# USING INFORMATION MODELS TO FORMALIZE GROUP EXPERTISE BASED ON SUBJECTIVE ESTIMATES

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**Abstract**

The paper analyzes the construction of aggregated estimates (of processes or process parameters) on the basis of expert opinion. A three-level information model is constructed, describing the connection between criteria, indicators and their scaling constants. Coefficients of concordance of expert opinion are also discussed, as well as estimates of the expert competence and knowledge in the analyzed field (or regarding a given criterion). The procedure to find the aggregated estimate is described in nine steps, which consecutively construct the model of the estimate and its components, i.e. indicators, weight coefficients, criteria, scaling constants.

**Key words:** information model, expertise, aggregated estimate, subjective information

## 1 Introduction

Usually the parameters of natural systems allow direct or indirect measurement using technical instruments. Unlike these, public processes usually do not allow direct measurement due to their specificity and the necessary information cannot be experimentally acquired. The only source of information is the subjective opinion and knowledge of experts, i.e. the analysis of public processes should rely on subjective data, and therefore it should be approached by the techniques of subjective statistics [8]. One such technique is the expertise, as it is a system of actions for subjective elicitation of parameters. It is a common approach in processes where the subjective elicitation dominates, e.g. in decision making, knowledge assessment in education, art, sports, etc.

The conviction of all interested parties in the reliability of the result $Q$ of the assessment is a key factor for the optimal control of public processes. The quantitative methods that employ the apparatus of subjective statistics allow finding solutions in this direction. They are implemented when the estimated variable $Q$ is ambiguously defined, it is related to output data that is not formalized and is dependent on a set of properties and characteristics. Many works focus on such problems and employ the formal expert models [6], [3], [4], [10]. Other works [12], [13] proposed a formalized procedure to analyze the opinion of an expert panel, called REPOMP, applied to find a waste treatment decision, as well as in an energy efficiency problem of an educational building.

The work [6] proposed methods and techniques to increase the credibility of the estimate using a two-stage procedure for multi-factor estimation, ranking of the competence of experts, differential approach to the opinion of the participants in a process, averaging while taking into account the significance of the criteria in the aggregated estimate, etc.

Assume that the aggregated estimate $Q$ is quantitatively dependent on the values of $m$ non-overlapping criteria (factors) $K_i$ ($i$ = 1, 2, …, $m$) and their positive scaling constants $k_i$, elicited by experts. The estimate $Q$ is increasing on each of its arguments. It is convenient to use one and the same scale for $K_i$ and $Q$, whereas the sum of the scaling constants $k_i$ should equal to one. Each expert from the expert set $\{E\} = \{e_1, e_2, \dots, e_N\}$ (Fig.1) defines values for the criteria using the elicitation scale $\{S\}$, e.g. $\{S\} = \{2, 3, 4, 5, 6\}$.

On the other hand, each of the criteria $K_i$ is quantitatively dependent on the aggregated estimates of $n$ indicators $\overline{P}_j$ ($j$ = 1, 2, …, $n$) and their scaling constants (significance coefficients) $b_{i,j}$ ($i$ = 1, 2, …, $m$). The criterion $K_i$ is an increasing function on each of its arguments. It is convenient to measure $\overline{P}_j$ and $K_i$ in one and the same scale, whereas the sum of the significance coefficients equals to one, i.e. $\sum_{j=1}^{n} b_{i,j} = 1$, $i$ = 1, 2, …, $m$. The selection of indicators is made using the cluster method [3] or the reduction on consistent similarity method, whereas the coefficients of significance $b_{i,j}$ of each indicator are defined by experts.

The values of $\overline{P}_j$ ($j$ = 1, 2, …, $n$) are elicited by $N$ experts, the $r$-th of which gives the estimates $I_{j,r}$ ($j$ = 1, 2, …, $n$). to each indicator. Let $a_{j,r}$ be the normalized coefficient of competence of the $r$-th expert regarding the $j$-th indicator. Then each $\overline{P}_j$ is quantitatively dependent of the expert estimates $I_{j,r}$ ($r$ = 1, 2,…, $N$) and their corresponding coefficients of competence $a_{j,r}$. The indicator $\overline{P}_j$ must be increasing on each of its arguments. It is convenient to measure $I_{j,r}$ and $\overline{P}_j$ in one and the same scale, whereas the sum of the coefficients of competence should be one: $\sum_{r=1}^{N} a_{j,r} = 1$, $j = 1, 2, \ldots, n$.

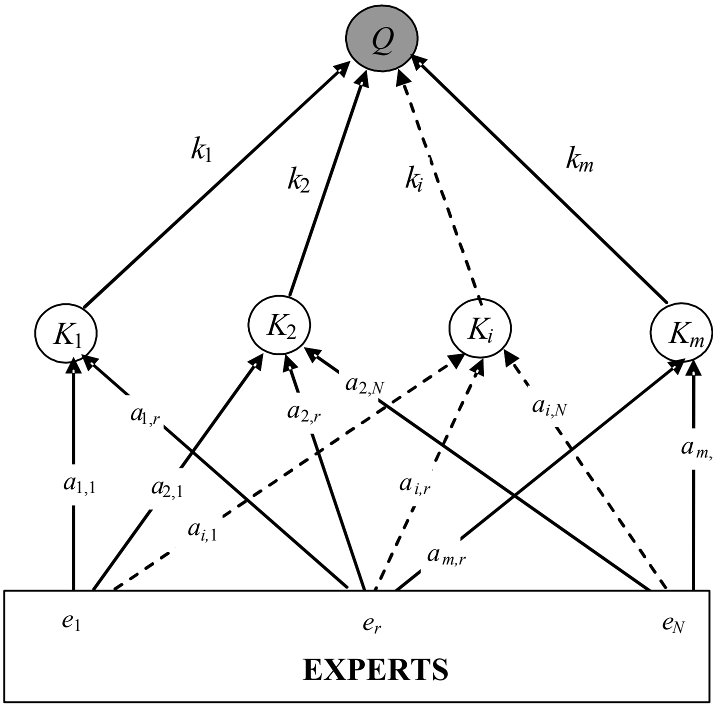


**Figure 1.** Graph of the influence of the criteria over the aggregated estimate

The purpose of this work is to propose ideas and mathematical (information) models to formalize the aggregated multi-criteria multi-person expert estimates. It is necessary to answer the following questions:

- How to define the weight coefficients of the criteria in the aggregated estimate?
- How to take into account the fact that the experts have different competence regarding the criteria they need to assess?
- How to achieve higher level of objectivity in the elicitation process with the help of the parameters of expertise?

Some initial discussion on these problems is proposed in [5].

Many works on the expertise problem exist. The multi-criteria methods [2], [4] are strongly related to the ideas proposed here. The works [1], [7] propose an instrumentalism for decision making in the field of multi-criteria multi-person multi-measurement tasks, whereas the problem of correlation of parameters in multi-criteria problems is discussed in [6].

The problems analyzed here have the following constrains:

- they do not refer to decision making, but to the definition of multi-criteria estimates of the status of a given object,
- the estimates are given using a predefined numerical scale,
- the experts do not choose among predefined alternatives, but define estimates for each criterion and its weight coefficient.

## 2 Algorithmic Representation of the Method

One of the main characteristics of the proposed approach for formalization of the subjective estimates is the three-level information model (Fig. 2) that contains an additional initial phase of multi-criteria multi-person elicitation (compared to the model from Fig. 1).

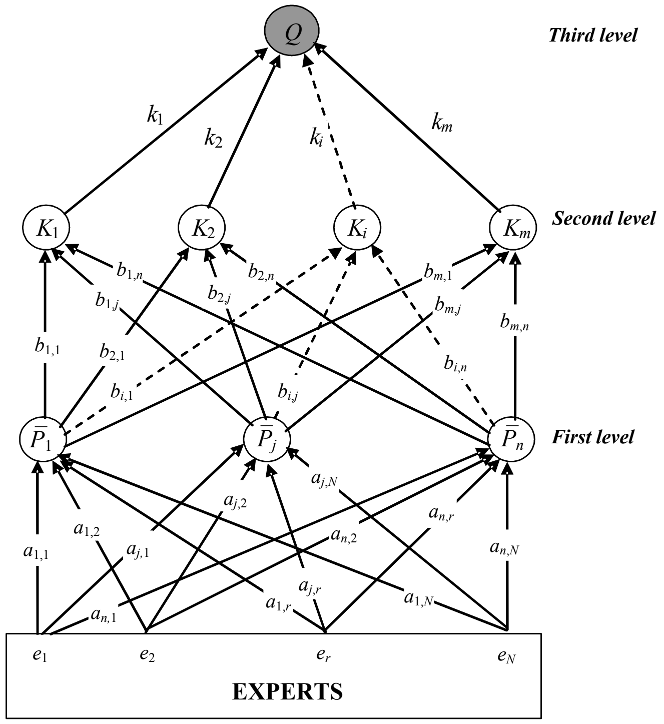


**Figure 2.** Three-level graph of indicators and criteria

The aggregated estimate $Q$ is at the highest (third) level. The second level comprises the criteria $K_i$, correlated with the indicators $\overline{P}_j$. In the general case, each criterion $K_i$ is influenced by the indicators and depends on the indicator values $\overline{P}_j$ ($j = 1, 2, \ldots, n$) and the significance coefficients $b_{i,j}$ that the indicator has regarding this factor. If such dependencies do not exist, then $b_{i,j} = 0$, and the corresponding arc in the graph does not exist. In a special case, each indicator influences only one criterion, and the graph is reduced to a unitary one.

As proposed in [5], the steps of the algorithm to find the aggregated estimate $Q$ comprise the consecutive definition of:

$1^0$ The group of experts;
$2^0$ The coefficients of competence of the experts;
$3^0$ Models of the expert estimate for $Q$, the criteria and the indicators;
$4^0$ The value of the indicators according to each expert and concordance check of their opinion;
$5^0$ Aggregated value of the indicators;
$6^0$ Coefficients of significance of the indicators for each criterion;
$7^0$ Values of the criteria;
$8^0$ Scaling constants of the criteria in the aggregated estimate $Q$;
$9^0$ Aggregated estimate $Q$.

### *$1^0$ Defining the group of experts*

The initial set of experts is defined usually by the public body in charge of the analysis. Each of the initially selected experts may in turn propose other members, judging upon their knowledge of the competence of colleagues. At the next stage, it is necessary to define the group of $l$ experts and recommended number of members $N < l$.

Each expert defines their opinion regarding the participation of the other experts by *"Yes"* or *"No"*. Assume that the $r$-th expert has collected $w_r \le l$ positive votes. Then it is possible to calculate the coefficients of mutual acceptance $K_{np,r} = w_r / l$ for each expert. The ones with the lowest calculated coefficients are taken out of the list until the required number of $N$ experts is achieved. The following algorithm applies:

1. Define the initial proposed set of experts.
2. Discuss the changes in the group proposed by the experts.
3. Define the final content of the expert group with an initial number of $l$ members.
4. Calculate the coefficients of mutual acceptance $K_{np,r} = w_r / l$, $r = 1, 2, \ldots, l$.
5. Rank the experts in descending order of their coefficient of acceptance.

6. Take the experts with the lowest coefficient from 4) until the condition $N < l$ is satisfied.
7. Form the final list of experts.

### ***2<sup>0</sup> Define the coefficients of competence of the experts***

It is necessary to find the coefficients of competence $a_{j,r}$ ($r = 1, 2, \ldots, N$) of the experts regarding the $j$-th indicator. Initially it is necessary to question $N \leq N$ experts regarding the relative competence of all experts. The questionnaire of the $r$-th expert is a square $N \times N$ matrix with elements $x_{ii,jj}^{j,r}$. If $x_{ii,jj}^{j,r} = 1$, then the $r$-th expert thinks the $jj$–th expert is more competent than the $ii$-th expert regarding the $j$-th indicator. If $x_{ii,jj}^{j,r} = 0$, then the $r$-th expert thinks the $ii$-th expert is more competent than the $jj$–th expert regarding the $j$-th indicator. The elements of the main diagonal are not filled in. Thus, the questionnaire transforms into a square $N \times N$ matrix of incidence. An example questionnaire from an expert for a given indicator from the investigation among 6 experts is given in Table 1.

**Table 1.** Ranking according to competence

| | | **EXPERTS *j*** | | | | | |
|---|---|---|---|---|---|---|---|
| | | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ |
| **EXPERTS *i*** | $e_1$ | | 1 | 1 | 1 | 1 | 1 |
| | $e_2$ | 0 | | 1 | 1 | 1 | 1 |
| | $e_3$ | 0 | 0 | | 0 | 1 | 0 |
| | $e_4$ | 0 | 0 | 1 | | 0 | 1 |
| | $e_5$ | 0 | 0 | 0 | 1 | | 0 |
| | $e_6$ | 0 | 0 | 1 | 0 | 1 | |
| | $\sum$ | 0 | 1 | 4 | 3 | 4 | 3 |

The analysis of results is performed in two iterations. In the first iteration, the average of the incidence matrix elements is calculated presuming equal importance of the experts. The values $\xi_{ii,jj}^{(1)j}$ of the first iteration are defined as non-weighted average of the data from the questionnaire according to the formula:

$$\xi_{ii,jj}^{(1)j} = \sum_{r=1}^{N'} x_{ii,jj}^{j,r} \Big/ N', \quad ii = 1,2,\ldots,N', jj = 1,2,\ldots,ii-1, ii+1,\ldots,N'. \tag{1}$$

If all participants share the same opinion, the respective matrix element $\xi_{ii,jj}^{(1),j}$ is either 1 or 0 depending on the opinion. If opinions do not entirely coincide then when calculating the non-weighted average, the elements of the matrix are real numbers in the interval (0; 1) and indicate the average opinion of all experts regarding the relative competence of each expert. Then a rank is defined for each expert. The non-weighted ranks $R_r^{(1),j}$ are defined as sums of the columns of the elements $\xi_{ii,jj}^{(1),j}$ :

$$R_r^{(1),j} = \sum_{ii=1}^{r-1} \xi_{ii,r}^{(1),j} + \sum_{ii=r+1}^{N'} \xi_{ii,r}^{(1),j}, \quad r = 1, 2,\ldots \tag{2}$$

The data in table 2 results from the summation using (2). The elements of the matrix are the average values of the expert answers.

**Table 2.** Resulting $\xi^{(1)}$ matrix from the first iteration

| | ***EXPERTS j*** | | | | | |
|---|---|---|---|---|---|---|
| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ |
| $e_1$ | | 0,83 | 0,67 | 0,83 | 0,83 | 0,67 |
| $e_2$ | 0,17 | | 0,67 | 0,67 | 0,67 | 0,83 |
| $e_3$ | 0,33 | 0,33 | | 0,50 | 0,67 | 0,50 |
| $e_4$ | 0,17 | 0,33 | 0,50 | | 0,00 | 1,00 |
| $e_5$ | 0,17 | 0,33 | 0,33 | 1,00 | | 0,33 |
| $e_6$ | 0,33 | 0,90 | 0,70 | 0,00 | 0,67 | |
| $R_r^{(1),j}$ | 1,17 | 1,89 | 2,20 | 2,17 | 2,01 | 2,66 |
| ***rank*** | **6** | **5** | **2** | **3** | **4** | **1** |

At the second iteration, the competence is given more precision by weighting the opinion of each expert in the questionnaire with the rank from the first iteration. The values $\xi_{ii,jj}^{(2),j}$ of the second iteration of the incidence matrix elements are defined as a weighted average of the questionnaires using the formula:

$$\xi_{ii,jj}^{(2),j} = \sum_{r=1}^{N'} R_r^{(1),j} x_{ii,jj}^{j,r} \Big/ \sum_{r=1}^{N'} R_r^{(1),j}, \quad ii = 1,2,\ldots,N', jj = 1,2,\ldots,ii-1,ii+1,\ldots,N'. \tag{3}$$

The weighted ranks of the experts $R_r^{(2),j}$ are defined as sums of the columns of elements $\xi_{ii,jj}^{(2),j}$ :

$$R_r^{(2),j} = \sum_{ii=1}^{r-1} \xi_{ii,r}^{(2),j} + \sum_{ii=r+1}^{N'} \xi_{ii,r}^{(2),j}, \quad r = 1,\ 2,\ldots \tag{4}$$

Finally, the weighted ranks are normalized to the fractional coefficients of competence regarding the *j*-th indicator:

$$a_{j,r} = \frac{R_r^{(2),j}}{\sum_{r=1}^{N} R_r^{(2),j}}, \quad r = 1,\ 2,\ldots \tag{5}$$

The results from the second iteration are given in Table 3.

The weight coefficients of the experts on this criterion finally are: 0,096, 0,110, 0,180, 0,192, 0,172, 0,251. It is obvious that after the competence analysis in the second iteration, experts $e_3$ and $e_4$ have changed places.

**Table 3.** Resulting $\xi^{(2)}$ matrix from the second iteration

| | ***EXPERTS j*** | | | | | |
|---|---|---|---|---|---|---|
| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ |
| $R_r^{(1),j}$ | **1,17** | **1,89** | **2,20** | **2,17** | **2,01** | **2,66** |
| $e_1$ | | 0,91 | 0,64 | 0,84 | 0,82 | 0,72 |
| $e_2$ | 0,09 | | 0,72 | 0,60 | 0,68 | 0,78 |
| $e_3$ | 0,36 | 0,28 | | 0,48 | 0,66 | 0,57 |
| $e_4$ | 0,16 | 0,40 | 0,52 | | 0,00 | 1,00 |
| $e_5$ | 0,18 | 0,32 | 0,34 | 1,00 | | 0,44 |
| $e_6$ | 0,28 | 0,22 | 0,43 | 0,00 | 0,56 | |
| $R_r^{(2),j}$ | 1,07 | 1.22 | 2,01 | 2,08 | 1,91 | 2,79 |
| $a_{j,r}$ | 0,096 | 0,110 | 0,180 | 0,192 | 0,172 | 0,251 |
| *rank* | **6** | **5** | **3** | **2** | **4** | **1** |

In the estimates that the experts make further on, each of them participates according to her competence, defined by her weight coefficient on the given indicator.

### $3^0$ *Models of the expert estimate for Q, the criteria and the indicators*

The aggregated estimate $Q$, the $m$ criteria $K_i$, as well as the $n$ indicators $\overline{P}_j$ are functions $y$ of $t$ variables $x_s$ and their scaling constants $c_s$. The structure of these $(m+n+1)$ functions is the following:

$$y = f\ (c_1,\ c_2,\ldots \qquad\qquad \ldots \tag{6}$$

The correspondence between the formal parameters $y$, $t$, $x_s$ ($s$=1, 2, …, $t$) and $c_s$ ($s$=1, 2, …, $t$) with the parameters, criteria and indicators is shown in Table 4.

**Table 4.** Correspondence between parameters

| Formal parameters | Aggregated estimate | Estimate of the $i$-th criterion | Aggregated estimate of the $j$-th indicator |
|---|---|---|---|
| $y$ | $Q$ | $K_i$ | $\overline{P}_j$ |
| $t$ | $m$ | $n$ | $N$ |
| $x_s$ | $K_i$ | $\overline{P}_j$ | $I_{j,r}$ |
| $c_s$ | $k_i$ | $b_{i,j}$ | $a_{j,r}$ |

The function $y$ is increasing on each of its arguments, $x_s$ and $y$ are measured in one and the same scale, whereas the sum of the scaling constants $c_s$ is equal to one. That is why $y$ is usually one of the measures of central tendency – mode, median, mean value, etc.

The so-called $\gamma$-averaging model is proposed in [6]:

$$y = \left( \sum_{s=1}^{t} c_s x_s^{\gamma} \right)^{\frac{1}{\gamma}}. \tag{7}$$

This is a general model which provides freedom in the choice of the form and has higher variability in the model of the aggregated estimate. If $\gamma = 1$ it coincides with the additive model. Another possible value for $\gamma$ is 0,5. The multiplicative model is also popular:

$$y = \prod_{s=1}^{t} x_s^{c_s}. \tag{8}$$

It is assumed that the responsible body is familiar with the methods, models and procedures, and depending on the case analyzed it is in position to decide which model to calculate the aggregated estimate $Q$ of the $m$ criteria $K_i$, as well as of the n indicators $\overline{P}_j$.

### $4^0$ *Define the value of the indicators according to each expert and conduct concordance check of their opinion*

The aggregated indicators $\overline{P}_j$ ($j$ = 1, 2, …, $n$) are different depending on the object of assessment, and also different are the experts who give the estimates $I_{jr}$ ($j$ = 1, 2, …, $n$; $r$ = 1, 2, …, $N$) to those indicators using a predefined ranking scale. The resulting estimates are based on the correspondence between the actual state and the imposed standards.

The estimates of the experts may substantially differ, and if averaged they can lead to inadequate values. That is why it is necessary to test for possible lack of concordance of expert opinion. It is necessary to calculate the coefficient of concordance $W$ [9]. The concordance in the opinion is accepted if $W \geq 0{,}8$. Otherwise it is necessary to apply methods for concordance of the opinion, e.g. the Delphi method [11]. In any case, the high variance of the estimates $I_{jr}$ ($r$ = 1, 2, …, $N$) shows that the $j$-th indicator is not well elicited and needs to be corrected by external experts.

The indicators may be referred to as subsections in the systematization and classification of the data regarding the assessment of the object in question.

There are several requirements to the indicators:

- they must be connected with the criteria by content,
- they must be measurable at least at an expert level,
- they should use data that is clear, valid and understandable,
- they should allow quantitative analysis.

The formulation (usually verbal) and the choice of the indicators $I_1, I_2, \ldots, I_n$ is performed using reduction on consistent similarity according to the cluster method [6] or other methods.

The numerical value of each indicator is not available, explicit and determined. The experts $\{E\} = \{e_1, e_2, \ldots, e_N\}$ define values for the indicators taking into account the information regarding the status judged upon the respective indicator. However, the experts have different competence. That is why it is necessary to calculate coefficients of competence $a_{ij}$ ($i$ = 1, 2, …, $N$; $j$ = 1, 2, …, $n$) of each expert in the aggregated estimates of the indicators $I_1, I_2, \ldots, I_n$.

Several examples may demonstrate the ideas. One interesting field of application is the assessment of universities, professional fields, specialties and curricula by Accreditation, Academic or Faculty Committee. The criteria $K_i$ (usually not more than 10) that influence the aggregated estimate $Q$ are preliminarily normalized and published. For example, $K_1$ – quality of the education activity, $K_2$ – quality of the research, and $K_3$ – quality of the managerial activity. Each of these depends on a set of indicators that are used to judge upon the value of the criterion. For example, student grades $I_1$, realization of the graduates $I_2$, incomes of the university $I_3$, etc. Each expert may define different values for the indicators, as well as different influence over the quality of the estimate (the three criteria $K_1$, $K_2$, $K_3$). For the administrator of the university ($e_i$) the student grades may be measured by the mean of the student grades, whereas for the employer ($e_j$) – by the practical abilities of graduates. As a result, the experts define different values for the indicator.

Another application area is the education capacity $Q$ of the university, which is defined by the education documentation $K_1$, the infrastructure $K_2$, the academic staff $K_3$, and the quality of education $K_4$. Several indicators may be defined, e.g. $I_1$ – number and type of the specialties, $I_2$ – number and qualification of lecturers, $I_3$ – education and research space, etc. However, different experts will give different values to some of these.

The third area of application is auction procedures. The complex estimate of each offer may be based on the following criteria: $K_1$ – economic effect, $K_2$ – reliability, and $K_3$ – executability. But the indicators may be very different: $I_1$ – originality of the technical solution, $I_2$ – element set used, $I_3$ – existing realizations of the product, etc.

### *$5^0$ Finding the aggregated value of the indicators*

The following algorithm may be employed to find the aggregated values of the indicators $\overline{P}_j$, $j = 1, 2, \ldots, n$:

1. Find the coefficients of competence of the experts $a_{j,r}$ ($j = 1, 2, \ldots, n$; $r = 1, 2, \ldots, N$) after applying the algorithm from $2^0$ for each indicator.
2. Find the estimates of the experts $I_{j,r}$ ($j = 1, 2, \ldots, n$; $r = 1, 2, \ldots, N$) according to section $4^0$.
3. If the concordance of the estimates $I_{j,r}$ is insufficient, then the estimates are corrected using Delphi (if all experts are individuals), or using super-experts (if part of the experts are groups).
4. Choose the models of the aggregated indicators for $j = 1, 2, \ldots, n$, using (6) or (7).
5. Replace the values of $I_{j,r}$ and $a_{j,r}$ in these models according to Table 4.

The values of the indicators serve to define the criteria.

### *$6^0$ Coefficients of significance of the indicators for each criterion*

It is now necessary to find the coefficients of significance $b_{i,j}$ ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$), which show the importance of each indicator for the criteria. As a rule, the experts that define the coefficients of significance $b_{i,j}$ are different from those in step $4^0$. It is important that these experts are knowledgeable in the specific area (problems) analyzed. The weights $b_{i,j}$ may be defined directly by the responsible body, and then no specific techniques need to be applied. Since $b_{i,j}$ are in fact scaling constants, they may be defined using algorithms for quantitative decision analysis [13].

The expert estimate method using ranking is proposed to define the weight coefficients. The experts rank the indicators according to the degree of influence over a criterion, and they are allowed to assign equal ranks. If there are $m$ criteria, then $m' \leq m$ ranks will be defined, and each indicator may fall within any of these ranks.

An example demonstrates how to rank the indicators according to their influence over the criterion $K_1$. Each expert fills in a row from the table and ranks their opinion regarding the influence of nine indicators over the criterion. After that the results are subjected to the rank correlation algorithm [9], and if there are ranks that coincide, the matrix is normalized. The results from the ranking are given in Table 5 (rank matrix) and Table 6 (normalized rank matrix).

**Table 5.** Rank matrix for the criterion $K_1$

| $K_1$ | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | $I_8$ | $I_9$ |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 1 | 1 | 5 | 8 | 4 | 3 | 2 | 5 | 7 |
| $e_2$ | 1 | 1 | 6 | 9 | 4 | 2 | 2 | 3 | 7 |
| $e_3$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 6 | 8 |
| $e_4$ | 1 | 2 | 7 | 6 | 5 | 5 | 9 | 5 | 3 |
| $e_5$ | 1 | 2 | 3 | 1 | 4 | 2 | 4 | 5 | 7 |
| $e_6$ | 1 | 2 | 3 | 8 | 4 | 5 | 7 | 3 | 6 |

**Table 6.** Normalized rank matrix

| $K_1$ | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | $I_8$ | $I_9$ |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 1,5 | 1,5 | 6,5 | 9 | 5 | 4 | 3 | 6,5 | 8 |
| $e_2$ | 1,5 | 1,5 | 7 | 9 | 6 | 3,5 | 3,5 | 5 | 8 |
| $e_3$ | 1 | 2 | 3 | 4 | 5 | 6,5 | 8 | 6,5 | 9 |
| $e_4$ | 1 | 2 | 8 | 7 | 5 | 5 | 9 | 5 | 3 |
| $e_5$ | 1,5 | 3,5 | 5 | 1,5 | 6,5 | 3,5 | 6,5 | 8 | 9 |
| $e_6$ | 1 | 2 | 3,5 | 9 | 5 | 6 | 8 | 3,5 | 7 |

The coefficients of concordance $w$ are employed to check the coherence in the opinion and the estimates of the experts:

$$w = \frac{12\sum_{i=1}^{n}(S_i - \overline{S})^2}{m^2(n^3 - n) - m\sum_{\lambda} T_{\lambda}}. \tag{9}$$

where:

$S_i$ – the sum of the ranks from the objects in all rankings; $\overline{S}$ - average sum of the ranks from an object;

$m$ –number of experts;

$n$ – number of ranked objects;

$T_\lambda = \frac{1}{12}\sum_{t_\lambda}(t_\lambda^3 - t_\lambda)$ , $t_\lambda$ – number of repeating ranks for each expert

The concordance coefficient calculated here is $w_k$ = 0,58. If an indicator is proven significant, then the weight coefficients $b_{ij}$ are calculated. When the number of the indicators is $n < 7$, the significance is tested using the $Z$-criterion, and if $n \geq 7$ – using the $\chi^2$ criterion. In this case, the statistical significance of the coordination in the opinion of the experts is tested using the $\chi^2$ criterion at a significance level $\alpha$ = 0,01 and degrees of freedom $\nu$ = 8. The calculated concordance coefficient $w_k$ is proven significant, which allows defining the weight coefficients $b_{ij}$ for each indicator. The defined weight coefficients and the ranks for each indicator for each criterion (in this example there are 5 criteria) are given in Table 7.

**Table 7.** Weight coefficients $b_{ij}$ ($i$ = 1, 2, …, 6; $j$ = 1, 2, …, 9) and ranks of the indicators

| | | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | $I_8$ | $I_9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $K_1$ | $W_{1j}$ | 0,215 | 0,192 | 0,097 | 0,067 | 0,100 | 0,118 | 0,074 | 0,090 | 0,046 |
| | Rank | **1** | 2 | 4 | 8 | 5 | 3 | 7 | 6 | 9 |
| $K_2$ | $W_{2j}$ | 0,193 | 0,118 | 0,213 | 0,137 | 0,083 | 0,061 | 0,026 | 0,085 | 0,084 |
| | Rank | 2 | 4 | **1** | 3 | 7 | 8 | 9 | 5 | 6 |
| $K_3$ | $W_{3j}$ | 0,035 | 0,207 | 0,028 | 0,203 | 0,155 | 0,127 | 0,113 | 0,03 | 0,101 |
| | Rank | **7** | **1** | 9 | 2 | 3 | 4 | 5 | 8 | 6 |
| $K_4$ | $W_{4j}$ | 0,033 | 0,087 | 0,052 | 0,139 | 0,165 | 0,16 | 0,063 | 0,207 | 0,094 |
| | Rank | 9 | 6 | 8 | 4 | 2 | 3 | 7 | **1** | 5 |
| $K_5$ | $W_{5j}$ | 0,028 | 0,045 | 0,063 | 0,073 | 0,167 | 0,17 | 0,148 | 0,102 | 0,204 |
| | Rank | 9 | 8 | 7 | 6 | 3 | 2 | 4 | 5 | **1** |

The results in Table 6 show the high information potential of the five criteria and nine indicators, since the highest ranks are distributed among the criteria at different indicators.

The work [6] proposes other methods for expert definition of weight coefficients, but the correlation analysis proved that the method proposed here is much more accurate and reliable. It rejects results for which the concordance coefficient is not significant. The higher complexity of the method is compensated by the more reliable results in the cases where the variance of the expert opinions is unknown and higher precision is needed.

### *$7^0$ Defining the values of the criteria*

In order to find the values of each criterion it is necessary to define models for the criteria, which as a rule differ from the models of the indicators.

The following algorithm may be applied to find the values of the criteria $K_i$, $i = 1, 2, \ldots, m$:

1. Find the coefficients of significance of the indicators $b_{i,j}$ ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$;) after applying step $6^0$ $m$ times (once for each criterion).
2. Find the estimates of the indicators $\overline{P}_j$, $j = 1, 2, \ldots, n$ according to $5^0$.
3. Choose the model of the criteria $i = 1, 2, \ldots, m$, using formulae (6) or (7).
4. Replace the values of $\overline{P}_j$ and $b_{i,j}$ according to Table 4.

### *$8^0$ Constants of the criteria in the aggregated estimate Q*

Here, it is necessary to find the constants $k_i$ ($i = 1, 2, \ldots, m$) of the criteria, which show what the weight of each criterion in the aggregated estimate $Q$ is. The experts that define the coefficients $k_i$ are usually representatives of higher management of the organization, as they are responsible for policy making and strategic management. The weights may also be defined by the responsible body (organization).

### *$9^0$ Aggregated estimate Q*

In order to define the aggregates estimate $Q$ it is necessary to apply the following algorithm:

1. Find the constants $k_i$ ($i = 1, 2, \ldots, m$) of the criteria by applying the procedures from $8^0$.
2. Find the values of the criteria $K_i$, $i = 1, 2, \ldots, m$ according to $7^0$.
3. Find the model of the aggregated estimates usually using formulae (6) or (7).
4. Replace the values of $K_i$ и $k_i$ in the chosen model according to Table 4.

## 3 Conclusions

The paper presented in detail and in examples the application of quantitative techniques to formalize the construction of an aggregated expert estimate regarding a process or its characteristics (quality). An appropriate application of those techniques could be higher education, curricula, majors, professional fields, universities, students, etc. A three-level model was constructed to demonstrate the basis of the approach. The proposed model also took into account the competence of the experts regarding each indicator, as well as defining the significance of the indicators in the criteria. Nine steps were outlined for the construction of the aggregated estimate and for finding its parameters (constants, coefficients, indicators, criteria). Each step consecutively approaches each part of the model, i.e. its parameters, which are needed to finally define the aggregated estimate.

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