEMS technology, the electromechanical energy harvesting via vibration is the subject of many scientific works [2]. One of the most important aspects of this field of energy harvesting is the construction of appropriate devices that obtain energy through vibrations.

In the literature on vibration problems in electromechanical energy harvesting, three most important processes can be distinguished: the harvester design stage, the research and analysis stage, and the structure optimization stage. In article [3], the Authors proposed piezoelectric energy harvester based on MEMS. The developed harvester used the dependence of multi-plate cantilever structures in which the vibrations of the first plate are transferred to the neighboring ones. FEM analyses were performed to determine the impact of coupling effect in the structures on energy storage. The cantilever structure design collecting energy from vibrations caused by the heart is presented in [4]. A helical piezoelectric structure was used due to the reduction of stiffness and resonant frequency. Various models differing in the number of turns of the spiral beam were analyzed. The work [5] analyzes the influence of the support beam shape on the energy harvester efficiency. The proposed cantilever beam was made of piezoelectric and metallic layers. Using the Galerkin approach, a reduced model was developed to determine the collected power and generated voltage. The problem of improving ready-made energy harvesters is a subject widely described in the literature. One of the most popular method used in this process is shape optimization (or modification). The design modification of the PVEH type device is presented in the article [6]. By modifying the profile thickness, the aim was to achieve uniform stress distribution along the length of the structure. After the tests, it was shown that the proposed modification causes a 20% increase in the generated power with a simultaneous reduction in the peak stress value. Studies on the effect of thickness variation on electromechanical energy harvesting are also presented in [7]. Using the FEM method, the model developed on the basis of Rayleigh-Ritz approximations was verified. Moreover, the influence of changing the beam inclination angle was determined in order to obtain the optimal deformation of piezoelectric elements. On the basis of the obtained results, it was confirmed that the most optimal cantilever section is the conical section, which significantly increases the efficiency of the structure. The modification of the cantilever harvester to obtain energy from the motion of sea waves is presented in [8]. A mathematical model was created through the Airy wave theory and the Bessel equations to calculate the stored power and the output charge. The modification of the harvester consisted of changing its cross-section, which allowed for the distribution of uniform surface strain and ultimately increased the efficiency of energy collection. The cantilever shape optimization based on Rayleigh's method was presented in [9]. The Authors proposed a new design of a monomorphic device enabling more efficient energy harvesting. The influence of the shape of trapezoidal beams of uniform thickness was also investigated and the most optimal structure was determined. The improvement of the performance of the PZEH harvester by changing the design is described in [10]. This change included two permanent magnets attached to the mass proof double beam structure. The absorbed structure improves the electrical power (through the magnetic field) and increases the throughput of the harvester. The impact of introducing modifications in the system on the improvement of the frequency bandwidth was also determined. The improvement of the efficiency of the bi-stable piezoelectric harvester of the BEH type was discussed in [11]. It was proposed to change the structure by adding four magnets (FBEH): two movable, tip and fixed. With the use of Hamilton's principle, a distributed parameter model was determined and the dependence of excitation amplitudes and frequencies was investigated. In the work [12], a piezoelectric floor plate enabling the collection of energy (EHFT – energy harvesting floor tile) in a pedestrian environment was proposed as an example of the low-vibration harvester application. The influence of individual parameters (body weight, walking pace and pedestrian traffic density) on the obtained energy was determined. The developed energy harvesting structure consisted of a piezoelectric cantilever and a system of springs. Based on the research, it was determined that the parameters of weight and the pace of walking have the greatest impact on electromechanical energy harvesting. The work [13] presents the influence of taking into account the construction of the magnet and the nonlinear boundary condition on the behaviour of a monomorphic piezoelectric cantilever. According to the Euler-Bernoulli theory, the energy harvester was modeled and the equations of motion were derived using Lagrange's equations. Based on the obtained results, it was found that the inclusion of two external magnets in the structure while maintaining the appropriate distance significantly increases the power and efficiency of the harvester. Moreover, it has been shown that distance also has a great influence on the resulting frequency range. The use of low-frequency vibrations for electromechanical energy harvesting is presented in the paper [14]. The system consisted of a piezoelectric pendulum spring and a binder clip. Such a harvester is able to remove ultra-low frequencies and multi-directional energies from vibration. The work [15] focuses on the remaining problems of harvester technology, such as energy dissipation or durability of the device. The hybrid nanogenerator developed in this work , the basic unit of which is the structure of magnetic levitation, shows resistance to energy dissipation and mechanical fatigue. In addition, such a device can be used in a wireless monitoring system.

Li *et al.* [11] tested the influence the nonlinearity intentionally provided by a set of additional permanent magnets (PMs) on the behaviour of the piezoelectric energy harvester (EH). The set of PMs is arranged in such a way that one is attached to the tip of the piezoelectric beam and three remaining PMs are attached to the basis. Positions of two of these PMs are adjustable that allows for obtaining the desired potential well of the system. It resulted in extending the operational frequency bandwidth and increment of the efficiency of the device, especially for lower frequencies. Other interesting example of improvement of the piezoelectric energy harvesting device was proposed in [10]. In this work the authors enhance EH by expanding operational frequency bandwidth. First, it is achieved by an increase in the number of degrees of freedom, due to double-beam structure, resulting in additional resonant peak. Second, the PMs placed on the tip end acting with each other which provides the nonlinearity. Authors' analysis shows that increment of the magnetic field intensity of the PMs causes expanding of the operational frequency bandwidth. Moreover, the proposed EH is relatively more efficient while keeping comparative operational frequency spectrum in relation to device size than other EHs proposed in the literature [16–18]. Nonlinear devices such as EHs often provide difficulties in derivation of the mathematical models. Hence, in [11] equations of motion were derived with aid of the Hamilton principle taking into account energy balance between mechanical, magnetic and electrical parts of the system [19]. Contrary to the system considered in [19] mathematical models proposed in [10, 11] and many other approaches, e.g. [20, 21] do not include nonlinear behaviour of the electrical circuit, hence energy and coenergy are not distinguished in [11]. It facilities system analysis when displacements in the system are relatively small. However, in other cases the consideration of nonlinear behaviour of the electrical circuit is unavoidable for proper description of the system dynamics. Ostrowski *et al.* examined electromechanical EH assuming that the PM moving inside an electromagnetic coil achieves significant velocities and amplitudes greater than the axial length of the electromagnetic coil [19]. In this case both electromechanical coupling and the coil inductance significantly depend on the PM position, contrary to earlier works where only electromechanical coupling is the function of PM position, e.g. in [22]. A rigorous mathematical model of the electromechanical dependences was derived with aid of the Hamilton principle. In such a case when the coil inductance, which is an analog of the mass, depends on magnet position. Hence, energy of the magnetic field self-induced by the coil, which is the analog of the kinetic energy, is not equal to corresponding magnetic coenergy [23]. Such derivation allows to find formulas describing reluctance force acting on the magnet and additional nonlinearity strongly disturbing the electrical response.

The mathematical tool described above can be used to design and optimize EH cooperating with a mechanism called mechanical amplifier that increases amplitude of vibration of EH resulting in significant magnet displacements [24]. The effect of the motion amplification is especially used for excitations at ultralow frequencies. An effect similar to the motion amplification was achieved by Fu *et al.* by employing rotary-translational motion of the magnet [25]. Due to the fact that magnetic field of PM moves along with PM, rotational component of the PM motion causes also rotation of the magnetic field. Hence, rates of the changes of the magnetic flux linkage are greater that allows to generate satisfactory voltage levels even at very low frequencies. Double-well characteristics achieved with two restoring magnets and spring bumpers also additionally expand operational frequency bandwidth.

The operational frequency bandwidth considered as any performance metric can be misleading since usually expanding of resonance peak causes also reduction of its height corresponding with the peak efficiency [26]. The solution to this problem are tunable EH systems that allow for shifting the resonance peak by change of their resonance frequency. Drawback of such devices is their complexity, however in many cases it can be compensated by achieved peak-efficiency at each frequency in the achievable bandwidth. The first self-sufficient precisely tunable energy harvesting system was described by Mösch *et al.* [27]. Due to the highly efficient electronics the energy consumed to monitor the excitation frequency is smaller than energy produced during operation in normal working conditions. The adaptation is realised by means of the tuning magnet rotated by stepper motor that provides the change of the total stiffness of the system. Other interesting and innovative concept of the tuning of energy scavenging process to vary environmental conditions adapt frequency of the vibration itself instead of the change of the natural frequency of EH. It is possible with semi-active modal control proposed by Ostrowski *et al.* [28]. The aim of such control is to precisely transfer the vibration energy between structural vibration modes by means of semi-actively lockable joints. Such a control is intended for both two possible applications: vibration attenuation or energy harvesting. Here, the attention is paid to the energy harvesting applications. The authors showed that it is possible to design a preliminary structure that plays the role of the adaptive energy buffer for the secondary structure that is EH. Transfer of the vibration energy from randomly excited vibration modes of the preliminary structure to the preselected vibration mode that well-cooperates with EH significantly enhances the energy scavenging process in broad frequency spectrum.

3. PARAMETER IDENTIFICATION OF VIBRATING STRUCTURES

Due to the increasing use of vibrations in energy harvesting, it is necessary to correctly determine the phenomenon and the potential risk associated with long-term exposure of the object to vibrations [29]. One of the most frequently used methods in this case is the modal identification or parametric identification. Correct determination of the location of structural damage is a complex issue in which various damage indicators as well as analytical or numerical techniques are used [30, 31]. Identification of the physical parameters of systems, structures or machines is a difficult task in which two approaches are often used: parametric and non-parametric [32]. The paper [32] presents a proposal for a new combination of both approaches (parametric and non-parametric) in order to determine the vibration modes. Two modes of vibration were analyzed: single- (SDOF) and multi-degree-of-freedom (MDOF). The parametric representation of the vibration modes was presented through parametric linear auto-regression and ARMAX models. The proposed methodology correctly determines the dynamics of the analyzed object while minimizing the necessary information about the model. A new parametric approach based on experimental data to develop parameters of nonlinear models was presented in [33]. This method involved introducing a strong, non-resonant harmonic excitation and checking the modulation of the system response. As an application of the proposed technique, the identification of nonlinear force coefficients of three selected objects was indicated. The use of parametric excitation was also used in [34] to determine the dynamic behaviour of a visco-elastic sandwich beam. In these tests, it was assumed that internal resonance case is considered in the beam and the excitation result from fluctuations in velocity and tension. A mathematical model based on first-order differential equations was developed using the multiple scales method. The obtained results of numerical simulations show the influence of the internal resonance of the sandwich beam and the introduced parametric excitation on object dynamics. The use of the algebraic method of identifying mechanical systems with multi degrees of freedom is presented in [35]. System parameters such as mass, damping and stiffness were determined by measuring the transient position and the steering force signal. The proposed method is universal and can be used both in the systems with symmetrical stiffness and asymmetrical.

In the research on the identification of damage, an important factor influencing the obtained results is the inclusion of the ambient vibrations in the model. It is an unknown source of excitation that generates stationary or nonstationary signals [36]. In the work [36] a technique was proposed based on the method of recognizing statistical patterns that enable the localization of damage in both behavioral conditions (stationary and nonstationary). The introduced technique is based on a hybrid algorithm that extracts statistical metrics in order to analyze the features. The location of the damage was carried out with the use of spectral functions and spectral measures. Identification of damage and modal parameters of a cracked cantilever beam, taking into account ambient vibration, is also presented in [37]. The numerical tests were performed using the finite element method in ANSYS software. Using the enhanced frequency domain decomposition (EFDD) and stochastic subspace identification (SSI) methods, dynamic characteristics were determined during environmental vibration tests. Moreover, an automated technique for updating the model by estimating Bayesian parameters has been proposed. Defect identification was studied through the application of recurrence analysis methods based on drilling signals were proposed for detecting and locating composite defects that were modeled as holes drilled with different diameter inside a composite material [38]. A comparison of methods for detecting defects in composites is discussed in the paper [39].

The development of optimization techniques made it possible to determine vibration parameters by applying or modifying heuristic methods [40]. The use of genetic algorithm for multi-criteria optimization of phonon structures is presented in [41]. The objective function was to minimize the frequency transmission of acoustic waves, the number of layers in structure and the presence of high transmission peaks within the band. Two types of genetic algorithms (GACL and GAVL), differing in the number of layers, were used. The transfer matrix method (TMM) algorithm was used to determine the transmission of quasi-one-dimensional systems. A new approach to acquiring and analyzing acceleration data in order to assess the condition of beams is presented in [42]. The assessment and location of the bridge girder damage were tested using three methods: stochastic subspace identification, system equivalent reduction process and particle swarm optimization algorithm. The proposed methodology was verified by numerical simulations and experimental research. On the basis of the obtained results, it was found that the proposed approach with high accuracy facilitates locating the damage and determining its quantification. The issue of meta-heuristic and damage detection in the structure of trusses and spatial frames is discussed in [43]. Based on the Bayesian data fusion, various failure rates were determined using the natural frequency and mode shape. Four failure indicators (DIMSE, FRFSEDR, FSEDR and RFBDI) were used to accurately determine the location of the failure. The proposed technique allows to shorten the computational time of the optimization algorithm by reducing the number of suspected structural failures. The use of the particle swarm optimization (PSO) algorithm to identify disproportionate damping parameters is presented in the work [44]. The objective function was to minimize the incomplete eigenvector which is influenced by the object external damping devices. Through simulated measurement data, the accuracy and efficiency of the PSO algorithm were determined. The proposed methodology was found to work correctly both with and without noisy data. Moreover, in the case of noisy data, this method has been shown to be a more favorable choice than the gradient method. The paper [40] presents the use of optimization techniques to identify the parameters of discrete-continuous systems. Using the Lagrange multiplier formalism and Timoshenko beam theory, the stepped cantilever beam parameters were identified. By using two non-deterministic algorithms, PSO and GA, the relative error (objective function) was minimized by comparing the experimental and numerical results. The coupling of FRF and optimization techniques in the damage identification of trusses and beams are discussed in [45]. Using the genetic algorithm (GA) and the bat algorithm (BA), the location and severity of damage were estimated. The damage phenomenon was modeled as a change in the stiffness of individual elements. As an objective function, the minimization of the differences in the values measured and calculated by FRF technique was determined. Based on obtained results, it was found that in terms of precision and computational time, the BA is the more favorable algorithm. An innovative approach to the damage identification resulting from temperature changes is proposed in [46]. Using a genetic algorithm, a non-destructive method was proposed to determine the location and scale of damage caused by temperature changes and noise. In the verification process, a system consisting of a span continuous beam and steel grid was used. In optimization studies, a complex objective function was adopted with different weighting factors depending on the frequency and shape of the modes.

In addition to optimization techniques, neural networks are also used to identify damage locations. In article [47], local changes in stiffness and mass for three objects are determined with the use of convolutional neural networks (CNNs). The raw input data for the neural networks was directly implemented as training and validation data. The proposed technique correctly determined the location of damage. The use of artificial neural network (ANN) also in research on damage detection is also presented in [48]. Modal strain energy, modal flexibility and modal curvature were used as techniques for locating damage in noisy conditions. The innovative approach was aimed at quantifying the extent of damage and reducing external noise at various levels.

Due to the motion of a rigid body, many elements are subject to vibration and identifying the deformation of the structural motion is a complex issue. The work [49] presents a combination of this problem involving the identification of vibration modes and the motion of a rigid body with the use of digital measurements. By using image processing algorithms and photogrammetry, the vibration was determined and the modal analysis was carried out, allowing for the determination of mode parameters.

4. MODAL ANALYSIS AND ITS APPLICATION IN MODERN VIBRATION ENGINEERING

Nowadays, the term modal analysis can represent one of the two complementary approaches [50]. The first one is theoretical (or numerical) modal analysis and the second one is modal testing, which can mean either experimental or operational modal analysis. In the following section recent advances in both approaches for modal analysis will be synthetically described. The Authors believe that such a comprehensive description will allow the Reader interested in using the modal analysis to find a method which suits the best his or her needs.

4.1. Theoretical (numerical) modal analysis

In theoretical modal analysis it is assumed that the knowledge of the geometry, material characteristics and boundary conditions of the investigated mechanical system are known. Based on this quantities mass and stiffness matrices of the system are built (using for example finite element method). Having these matrices system modal parameters (i.e. natural frequencies, damping coefficients and mode shapes) are determined using one of the widely available eigensolvers [51].

4.1.1. Linear modal analysis

Modal analysis of linear systems constitutes historically the most widely explored type of modal analysis. However, even for linear, but a large and complex mechanical system specialized substructuring techniques are frequently used [52]. These techniques allow us to divide the system into smaller subsystems (components), which can be initially processed in parallel to obtain the so-called component modes and then assembled together to form approximate model of the overall system. Such an approach is computationally very efficient and allow to reduce the number of equations needed to describe the overall system dynamics.

– *Modal analysis of linear structural systems*

Component mode synthesis (CMS) is a model reduction technique applied for large structural models. Generally, this technique involves the following four major steps:

i. First, the overall structure is divided into a number of substructures, so-called components.
ii. Then, numerical method such as the finite element method (FEM) is utilized to formulate a discrete model for each component. This discrete model is reduced and consists of partial physical coordinates of the full model and a set of generalized coordinates (modes).
iii. Next, all individual reduced order models are assembled to formulate a global model for the overall structure. The global model has a much smaller size than the original model directly obtained from the FEM.
iv. Finally, the responses in the physical coordinates may be computed using back-substitution technique.

According to the boundary conditions applied for individual components CMS may be classified as either [53]:

- fixed-interface methods,
- free-interface methods.

The results of linear modal analysis cannot be transferred in straightforward way to nonlinear systems such as flexible multibody systems, however, is it possible to take advantage of the component mode synthesis method applying linearization to the dynamics of multibody system.

– *Modal analysis of flexible multibody systems*

Four different approaches are available to model and simulate the dynamics of flexible multibody systems [54]:

a. Floating frame of reference formulation (FFRF), which is applied for the flexible multibody systems undergoing the large overall motions and small deformations.
b. Large rotation vector formulation, which has limited application in the redundant description of the large rotation of the cross section of a beam.
c. Incremental finite element formulation, which is able to describe the large deformations, but may not model well the large rotation with zero strains.
d. Absolute nodal coordinate formulation (ANCF), which is an effective approach for the flexible systems subjected to large rotations and large deformations.

Interesting overview of various aspects of flexible multibody dynamics, including the selection of reference frames, the flexible body kinematics descriptions, the model reduction techniques and the modeling methods of the contact has been described by Rong *et al*. [55].

Particular emphasis on modal reduction procedures for flexible multibody systems by means of different methods of component mode synthesis has been placed in the paper by Sonneville *et al*. [56]. Theoretical considerations presented in the study have been validated on interesting example of the tendon-actuated lightweight in-space manipulator (TALISMAN).

Augmented formulation of the floating reference frame for analysis of a slider–crank system has been proposed by Kim *et al*. [57]. In this formulation, the constraint and fixed-interface normal modes have been corrected by considering the truncated modal effect with the residual flexibility. The proposed method offered a more precise reduced system leading to a more accurate and efficient flexible multibody simulation.

A different approach for planar and spatial mechanisms has been proposed by Cammarata [58]. This approach, instead of local modes, involved a reduction based on global flexible modes of the whole mechanism. Through the use of global modes obtained for the linearized dynamic system around a certain configuration it was possible to obtain a modal basis for the flexible coordinates of the multibody system.

An efficient method for model order reduction of a flexible multibody system undergoing both large overall motions and large deformations has been proposed by Tang *et al.* [59]. The dynamical system was initially modeled by using the nonlinear finite elements of absolute nodal coordinate formulation and then locally linearized at a series of quasi-static equilibrium configurations. Three examples were presented to validate the accuracy and efficiency of the proposed method. They were: rotating flexible beam, flexible cable and flexible cable net.

In the paper by Palomba and Vigoni [60] a parametric modal analysis approach for flexible multibody systems has been proposed. This approach allowed to derive an analytical polynomial expression for the eigenpairs as function of the system configuration, by solving a single eigenvalue problem and using only matrix operations. Effectiveness of the approach has been demonstrated on an example of open-chain, planar mechanism with two flexible links and two revolute joints.

4.1.2. Nonlinear modal analysis

Alternative approach to the dynamic analysis of the nonlinear mechanical system is a direct application of the nonlinear modes theory. This theory is still under development, however in this subsection a few recent advances of this theory will be presented. As it was formulated by Worden and Green [61] "*nonlinear modal analysis theory extends the linear theory to encompass objects which are amplitude invariants*". Engineers are generally working with two main ideas regarding the concept of the mode:

- A coherent (periodic) motion of the structure.
- A decomposition into lower-dimensional dynamical systems the motions of which is operating within invariant subspace.

Obviously, it is not possible to retain all the properties of a linear modal analysis when passing to a nonlinear theory. Adopting the first definition led to the idea of a Rosenberg normal mode and second one to a Shaw-Pierre normal mode.

Three important properties that distinguish nonlinear modes from linear ones are:

a) frequency-energy dependence (the first property means that frequency response functions of nonlinear systems are no longer invariant),
b) internal resonance (the second property tells us that nonlinear modes may interact during a general motion of the system),
c) mode bifurcations and stability (this property results in a fact that the number of nonlinear modes may exceed the number of degrees of freedom of the system).

An interesting introduction into the topic of nonlinear modal analysis can be found in book edited by Kerschen [62].

– *Nonlinear modes of systems with constant inertia tensor*

A machine learning approach to inversion of the modal transformation equivalent to a nonlinear form of modal superposition was presented by Worden and Green [61]. It was shown that issues can arise if the forward transformation is a polynomial and can thus have a multi-valued inverse. The proposed approach was demonstrated on simulated and experimental data representing a three-storey base-excited model of a shear building. The application of nonlinear normal mode theory to the analysis of real-world aerospace structure, which was airframe of the Morane–Saulnier Paris aircraft, was presented by Kerschen *et al.* [63]. The nonlinear normal modes of this aircraft were computed from a reduced-order nonlinear finite element model using a numerical algorithm combining shooting and pseudo-arc length continuation. It was shown that modes with distinct linear frequencies can interact and generate additional nonlinear modes with no linear counterpart.

– *Nonlinear modal analysis of multibody systems*

An attempt to extend the theory of oscillatory normal modes to general multi-body mechanical systems has been undertaken by Albu-Schäffer and Della Santina [64]. The main motivation of their work was to provide theoretical foundations for oscillation based locomotion, which comprises stance and a flight phases, single or multi-leg ground contact, etc. This leads however, to complex hybrid nonlinear dynamical systems, having nonholonomic constraints and being underactuated in the flight phase. However, as concluded by Albu-Schäffer and Della Santina providing answers for all posed questions in their paper requires further intensive work.

4.2. Modal testing

Contrary to theoretical (numerical) modal analysis in modal testing the system matrices are not known and the approach starts from measurements of dynamic responses of the investigated system [65]. Then, depending on the information about input forces we distinguish between experimental or operational modal analysis. In experimental modal analysis we determine frequency response functions (ratio between output and input as a function of frequency) and in operational modal analysis correlations or power spectral densities are used. Comparison of the two testing techniques has been presented by Orlowitz and Brand [66].

4.2.1. Experimental modal analysis (input-output)

There is presently a wide variety of input-output modal identification methods, whose application relies either on estimates of a set of frequency response functions (FRFs) relating the applied force and the corresponding response or on the impulse response functions (IRFs), which can be obtained through the inverse Fourier Transform. These methods try to perform some fitting between measured and theoretical functions and employ different optimization procedures and different levels of simplification. They are usually classified according to the following criteria:

i. Type of domain (time or frequency domain).
ii. Type of formulation (Indirect or direct methods).
iii. Number of modes or degrees of freedom (DOFs).
iv. Number of inputs and outputs (single input single output (SISO) or multiple input multiple output (MIMO)).

The first methods of identification were developed in the frequency domain. In the simpler SDOF formulations, a fitting between a measured and a theoretical FRF of a SDOF system in the vicinity of each resonant frequency is developed, neglecting the contribution of resonant modes. In the more sophisticated MDOF methods, the fitting between measured and theoretical FRFs is made globally in a wide range of frequencies. Time domain methods, which tend to provide the best results when a large frequency range or a large number of modes exist in the data, began to be developed as consequence of some limitations in terms of spectral estimates frequency resolution, as well as leakage errors in the estimates.

4.2.2. Operational modal analysis (output-only)

Output-only modal identification methods assume the excitation input as a zero mean Gaussian white noise, which means that the real excitation can be interpreted as the output of a suitable filter excited with that white noise input. Modelling the behaviour of the filter-structure system, one may conclude that some additional computational poles, without structural physical meaning, appear as consequence of the white noise assumption. There are two main groups of output-only modal identification methods:

- parametric methods in time domain,
- non-parametric methods essentially developed in frequency domain.

The frequency domain methods lead to estimates of operational mode shapes based on the construction of average normalized power spectral densities and ambient response transfer functions. The time domain methods involve the choice of an appropriate mathematical model to idealize the dynamic structural behaviour and the identification of the values of the modal parameters so as that model fits as much as possible the experimental data, following some appropriate criterion. These methods can be directly applied to discrete response time series or, alternatively, to response correlation functions. Comprehensive overview of the different methods for operational modal analysis can be found in the paper by Zahid *et al.* [67].

5. MECHANICAL BEHAVIOUR OF GRANULAR MATERIALS

Granular materials exhibit peculiarities that differ from solids and liquids on a macroscopic level and are absent in the smaller scales that emphasize the multiscale nature of the granular complex. The latest research on the properties of granular materials focuses on modeling the grain surface structures in relation to numerical methods. The in-depth analysis must be based on building a model of grains as close as possible to the real ones, and then by comparing the results obtained numerically with the results from experimental tests. Experimental testing of granular materials may, however, be difficult due to access to research facilities or due to specialized operation of equipment and parameters. Therefore, in examining the properties of granular materials, empirical methods are used, on the basis of which it is possible to obtain the results of material characteristics consistent with the results of analogue methods [68]. This lack of experimental data results in the lack of an accurate description of the relaxation dynamics of granular materials. As a result of the observations contained in [69], it was indicated that on the particle level the dynamic behaviour of granular systems is similar to that of complex fluids. Particulate materials can loosen even with a light load during a slight disturbance, despite the fact that if not driven, they jam and are then treated like solids. The granular materials are characterized by the fact that the individual grains have an irregular shape, and therefore the linear dimensions vary depending on the direction. The specific surface area determines the surface properties of the material, which affects its susceptibility to enrichment, crystallization, filtration, drying or dust removal of the material. Due to the irregular structure and the fractal nature of the granular materials, it is not possible to define the surface unequivocally. For this reason, it is necessary to point the specific surface area in research on the determination of various granular parameters. Due to the wide scope of application of granular materials, on the basis of the latest research results, parameters, such as flowability and the associated friction values of particles or the tendency to clump, can be determined by using computer-aided methods. Granular materials are used in various engineering fields. They exhibit peculiar features that differ from solids and fluids at the macroscopic level and are absent in the smaller scales that emphasize the multiscale nature of the granular complex.

The voids between the grains can affect the porosity of the material charge. The size of the grains in most cases determines the possibilities and scope of application when selecting further material processing. Due to the friction between the grains of the medium, granular materials can carry shear stresses. The angle of internal friction determines the ability of the material to flow in the presence of a compressive force applied perpendicular to the direction of the shear stress. The wall friction angle determines the flow along the wall with perpendicular compressive forces depending on the wall surface. The friction force varies linearly with the applied normal force [70]. Inter-particle friction for both natural grains and engineering materials is a function of Young's modulus and surface roughness. However, in the study [71] it was shown that shear damage is not related to slippage, but to cell opening due to loss of contact, which was demonstrated in the stress analysis of the earth dam. Traditional methods of testing granular materials are insufficient due to local discontinuities that can result in anisotropy or instability in the material. Hence, in order to take into account the nature of the material, one of the most efficient tools for simulating the behaviour of grained materials is the discrete element method DEM [72]. In essence, the method rely on the separate modeling of grains as rigid particles, and the deformation of the material is represented by the interaction between them. It is a very useful tool for the study of particle groups, the analysis of models that include a million particles significantly extends the computational time of detecting contact between successive grains. The applied DEM models use certain simplifications. In [73], a simplification of rolling friction was adopted, taking into account simplicity and lower computational requirements. The rolling resistance moment was applied using the limited rolling friction model (BROF). The spherical-shaped particles are prevented from undesirable rotation in resting state. Based on the model in [74], the shear anisotropy found in granular materials was used. On this basis, a comparison was obtained between the rolling resistance and the influence of the particle shape, which allowed to capture the bimodal nature of the material. The mechanical transition point in the contact force networks has shifted slightly below the average contact with rolling resistance. In numerical simulation, it is possible to adjust the shape of the particles, which significantly reduces the complexity of the structures and ensures the feasibility of studying the influence of various morphological factors of particles on the mechanical behaviour. The study [75] investigated the effect of roundness on the macroscopic and microscopic behaviour of particles under quasi-static shear conditions. DEM modeling is used to study the mechanisms of particle fracture as well. In the study [76] experiments of the material were carried out in relation to dynamic and quasi-static compression. Distribution curves contain fractal distribution data, and the fractal dimension and characteristic particle size decrease exponentially as stress increases.

Based on the model presented in [77], an innovative methodology was presented, on the basis of which it was found that the dynamics of the flow of granular material can be determined by measuring velocity in the plane using digital image correlation. The proposed model based on the PFEM particle finite element method accurately records the flow dynamics and can be used to model the transient flow of granular materials. In the work [78], also with the use of CT, the results of validation data of grain contact models corresponding to the modes of contact deformation of slip, twisting and rolling based on numerical simulations were obtained. On their basis, it was found that the energy dissipated is much more inhomogeneous than the normal forces between the particles. The research results included in the work [79] were obtained on the basis of a model using the CT and DEM methods. The research included in the study concerned the influence of the particle type on the shear strength through the anisotropic coefficient. Normal force contact anisotropy and normal contact anisotropy determine the shear strength of granular materials. In the work [80] the CT method was adapted, taking into account thermal conductivity networks next to the contact force network. By adding thermal conductivity at the edges of the network, a system of heat transfer from the voids between the grains can be constructed. In the work [80] the CT method was adapted, taking into account thermal conductivity networks next to the contact force network. By adding thermal conductivity at the edges of the network, a system of heat transfer from the voids between the grains can be constructed. Using SMT computed microtomography in [81], a new experimental approach has been proposed to calculate particle translation ratios relative to adjacent particles. Additionally, the use of 3D printing technology in [82] presents a model of generating repetitive data in order to support the calibration and validation of discrete mechanics models in granular materials. Cracking of the particles has no effect on the strength and deformation of the granular aggregates. In addition to mechanical factors, physical properties such as material homogeneity due to density also have an influence on particle fracture. However, the effect of density is not conclusive. It was noted in [83] that the effect of grain breaking during the sample preparation process has so far been neglected. Layered compaction was applied in triaxial samples. It was found that a lower degree of cracking is observed in a well-grained material, and the increase in the degree of particle breakage is due to an increase in the relative density.

Due to the moisture absorption capacity of granular porous materials, swelling can occur. In [84] a particle model for simulating the swelling of granular materials was presented. The DEM method was combined with the RFU pore finite volume method. On the basis of the conducted research and in comparison with the data obtained from the experimental tests, it has been shown that the reduction of the friction coefficient or the stiffness of the particles lowers the porosity. The angle of repose plays a very important role in the research on the behaviour of granular materials. It can be difficult to compare different measurement methods for this parameter as each measurement method simulates a specific application. The importance of the angle of repose was emphasized by the authors of [85] in a wide range of applications. The correct measurement of the angle of repose is greatly influenced by the preparation of the sample and the conditions related to the moisture content, dry mass density or particle size. The angle of repose does not always equal the peak or residual angle of internal friction. The authors emphasize that slight differences in the conditions of sample preparation may result in measurement errors. The latest work in the field of research on granular materials is the work [86], in which the authors proposed a design of a collision energy absorber based on a granular jamming mechanism. Using the modeling methodology described in the work, it is possible to model complex devices. By means of the vacuum pressure parameters, it is possible to control the dissipation of the impact energy within a wide range by means of the granulate jamming mechanism. This has been well demonstrated in [87], where a modified Bouc-Wen hysteresis model was used to describe the nonlinear properties of the tested samples, and the identification of the model parameters was carried out with the use of a genetic algorithm. As a result of the conducted research, the authors found a good agreement of the model with the experimental data. Due to the fact that the engineering applications of VPP are much ahead of their theoretical description. Work is undertaken on the mathematical description of complex mechanical mechanisms observed in these unconventional constructions [88].

6. CONCULSIONS

In the current paper recent trends and applications of mechanical vibration theory have been discussed. In particular four topics have been selected for detailed overview. These four research topics are: energy harvesting, parametric identification, modal analysis and granular materials. Application of energy harvesting devices discussed as the first topic allows to save significant amount of energy, and thus reduce the amount of greenhouse gases. The next topic helps in better design of structures subjected to vibrations and works out the better algorithms allowing to identify and search for optimal parameters for the mechanical devices. The third topic describes vibration in multibody systems and modal analysis, which are key to understanding the dynamic phenomena in vibrating machines. Finally, the last topic describes the properties of granulated materials from which modern, intelligent vacuum-packed particles are made. A potential application of this technology are intelligent vibration damping devices. In the Authors' opinion in near future we can still expect increasing number of publications on the above topics since there are still many open questions in all of them and answering these questions will provide the better understanding of various phenomena related to mechanical vibrations.

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