Structurally R-Controllable and Structurally R-Observable Descriptor Linear Electrical Circuits

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Abstract:
Structurally R-controllable and structurally R-observable descriptor linear electrical circuits are investigated. Sufficient conditions are given under which the R-controllability and R-observability of descriptor linear electrical circuits are independent of their parameters.

Keywords: *structural, R-controllability, R-observability, descriptor, linear, electrical circuit*

1. Introduction

A dynamical system is called descriptor (singular) if its mathematical model is represented by a combined set of differential and algebraic equations. Descriptor linear systems have been investigated in [3, 5–8, 16, 18]. The computation of Kronecker's canonical form of a singular pencil has been analyzed in [17].

The notion of controllability and observability and the decomposition of linear systems have been introduced by Kalman [11, 12]. These notions are the basic concepts of the modern control theory [1, 2, 8, 10, 14, 15]. They have been also extended to descriptor linear systems [4, 6, 8, 13, 19]. It is well-known that the controllability and observability of linear systems are generic properties of the systems [14].

In this paper structurally R-controllable and structurally R-observable descriptor linear electrical circuits will be investigated.

The paper is organized as follows. In Section 2 the basic definitions and theorems concerning descriptor linear electrical circuits and their controllability and observability are recalled. In Section 3 structural R-controllability of the descriptor linear electrical circuits is introduced and investigated. Similar results for structural R-observability are presented in Section 4. Concluding remarks are given in Section 5.

The following notation will be used: $\mathbb{R}$ - the set of real numbers, $\mathbb{R}^{n\times m}$ - the set of $n \times m$ real matrices and $\mathbb{R}^n = \mathbb{R}^{n\times 1}$, $\mathbb{C}$ - the field of complex numbers.

2. Descriptor Linear Electrical Circuits

2.1. Preliminaries

Consider the descriptor linear electrical circuit composed of resistors, coils, capacitors and source voltages described by the equations

$$E\dot{x} = Ax + Bu, \qquad (1a)$$

$$y = Cx, \qquad (1b)$$

where $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}^m$, $y = y(t) \in \mathbb{R}^p$ are the state, input and output vectors and $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$.
It is assumed that $\det E = 0$, $\operatorname{rank} B = m$ and the pencil is regular, i.e.

$$\det[Es - A] \neq 0 \text{ for some } s \in \mathbb{C}. \qquad (2)$$

Usually as the state variables $x_1, \ldots, x_n$ (the components of the state vector x) the currents in the coils and voltages on the capacitors are chosen.
It is well-known [9] that:

1) every electrical circuit is a descriptor system if it contains at least one mesh consisting of capacitors and voltage sources only or at least one node with branches with coils;
2) every linear descriptor electrical circuit is a linear system with regular pencil (the condition (2) is satisfied).

2.2. Elementary Matrix Operations

The following elementary operations on matrices will be used:

1) Multiplication of the i-th row (column) by a real number c. This operation will be denoted by $L[i \times c]$ ($R[i \times c]$).
2) Addition to the i-th row (column) of the j-th row (column) multiplied by a real number c. This operation will be denoted by $L[i + j \times c]$ ($R[i + j \times c]$).
3) Interchange of the i-th and j-th rows (columns). This operation will be denoted by $L[i, j]$ ($R[i, j]$).

2.3. Controllability and Observability

For descriptor linear electrical circuits different notions of controllabilities and observabilities can be introduced, such as C-controllability, R-controllability, I-controllability and C-observability, R-observability, I-observability [6, 8].

Definition 1. [8] *The descriptor linear electrical circuit (1) is called completely controllable (in short C-controllable) if for every initial state $x_0 = x(0) \in \mathbb{R}^n$ and every finite state $x_f \in \mathbb{R}^n$ there exist a time $t_f > 0$ and input $u(t) \in C^q$ (the set of q times piecewise continuously differentiable functions) in $[0, t_f]$ such that $x(t_f) = x_f$.*

Theorem 1. [8] *The descriptor linear electrical circuit (1) is C-controllable if and only if*

$$\operatorname{rank}\begin{bmatrix} Es - A & B \end{bmatrix} = n \text{ for all } s \in \mathbb{C} \qquad (3a)$$

and

$$\operatorname{rank}\begin{bmatrix} E & B \end{bmatrix} = n. \qquad (3b)$$

Definition 2. [8] *The descriptor linear electrical circuit (1) is called completely observable (in short C-observable) if there exists a finite time $t_f > 0$ such that for given $u(t)$ and $y(t)$ in $[0, t_f]$ it is possible to find its unique initial condition $x_0 = x(0)$.*

Theorem 2. [8] *The descriptor linear electrical circuit (1) is C-observable if and only if*

$$\text{rank}\begin{bmatrix} Es - A \\ C \end{bmatrix} = n \text{ for all } s \in \mathbb{C} \tag{4a}$$

and

$$\text{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n. \tag{4b}$$

Neglecting the impulse part of the descriptor linear electrical circuit we obtain the R-controllability (R-observability), i.e. controllability (observability) within the reachable set.

Theorem 3. [8] *The descriptor linear electrical circuit (1) is R-controllable if and only if the condition (3a) is satisfied.*

Theorem 4. [8] *The descriptor linear electrical circuit (1) is R-observable if and only if the condition (4a) is satisfied.*

Similarly, neglecting the standard part of the descriptor linear electrical circuit we obtain the impulse controllability (observability), in short I-controllability (I-observability).

Theorem 5. [6] *The descriptor linear electrical circuit (1) is I-controllable if and only if*

$$\text{rank}\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = n + \text{rank}E. \tag{5}$$

Theorem 6. [6] *The descriptor linear electrical circuit (1) is I-observable if and only if*

$$\text{rank}\begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = n + \text{rank}E. \tag{6}$$

In this paper we focus on the R-controllability and R-observability.

3. Structurally R-Controllable Electrical Circuits

In this section structural R-controllability of the descriptor linear electrical circuits will be introduced and investigated.

Definition 3. *The descriptor electrical circuit (1) is called structurally R-controllable if its R-controllability is independent of its resistances, inductances and capacitances.*

Theorem 7. *The descriptor electrical circuit (1) is structurally R-controllable if the linearly independent meshes contain only resistances, inductances and source voltages and the number of linearly independent meshes containing only capacitances and source voltages is equal to the number of its capacitances.*

Proof. If the assumptions are satisfied then for each row of the matrix $Es - A$ containing entries with resistances, inductances and capacitances the corresponding rows of the matrix B have nonzero entries independent of the circuit parameters. Using elementary column operations it is possible to eliminate all parameter-dependent entries in these rows of the matrix $\begin{bmatrix} Es - A & B \end{bmatrix}$. After this elimination procedure, which does not change the rank of the matrix, we obtain full row rank matrix with entries independent of the resistances, inductances and capacitances. □

Example 1. *Consider the descriptor linear electrical circuit shown in Figure 1 with given resistances R_1, R_2, R_3, inductances L_1, L_2, L_3, capacitance C and source voltages e_1, e_2.*

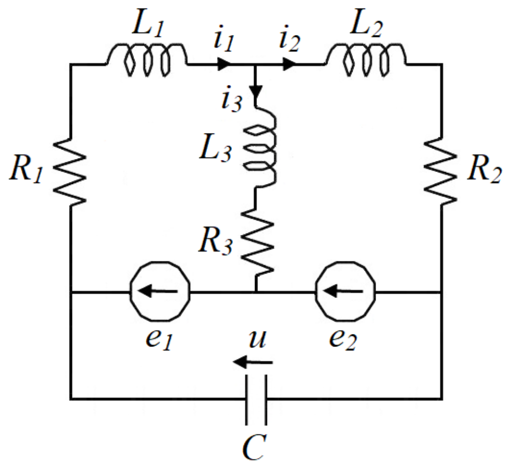

Fig. 1. Descriptor electrical circuit of Example 1

Using Kirchhoff's laws we may write the equations

$$\begin{aligned} e_1 &= L_1\frac{di_1}{dt} + R_1 i_1 + L_3\frac{di_3}{dt} + R_3 i_3, \\ e_2 &= L_2\frac{di_2}{dt} + R_2 i_2 - L_3\frac{di_3}{dt} - R_3 i_3, \\ i_3 &= i_1 - i_2, \\ u &= e_1 + e_2. \end{aligned} \tag{7}$$

The equations (7) can be written in the form

$$E\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ u \end{bmatrix} = A\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ u \end{bmatrix} + B\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \tag{8a}$$

where

$$E = \begin{bmatrix} L_1 & 0 & L_3 & 0 \\ 0 & L_2 & -L_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} -R_1 & 0 & -R_3 & 0 \\ 0 & -R_2 & R_3 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}. \tag{8b}$$

Using (8b) we obtain

$$\begin{bmatrix} Es-A & B \end{bmatrix} = \begin{bmatrix} sL_1+R_1 & 0 & sL_3+R_3 & 0 & 1 & 0 \\ 0 & sL_2+R_2 & -sL_3-R_3 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}. \tag{9}$$

By Theorem 7 the R-controllability of the electrical circuit is independent of its resistances, inductances and capacitance. Using on the matrix (9) elementary column operations $R[1+5\times(-sL_1-R_1)]$, $R[3+5\times(-sL_3-R_3)]$, $R[2+6\times(-sL_2-R_2)]$, $R[3+6\times(sL_3+R_3)]$, $R[1+4\times(sL_1+R_1)]$, $R[2+4\times(sL_2+R_2)]$, *or equivalently postmultiplying the matrix (9) by*

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ sL_1+R_1 & sL_2+R_2 & 0 & 1 & 0 & 0 \\ -sL_1-R_1 & 0 & -sL_3-R_3 & 0 & 1 & 0 \\ 0 & -sL_2-R_2 & sL_3+R_3 & 0 & 0 & 1 \end{bmatrix}, \tag{10}$$

and taking into account (3a) we obtain

$$\begin{aligned} \operatorname{rank}\begin{bmatrix} Es-A & B \end{bmatrix} &= \operatorname{rank}\left\{\begin{bmatrix} Es-A & B \end{bmatrix} Q\right\} \\ &= \operatorname{rank}\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} = 4 \\ &\text{for all } s \in \mathbb{C}. \end{aligned} \tag{11}$$

Therefore, the electrical circuit is structurally R-controllable.

Example 2. *Consider the descriptor linear electrical circuit shown in Figure 2 with given resistance R, capacitances* C_1, C_2, C_3 *and source voltages* e_1, e_2.

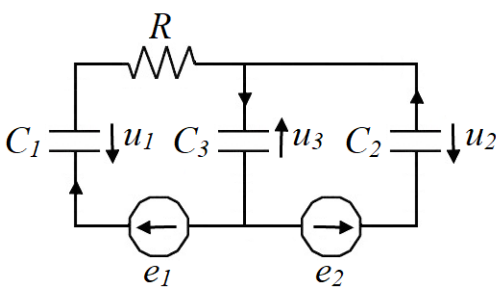

Fig. 2. Descriptor electrical circuit of Example 2

Using Kirchhoff's laws we may write the equations

$$\begin{aligned} e_1 &= R_1C_1\frac{du_1}{dt} + u_1 + u_3, \\ C_1\frac{du_1}{dt} &+ C_2\frac{du_2}{dt} = C_3\frac{du_3}{dt}, \\ e_2 &= u_2 + u_3, \end{aligned} \tag{12}$$

which can be written in the form

$$E\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = A\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + B\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \tag{13a}$$

where

$$E = \begin{bmatrix} RC_1 & 0 & 0 \\ C_1 & C_2 & -C_3 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{13b}$$

Using (3a) and (13b) we obtain

$$\begin{aligned} \operatorname{rank}\begin{bmatrix} Es-A & B \end{bmatrix} &= \operatorname{rank}\begin{bmatrix} sRC_1+1 & 0 & 1 & 1 & 0 \\ sC_1 & sC_2 & -sC_3 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\ &= \begin{cases} 2 & \text{for} \quad s=0, \\ 3 & \text{for} \quad s\neq 0, \end{cases} \end{aligned} \tag{14}$$

By Theorem 7 the R-controllability of the electrical circuit depends of its resistance and capacitances. Parameter-dependent entries of the matrix (14) can not be eliminated using elementary column operations. Therefore, the electrical circuit is not structurally R-controllable.

4. Structurally R-Observable Electrical Circuits

In this section structural R-observability of the descriptor linear electrical circuits will be introduced and investigated.

Definition 4. *The descriptor electrical circuit (1) is called structurally R-observable if its R-observability is independent of its resistances, inductances and capacitances.*

Theorem 8. *The descriptor electrical circuit (1) is structurally R-observable if its outputs are linearly independent combinations of those state variables which are expressed by ordinary differential equations in the circuit model, i.e. state variables related to linearly independent meshes containing either resistances and inductances or resistances and capacitances.*

Proof. Taking into account duality of notions of controllability and observability we can accomplish the proof in a similar way to the one of Theorem 7. If the assumptions are satisfied then for each column of the matrix $Es-A$ containing entries with resistances, inductances and capacitances the corresponding columns of the matrix C have nonzero entries independent of the circuit parameters. Using elementary row operations it is possible to eliminate all parameter-dependent entries in these columns of the matrix $\begin{bmatrix} [Es-A]^T & C^T \end{bmatrix}^T$. After this elimination procedure, which does not change the rank of the matrix, we obtain full column rank matrix with entries independent of the resistances, inductances and capacitances. □

Example 3. *(Continuation of the Example 1) The R-observability of the linear electrical circuit shown in Figure 1 will be analyzed for the following two cases:*

Case 1. $C = \begin{bmatrix} C_1 & 0 \end{bmatrix} \in \Re^{2\times 4}$, $\det C_1 \neq 0$, *satisfying Theorem 8.*

Case 2. $C = \begin{bmatrix} 0 & C_2 \end{bmatrix} \in \Re^{2\times 4}$, $\det C_2 \neq 0$, *not satisfying Theorem 8.*

In Case 1 we have the following results. Assuming

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{15}$$

and using (8b) and (15) we obtain

$$\begin{bmatrix} Es - A \\ C \end{bmatrix} = \begin{bmatrix} sL_1 + R_1 & 0 & sL_3 + R_3 & 0 \\ 0 & sL_2 + R_2 & -sL_3 - R_3 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{16}$$

By Theorem 8 the R-observability of the electrical circuit is independent of its resistances, inductances and capacitance. Using on the matrix (16) elementary row operations $L[1+3\times(-sL_3-R_3)]$, $L[1+5\times(-sL_1-sL_3-R_1-R_3)]$, $L[1+6\times(sL_3+R_3)]$, $L[2+3\times(sL_3+R_3)]$, $L[2+5\times(sL_3+R_3)]$, $L[2+6\times(-sL_2-sL_3-R_2-R_3)]$, *or equivalently premultiplying the matrix (16) by (17) and taking into account (4a) we obtain*

$$\begin{aligned} \operatorname{rank}\begin{bmatrix} Es - A \\ C \end{bmatrix} &= \operatorname{rank}\left\{P\begin{bmatrix} Es - AC \end{bmatrix}\right\} \\ &= \operatorname{rank}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = 4 \\ &\text{for all } s \in \mathbb{C}. \end{aligned} \tag{18}$$

Therefore, the electrical circuit is structurally R-observable.
In Case 2 we have the following results. Assuming

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

and using (8b) and (19) we obtain

$$\begin{bmatrix} Es - A \\ C \end{bmatrix} = \begin{bmatrix} sL_1 + R_1 & 0 & sL_3 + R_3 & 0 \\ 0 & sL_2 + R_2 & -sL_3 - R_3 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{20}$$

By Theorem 8 the R-observability of the electrical circuit depends of its resistances and inductances. Parameter-dependent entries of the matrix (20) can not be eliminated using elementary row operations. Therefore, the electrical circuit is not structurally R-observable.

Example 4. *(Continuation of the Example 2) The R-observability of the linear electrical circuit shown in Figure 2 will be analyzed for the following three cases:*

Case 1.

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \text{ satisfying Theorem 8.} \tag{21}$$

Case 2.

$$C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \text{ not satisfying Theorem 8.} \tag{22}$$

Case 3.

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \text{ not satisfying Theorem 8.} \tag{23}$$

In Case 1 we have the following results. From (8b) and (21) we obtain

$$\begin{bmatrix} Es - A \\ C \end{bmatrix} = \begin{bmatrix} sRC_1 + 1 & 0 & 1 \\ sC_1 & sC_2 & -sC_3 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}. \tag{24}$$

By Theorem 8 the R-observability of the electrical circuit is independent of its resistance and capacitances. Using on the matrix (24) elementary row operation $L[1+4\times(-sRC_1)]$, *neglecting its second row and taking into account (4a) we obtain*

$$\operatorname{rank}\begin{bmatrix} Es - A \\ C \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = 3 \quad \text{for all } s \in \mathbb{C}. \tag{25}$$

Therefore, the electrical circuit is structurally R-observable.

$$P = \begin{bmatrix} 1 & 0 & -sL_3 - R_3 & 0 & -s(L_1 + L_3) - R_1 - R_3 & sL_3 + R_3 \\ 0 & 1 & sL_3 + R_3 & 0 & sL_3 + R_3 & -s(L_2 + R_3) - R_2 - R_3 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{17}$$

In Case 2 we have the following results. From (8b) and (22) we obtain

$$\begin{bmatrix} Es - A \\ C \end{bmatrix} = \begin{bmatrix} sRC_1 + 1 & 0 & 1 \\ sC_1 & sC_2 & -sC_3 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \quad (26)$$

By Theorem 8 the R-observability of the electrical circuit depends of its resistance R and capacitance C_1. The entry sRC_1 of the matrix (26) can not be eliminated using elementary row operations. Therefore, the electrical circuit is not structurally R-observable.
In Case 3 we have the following results. From (8b) and (23) we obtain

$$\begin{bmatrix} Es - A \\ C \end{bmatrix} = \begin{bmatrix} sRC_1 + 1 & 0 & 1 \\ sC_1 & sC_2 & -sC_3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}. \quad (27)$$

The analysis is similar to the one presented in Case 2. By Theorem 8 the electrical circuit is not structurally R-observable.

5. Conclusion

Structurally R-controllable and structurally R-observable descriptor linear electrical circuits have been investigated. Sufficient conditions under which the R-controllability and R-observability of descriptor linear electrical circuits are independent of their parameters have been given (Theorems 7 and 8). The considerations have been illustrated by examples.

An open problem is an extension of presented approach to C-controllability (C-observability) and I-controllability (I-observability).

The considerations can be also extended to descriptor fractional systems with different fractional orders.

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