# **FRICTION COMPENSATION IN NONLINEAR DYNAMICAL SYSTEMS USING FAULT-TOLERANT CONTROL METHODS**

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#### Summary

This paper describes the application of differential geometry and nonlinear systems analysis to the estimation of friction effects in a class of mechanical systems. The proposed methodology relies on adaptive filters, designed with a nonlinear geometric approach to obtain the disturbance de-coupling property, for the estimation of the friction force. Thanks to accurate estimation, friction effects are compensated by injecting the on-line estimate of friction force to the control action calculated by a standard linear state feedback. The inverted pendulum on a cart is considered as an application example and the proposed approach is compared with a commonly used friction compensation strategy, based on an explicit model of the friction force.

Key words: Fault-Tolerant Control, Friction Compensation, Nonlinear Systems, Differential Geometry

### **INTRODUCTION**

Feedback control systems for engineering applications are often affected by friction in mechanical components. Detailed modeling of friction effects has been the subject of many research activities [1] and the inverted pendulum system is commonly adopted as a benchmark for control design including friction compensation [2]. The most challenging problem for on-line estimation of friction is the identification of modelling parameters, which are mostly not directly measurable and possibly varying in time. Therefore, there is a growing demand for robust and adaptive algorithms for friction estimation and compensation.

This problem has been addressed in literature also as a fault-tolerant control problem [3]. A faulttolerant control system can include a Fault Detection and Diagnosis (FDD) module [4], which is mainly used to fulfil the requirement of fault estimation and allow the controller to react to the system component failures actively, by reconfiguring its actions so that the stability and acceptable performance of the entire system can be maintained. This kind of structure is classified as an Active Fault-Tolerant Control Scheme (AFTCS).

This paper is focused on the application of an AFTCS to address the problem of friction compensation. The proposed AFCTS integrates a reliable and robust friction estimation module, implemented according to an FDD procedure relying on adaptive filters. The controller reconfiguration exploits a second control loop depending on the on-line estimate of the friction force. The advantages of this strategy are that a structure of logic-based switching controller is not required and, instead, an existing controller can be preserved and enhanced.

The FDD method is based on the Nonlinear Geometric Approach (NLGA) developed in [5]. By means of this framework, a disturbance de-coupled adaptive nonlinear filter providing the fault identification is designed. It is worth observing that the original NLGA FDD scheme of [5], based on residual signals, cannot provide fault size estimation.

Both the NLGA Adaptive Filters (NLGA-AF) and the AFTCS strategy are applied to an inverted pendulum on a cart (also called cart-pole system), an underactuated mechanical structure that is commonly used as a benchmark system for control design and mechatronics prototyping. A simulation model for the complete AFTCS loop has been implemented in the Matlab/Simulink® environment, and tested in the presence of nonlinear friction, disturbances, measurement noise and modelling errors.

The work proceeds with Section 1, providing the description of the cart-pole nonlinear benchmark system, Section 2, describing the implementation of the FDD scheme and the structure of the AFTCS strategy, and Section 3, in which stability, robustness and advantages of the AFTCS method over simpler friction compensators are investigated with simulations. Section 5 summarises contributions and achievements of the paper.

#### **1. THE CART-POLE NONLINEAR MODEL**

The dynamic model of a pendulum (or pole) on a cart shown in Fig. 1 is a classical benchmark in Systems and Control Theory.



The interest in this mechanical system is motivated by the similarity between its dynamic properties and those of several real-world engineering applications like, for example, aerospace vehicles during vertical take-off, cranes, and many others. Assuming that the cart has mass M and the pendulum mass m is concentrated at the tip of a pole, with neglectable inertia, of length L, the dynamic model obtained using Hamilton's principle  $\frac{1}{2}$  is the following: .. .

$$
\begin{cases}\n(M+m)\ddot{x}_p + mL\ddot{\theta}\cos\ddot{\theta} - mL\dot{\theta}^2\sin\ddot{\theta} = F_a - F_{fric} \\
m\ddot{x}_p\cos\ddot{\theta} + mL\ddot{\theta} - mg\sin\ddot{\theta} = \tau\n\end{cases}
$$
\n(1)

in which *g* is the gravity constant, *Fa* is the controllable actuator force,  $F_{\text{fric}}$  is the friction involved by the contact between cart and ground, and  $\tau$  is a torque acting directly at the base of the pole. The state variables are  $x=[x_1 \ x_2 \ x_3 \ x_4]^T$  =  $[x_p \dot{x}_p \theta \dot{\theta}]^T$  and the control input is  $u = F_a$ , while

 $d = \tau$  is considered as a disturbance, so that the model can be rewritten in the following state-space input affine form: .

$$
\begin{cases}\n\dot{x}_1 = x_2 \\
\dot{x}_2 = \frac{mLx_4^2 \sin x_3 - \text{mgsin } x_3 \cos x_3}{M + m \sin^2 x_3} + \\
& + \frac{u - F_{fric}}{M + m \sin^2 x_3} - \frac{d}{(M + m \sin^2 x_3)L} \\
\dot{x}_3 = x_4 \\
\dot{x}_4 = \frac{(M + m) \text{gsin } x_3 - mLx_4^2 \sin x_3 \cos x_3}{(M + m \sin^2 x_3)L} + \\
& - \frac{(u - F_{fric}) \cos x_3}{(M + m \sin^2 x_3)L} + \frac{(M + m)d}{m(M + m \sin^2 x_3)L^2}\n\end{cases}
$$
\n(2)

It is worth noting that considering  $\tau$  (i.e. the torque acting at the base on the pole) as a disturbance terms represents a realistic situation, since it may be related to the effect of an impact between the pole and some kind of obstacle.

#### **3. FDD AND AFTCS DESIGN**

The presented FDD scheme belongs to the NLGA framework, that allows to de-couple disturbances by means of a nonlinear coordinate transformation. Such a transformation is then the starting point to design a set of adaptive filters that are able to both detect additive fault acting on a single actuator and estimate the magnitude of the fault. It is worth observing that in this paper, we can consider the effect of friction as a fault affecting the actuator, so that thanks to the NLGA approach the friction estimate is de-coupled from disturbance d.

The proposed approach can be applied to nonlinear systems in the form:

$$
\begin{cases}\n\dot{x} = n(x) + g(x)u + l(x)f + p_d(x)d \\
y = h(x)\n\end{cases}
$$
\n(3)

where the state vector  $x \in X$  (an open subset of  $R^n$ ),  $u \in R^{l_c}$  is the control input vector,  $f \in R$  is the fault,  $d \in R^{l_d}$  the disturbance vector and  $y \in R^{l_m}$ the output vector, whilst  $n(x)$ ,  $l(x)$ , the columns of  $g(x)$ , and  $p_d(x)$  are smooth vector fields and h(x) is a smooth map. The model of Eq. 2 can be related to the form of Eq. 3 setting:

$$
n(x) = \begin{bmatrix} x_2 \\ \frac{mLx_4^2 \sin x_3 - \text{mgsin } x_3 \cos x_3}{M + m \sin^2 x_3} \\ \frac{(M + m) \text{gsin} \vec{x}_3 - mLx_4^2 \text{sin} \vec{x}_3 \cos x_3}{(M + m \sin^2 x_3)L} \end{bmatrix}
$$

$$
g(x) \equiv l(x) = \begin{bmatrix} 0 \\ \frac{1}{M + m \sin^2 \left(\frac{x}{M}\right)} \\ 0 \\ -\frac{\cos x_3}{\left(M + \sin^2 \left(\frac{x}{M}\right)\right)} \end{bmatrix}
$$

$$
p_d(x) = \begin{bmatrix} 0 \\ -\frac{\cos x_3}{(M + m\sin^2 x_3)L} \\ 0 \\ \frac{M + m}{m(M + \sin^2 x_3)L^2} \end{bmatrix}
$$

Assuming in addition that the full state vector is measurable (i.e.  $h(x)=I_4x$ ), the design of the strategy for the diagnosis of the fault  $f$  with disturbance de– coupling is organised as follows:

- computation of  $\sum_{i=1}^{P}$ , i.e. the minimal conditioned invariant distribution containing *P* (where *P* is the distribution spanned by the columns of  $p_d(x)$ ;
- computation of  $\Omega$ , i.e. the maximal observability codistribution contained in  $(\sum^{\mathrm{P}}_*)^{\perp}$  $_{*}^{\mathrm{r}}$  )<sup> $\pm$ </sup> ;
- if  $l(x) \notin (\Omega^*)^{\perp}$ , the fault is detectable and a suitable change of coordinate can be determined.

The computation of  $\sum_{i=1}^{p}$  can be obtained by means of the following recursive algorithm:

$$
\begin{cases}\nS_0 = \overline{P} \\
S_{k+1} = \overline{S} + \sum_{i=0}^m [g_i, \overline{S}_k \cap \ker\{dh\}] \n\end{cases}
$$
\n(4)

where *m* is the number of inputs,  $\overline{S}$  is the involutive closure of *S*,  $[g, \Delta]$  is the distribution spanned by all vector fields  $[g, \tau]$  in which  $\tau \in \Delta$ , and  $[g, \tau]$  is the Lie Bracket of *g* and  $\tau$ . It can be shown that if there exists a  $k \geq 0$  such that  $S_{k+1}=S_k$ , the algorithm of Eq.4 stops and  $\sum_{*}^{P} = S_k$ .

Once  $\sum_{i=1}^{P}$  is determined,  $\Omega^*$  can be obtained with the following algorithm:

$$
\begin{cases}\nQ_0 = (\Sigma_*^p)^\perp \cap \text{span}\{dh\} \\
Q_{k+1} = (\Sigma_*^p)^\perp \cap \sum_{i=0}^m [L_{g_i}Q_k + \text{span}\{dh\}] \n\end{cases}
$$
\n(5)

where  $L_g \Gamma$  denotes the codistribution spanned by all covector fields  $L_g\omega$ , with  $\omega \in \Gamma$ , and  $L_g\omega$ is the derivative of  $\omega$  along g. If there exists an integer  $k^*$ , such that  $Q_{k^*+1} = Q_{k^*}$ , then  $Q_{k^*}$  is indicated as o.c.a.( $(\sum_{i=1}^{P} )^{\perp}$  $\binom{F}{*}^{\perp}$ ), where o.c.a. stands for *observability codistribution algorithm.*

It can be shown that  $Q_{k^*} = \text{o.c.a.}(\ (\Sigma^P)^{\perp})$  $_{*}^{\mathrm{r}}$  )<sup> $_{+}^{\mathrm{r}}$ </sup>) represents the maximal observability codistribution containd in  $P^{\perp}$ , i.e.  $\Omega^*$  (see [5]). Therefore, when  $l(x) \notin (\Omega^*)^{\perp}$  the disturbance *d* can be de-coupled and the fault *f* is detectable.

As mentioned abote, the considered NLGA to the fault diagnosis problem, described in [5], is based on a coordinate change in the state space and in the output space, respectively denoted as  $\Phi(x)$ and  $\Psi(\nu(x))$ , which are local diffeomorphisms structured as follows:

$$
\begin{cases}\n\Phi(x) = \begin{pmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \overline{x}_3 \end{pmatrix} = \begin{pmatrix} \Phi_1(x) \\ H_2h(x) \\ \Phi_3(x) \end{pmatrix} \\
\Psi(y) = \begin{pmatrix} \overline{y}_1 \\ \overline{y}_2 \end{pmatrix} = \begin{pmatrix} \Psi_1(y) \\ H_2y \end{pmatrix}\n\end{cases}
$$
\n(6)

such that  $\Omega^* \cap \text{span} \{ dh \} = \text{span} \{ d(\Psi_1 \circ h) \},$  $\Omega^* = \text{span}\{\text{d}\Phi_1\}$  and the rows of  $H_2$  are a subset of the rows of the identity matrix. By using the new local coordinates  $(\bar{x}, \bar{y})$ , the system 4 is transformed so that it exhibits an observable subsystem that is affected by the fault and not affected by the disturbance, as described in [5]:

$$
\begin{cases}\n\bar{x}_1 = n_1(\bar{x}_1, \bar{x}_2) + g_1(\bar{x}_1, \bar{x}_2)c + \\
+ l_1(\bar{x}_1, \bar{x}_2, \bar{x}_3)f \\
\bar{y}_1 = h(\bar{x}_1)\n\end{cases}
$$
\n(7)

In the case of the cart-pole system, the following result is obtained:

$$
S_0 = P = cl(p_d(x)) =
$$
  
= 
$$
cl \left( \begin{bmatrix} 0 \\ -\frac{\cos x_3}{(M + m \sin^2 \overline{x_3})L} \\ 0 \\ \frac{M + m}{m(M + \sin^2 \overline{x_3})L^2} \end{bmatrix} \right) \equiv p_d(x)
$$
 (8)

Assuming that the full cart-pole state is measurable,  $\text{ker} \{\text{d}h\} = \emptyset$ , so that it follows  $\sum_{i=1}^{p} P_i = \overline{P}$  $E_*^F = P$  as  $S_0 \cap \text{ker} \{dh\} = \emptyset$ . Thus, the algorithm of Eq.3 stops with immediately (*k*=0).

Proceeding with the algorithm of Eq. 4, it can be observed that:

$$
(\overline{P})^{\perp} = \begin{pmatrix} 0 \\ -\frac{\cos x_3}{(M + m \sin^2[\vec{x}_3)L} \\ 0 \\ \frac{M + m}{m(M + m \sin^2[\vec{x}_3)L^2} \end{pmatrix}^{\perp} = \begin{pmatrix} 9 \\ -\frac{M + m}{M + m} \\ 0 & 0 \\ 0 & 1 - \frac{mL x_4 \sin[\vec{x}_3]}{M + m} \\ 0 & 1 - \frac{mL x_4 \sin[\vec{x}_3]}{M + m} \end{pmatrix}
$$

and  $\text{span} \{ \mathrm{d} h \} = I_4$ . From the first step of the algorithm it follows that  $\Omega^* = (\Sigma^P_*)^{\perp} = (\overline{P})^{\perp}$ and that  $(\Omega^*)^{\perp} = \Sigma^P_* = \overline{P}$ \*  $(\Omega^*)^{\perp} = \Sigma^P_* = \overline{P}$ . Therefore, the fault (i.e. the friction force) is detectable, since  $l(x) \notin (\Omega^*)^{\perp}$ .

Since the dimension of  $\Omega^*$  and of  $\Omega^* \cap \text{span} \{ dh \} = \text{span} \{ d(\Psi_1 \circ h) \}$  is 3, it follows that  $\Phi_1(y): \mathbb{R}^4 \to \mathbb{R}^3$  and that  $H_2$ y:  $R^4 \rightarrow R^1$  (the component  $\Phi_3(x)$  is not present in this case). Thus, as  $h(x)=I_4x$  the surjection  $\Psi(v(x))$  can be defined as follows:

$$
\Psi(y(x)) = {\binom{\Psi_1(x)}{H_2 x}} = \left( \begin{bmatrix} x_2 + \frac{mLx_4 \cos[\bar{x}_3]}{M+m} \\ x_1 \\ x_3 \\ x_4 \end{bmatrix} \right)
$$

where  $H_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$ . Moreover,  $\overline{X}_1 = \Phi_1(x) = \Psi_1(x)$  is the state variable of the observable subsystems that is decoupled from the disturbance.

Given this result, the NLGA-AF can be designed introducing the following additional constraint [6]: there exists a proper scalar component  $\overline{X}_{1s}$  of the state vector  $\overline{x}_1$  such that the corresponding scalar component of the output vector  $\overline{y}_{1s} = \overline{x}_{1s}$  and the following relation holds: .

$$
\overline{y}_{1s}(t) = M_1(t) \cdot f + M_2(t)
$$
 (10)

which means that the effect of the fault signal can be properly isolated. The functions  $M_1$  and  $M_2$  are generally computed from both inputs and outputs measurements of the system under diagnosis. The fault estimation is then obtained thanks to the following adaptive filter based on the least-squares algorithm with forgetting factor described in [7]: .

$$
\begin{cases}\n\dot{P} &= \beta P - \frac{1}{N^2} P^2 \vec{M}_1^2, & P(0) = P_0 > 0 \\
\vdots & \quad \hat{f} = P \epsilon \vec{M}_1, & \quad \hat{f}(0) = 0\n\end{cases}
$$
\n(11)

with the following equations describing the output estimation and the corresponding normalized estimation error:

$$
\begin{cases}\n\widehat{\overline{y}}_{1s} &= \overline{M}_1 \widehat{f} + \overline{M}_2 + \lambda \, \overline{\overline{y}}_{1s} \\
\epsilon &= \frac{1}{N^2} (\overline{y}_{1s} - \widehat{\overline{y}}_{1s})\n\end{cases}
$$
\n(12)

Finally, the signals  $\overline{M}_1, \overline{M}_2, \overline{\overline{y}}_{1s}$  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  are obtained by means of low-pass filtering of the signals  $M_1, M_2, \overline{y}_{1s}$ . The design parameters that must be properly tuned for the desired application are:

- $\lambda > 0$  is the bandwidth of the low-pass filters required by the computation of  $\rm\,M_{1},\rm M_{2},\bar{\bar{y}}_{1s}$
- $\beta > 0$  is the forgetting factor of the adaptation algorithm  $\overline{\phantom{0}}$
- $\bullet$  $N^2 = 1 + \breve{M}_1^2$  $= 1 + \tilde{M}_1^2$  is the normalisation factor of the least-squares algorithm.

In order to de-couple the effect of the disturbance d from the fault (i.e. friction) estimator, it is necessary to select from the observable subsystem the following state:

$$
\overline{x}_{1s} = \overline{x}_{11} = x_2 + \frac{mLx_4 \cos \overline{x}_3}{M+m}
$$
 (13)

whose differentiation allows to compute the full expression of  $M_1$  and  $M_2$ , namely:

$$
M_1 = \frac{1 - m \cos^2 x_3 / (m + M)}{M + m \sin^2 \overline{x}_3}
$$
 (14)

$$
M_2 = \frac{\left(1 - \frac{m \cos^2 x_3}{m + M}\right) mL x_4^2 \sin x_3}{M + m \sin^2 \left(\frac{m}{2}\right)} - \frac{m L x_4^2 \sin x_3}{m + M} + \frac{1 - m \cos^2 x_3 / (m + M)}{M + m \sin^2 \left(\frac{m}{2}\right)} u
$$
\n(15)

#### **4. SIMULATION RESULTS**

To compute with Matlab/Simulink® the simulations results described in this section, the AFTCS design has been completed by means of an optimal state feedback control law, on the basis of the linear approximation of a frictionless version of Eq. 2, in a neighborhood of  $x_0 = [x_{1d} \ 0 \ 0 \ 0]$ T, in which  $x_{1d}$  can be any value. In fact, the linear approximation is independent from x1, so that the input vector of the optimal controller can be calculated as  $x_e = [(x_1 - x_{1d}) x_2 x_3 x_4]$  and the cart-pole system will be stabilised in the upright position at any linear position reference. The full control loop is therefore the one shown in Fig. 2.



Fig. 2: Control loop with proposed AFTCS

The following values of the system parameters have been assumed:  $M = 1$  kg;  $m = 0.1$  kg;  $L = 0.3$ m;  $g = 9.81$  m/s<sup>2</sup>. The optimal controller has been designed using the LQR approach in order to minimize the standard quadratic cost function of state and input, with  $Q = 10 I_4$  and  $R = 1$ .

The simulation of the mechanical system has been completed by a nonlinear model of friction affecting the linear motion of the cart, including viscous friction and Stribeck friction, with Coulomb and static part, using the following mathematical description:

$$
F_{fric} = [F_c + (F_s - F_c)e^{-\left(\frac{|\dot{x}_p|}{v_s}\right)^2}]sign(\hat{x}_p) + F_x \dot{x}_p \qquad (16)
$$

in which  $F_x = 0.6$  N/m/s is the viscous coefficient,  $F_c$ =0.25 N is the Coulomb coefficient,  $F_s$ =0.4 N is the static part coefficient and  $v_s = 0.02$  m/s is the Stribeck velocity. This friction model is commonly accepted as compatible with experimental observations (see [1] and [2]). However, many industrial motion control systems adopts simpler models for direct compensation of the friction force. In particular, the most used approximation of the model in Eq. 16, as implemented in industrial controllers, disregard the Stribeck effect and include

only viscous and Coulomb friction, obtaining the following model:

$$
F_{fric} = F_c \operatorname{sign}(\hat{x}_p) + F_x \dot{x}_p \tag{17}
$$

 The plot in Fig. 3 shows the different shapes of the two nonlinear friction models of Eq. 16 and Eq. 17.



Fig. 3: Friction model with Stribeck effect and model used by standard compensation algorithms of industrial controllers

As can be seen, the simplified model matches the more detailed one if the velocity is sufficiently large, while it deviates significantly at low velocities. Moreover, the friction compensation based on a direct model of the friction effect requires necessarily a long and accurate tuning procedure, based on experimental data. The design of experiments themselves is also quite critical for an effective identification of friction parameters, as described in [1] and [2].

With approach based on the proposed AFTCS, instead, the friction force is adaptively estimated online. In the particular case of the cart-pole system, only mechanical parameters that can be quite easily measured (i.e. the masses *m* and *M* and the pole length *L*) are required.

The effectiveness of the methodology has been evaluated as follows. The NLGA-AF used as a friction estimator has been designed assuming the nominal model of the cart-pole, but the simulated mechanical system included a mismatch of 10% in the values of *M* and *m*, a random disturbance torque  $d$  (which is nevertheless de-coupled by design) and measurement noise on the state feedback signals. As can be seen from Fig. 4, the proposed NLGA-based estimator provides an accurate and robust measure of the actual friction force, even if the mathematical model of friction effects is not explicitly included in the design of the estimator. Such effective performance is achieved by means of a proper tuning of the adaptation mechanism of the filter, in particular  $\beta = 12$  and  $\lambda = 8:8$  have been fixed for the simulated case.

The same Fig. 4 shows also the friction force estimated by the model of Eq. 17, assuming instead that a perfect tuning of the parameters  $F_x$  and  $F_c$  can be achieved. As can be seen, since the simulated experiment is performed at relatively low velocities (which is a common operating mode for the considered mechanical system, since higher velocities would make more challenging the stabilization of the inverted pendulum), the estimation is not capable of tracking accurately the real friction force. Moreover, since the velocity measurement is assumed to be noisy, a low-pass filter (with the same bandwidth  $\lambda$  used for the AFTCS) is applied before the calculation of the friction model. Otherwise, at low velocities the noise could even reverse inconsistently the sign of the measurement and, therefore, change the sign of the estimated force, which would have a great impact on the performance of the control system if such an estimate is used for compensation.



Fig. 4: Friction force estimation obtained with the proposed AFTCS (blue) and with the model used by standard compensation algorithms of industrial controllers (red)

Thanks to its reconstruction, friction can in fact be compensated by simply adding its estimated value to the output of the optimal controller. As a result, the tracking of a time-varying linear cart position reference is dramatically improved, as shown in Fig. 5. The figure shows that in the first half of the plot the optimal controller by itself is not robust with respect to the nonlinear friction disturbing action, while in the second part of the simulation, friction compensation on the basis of either the NLGA-AF estimation or the simplified model of Eq. 17 is introduced and tracking error is reduced.

The performance of the two kind of compensators is not significantly different in this case, though a magnified plot shown in Fig. 6 reveals a small advantage of the AFTCS. However, it should be noted that the latter is adaptive by design, so that components aging and absence of lubrication or maintenance would never affect its performance in a practical application, as is instead

commonly the case when a model-based friction compensator is used.



Fig. 5: Tracking of a sawtooth reference for the linear position of the cart, without (right) and with (left) friction compensation. In the latter case, friction is compensated using either the NLGA-AF system (blue) or the simplified direct model (red)



Fig. 6: Detail of the tracking plot for the linear position of the cart, with friction compensated by the NLGA-AF (blue) or the simple model (red)

## **5. CONCLUSION**

This paper described the development of an active fault tolerant control scheme for the purpose of friction compensation in mechanical systems, which integrates a robust fault diagnosis method providing accurate estimation of friction effects. The methodology relies on disturbance de-coupled adaptive filters designed via the nonlinear geometric approach. The fault tolerant strategy has been applied to a classical control design benchmark, namely the inverted pendulum on a cart, which was simulated in presence of nonlinear friction, disturbing torque acting on the pole pivot, measurement noise and modelling errors.

It is worth observing that the suggested active fault tolerant control was already developed in works by the same authors, but applied to aerospace examples. Thus, the contribution of this paper consists of the application of the active fault tolerant control scheme to the well-known benchmark, in order to highlight the computational and mathematical aspects of the nonlinear disturbance de–coupling design, and hence it can be considered also as a tutorial for researchers working in mechanical systems monitoring and fault diagnosis, as well as fault tolerant control and friction compensation.

The proposed fault tolerant scheme allows to maintain the existing controller, since a further loop is added to the original scheme, thus providing the feedback of the adaptive friction estimate provided by the nonlinear geometric approach diagnosis module. The final performances of the developed fault tolerant control strategy are mainly due to the fact that the estimate is unbiased, thanks to disturbance de-coupling method. Moreover, when compared with other commonly used friction compensation methods, based on explicit models of the friction force, the proposed methodology has the advantage of being insensitive to parameter variation in the friction effect. The application of the proposed AFTCS is currently under evaluation on real case studies represented by didactical laboratory systems and industrial robotic manipulators.

### **REFERENCES**

- [1] Olsson, H., Astrom, K., de Wit, C., Gafvert, M., Lischinsky, P., *Friction models and friction compensation.* European Journal of Control Vol. 4, N. 3, pp. 176-195 (1998).
- [2] Campbell S, Crawford S, Morris K. *Friction and the Inverted Pendulum Stabilization Problem.* Journal of Dynamic Systems, Measurement and Control, Vol. 130, N. 5, 2008.
- [3] Patton, R., Putra, D., Klinkhieo, S., *Friction compensation as a fault-tolerant control problem.* International Journal of Systems Science, Vol. 41, N. 8, 2010.
- [4] Isermann, R.: *Fault±Diagnosis Systems: An Introduction from Fault Detection to Fault Tolerance*, Springer-Verlag, 2005.
- [5] De Persis, C., Isidori, A., *A geometric approach to non±linear fault detection and isolation.* IEEE Transactions on Automatic Control, Vol. 45, N 6, pp. 853–865, 2001.
- [6] Castaldi, P., Geri, W., Bonfè, M., Simani, S., Benini, M., *Design of residual generators and adaptive filters for the FDI of aircraft model sensors.* Control Engineering Practice, Vol. 18, N. 5, pp. 449-459, 2010.
- [7] Ioannou, P., Sun, J., *Robust Adaptive Control*. Prentice-Hall, 1996.



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