



# Rationalization of Manual Toll Collection System Maintenance Process

**Z. KASPRZYK**

WARSAW UNIVERSITY OF TECHNOLOGY, FACULTY OF TRANSPORT, Koszykowa 79, 02-008 Warsaw, Poland  
EMAIL: zka@wt.pw.edu.pl

## ABSTRACT

The research problem picked up in the article was determined by assessing the reliability of highway toll collection system based on the analysis of its operation. The complexity and diversity of the factors limiting the processes operating in the transport process significantly hinders the use of a universal indicator reliability assessment manual toll collection systems. Therefore, the evaluation of the reliability of manual toll collection system must be carried out in a comprehensive manner by presenting all aspects of the process are important from the point of view of the highway and the user operates the system. The research method adopted evaluation manual toll system and its components using the characteristics of reliability in the form of operational, the average time between subsequent damage, mean time to repair. In addition to the assessment capabilities of the system adopted in the payment of the corresponding number of vehicles per unit of time and the corresponding levels on the maximum waiting times for service and the maximum length of the queue of vehicles in the system.

**KEYWORDS:** manual toll collection system, operational reliability modeling

## 1. Introduction

Toll collection should be an uninterrupted process, assuring the highest possible continuity of transportation and adequate service level to a toll road user. Motorists consent to paying a toll under the condition they are guaranteed a comfortable journey. That includes being provided a good service and assurance of continuous transport services. Because the toll collection system is incoherent, it requires road management to use two manual toll collection systems alongside an electronic road toll system, increasing consequently operating costs and rendering impossible assuring a consistently good toll collection service for every type of vehicle [3], [4].

Hence the reliability of a toll collection system hinges on providing a continuous transportation service and revenues it generates [11]. For the toll collection system to be highly reliable is especially important in places where safe and fast transportation of people and goods is paramount. Downtimes of the toll

collection system causes traffic congestion and break continuity of transportation services.

Toll collection systems operate under a wide range of conditions. Hence not only do they require their individual elements to be reliable, but also correct maintenance strategies to be in place in order for them to function properly [3], [4], [5]. Thus there is a need to analyse reliability and maintenance of manual toll collection systems. Following from that effective improvements need to be devised.

## 2. Reliability evaluation method for manual toll collection system

Centrepiece to reliability of transport telematic systems is their technical condition, availability, outlays incurred to provide services and profits they generated. The most important in case of manual

toll collection system is availability of that system i.e. its availability to provide all necessary functions (full operational ability) related to requirements concerning reliability and availability, road user service, throughput and continuity of transportation service as well as revenues generated by the aforementioned services.

In order to evaluate a manual toll collection system and its elements the following indicators were taken into account: system availability, mean time between failures, number of vehicles cleared per unit of time, maximum waiting times and maximum length of queue of vehicles pending passage [5], [10], [12].

The sheer complexity and diversity of factors impeding transport processes makes difficult using a single, universal evaluation indicator to determine reliability of manual toll collection systems [9]. Hence the reliability of manual toll collection system needs to be evaluated in a comprehensive manner in order to capture all aspects of processes important from both user's perspective and the person operating the system, especially when it comes to availability for executing the aforementioned functions.

An all-around evaluation of manual toll collection system is possible by correctly designing the manual toll collection evaluation method based on inter-linked indicators. They facilitate setting out a maintenance strategy for analysed system which improves its operation and decreases operating costs [7], [8].

Table 1 shows projected reliability parameters produced through evaluating reliability of manual toll collection system presented in paper [2]. Reliability of a system is evaluated as a control measure. It is expected to show how reliability parameters satisfy requirements concerning reliability and availability provided in literature.

**Table 1. Projected and required values of reliability parameters for manual toll collection system**

Projected values of reliability parameters		
Description	MTBF [h]	A
Toll booth	$3,93 \cdot 10^3$	0,9994912
Manual toll collection using k-out-of-n reliability structure	$3,07 \cdot 10^3$	0,9993508
Manual toll collection using parallel reliability structure	$8,975 \cdot 10^3$	0,9997772
Required values of reliability parameters		
Toll booth	$4,38 \cdot 10^3$	0,9995436
Manual toll collection system	$8,760 \cdot 10^4$	0,9999772

Reliability analysis of manual toll collection system proved that projected reliability parameters determining mean up time MTBF and availability A, both for toll booth and the entire system, fail to comply with standards and regulations concerning reliability and availability of a system. In order for the system to become consistent with those requirements, the maintenance process and system for the manual toll collection would need to change.

### 3. Rationalization of the manual toll collection system

Reliability analysis of the system proved it does not comply with requirements in terms of reliability and availability of the manual toll collection system. Hence the maintenance process of the manual toll collection system needs to be analysed since it determines all technical and organisational activities executed with relation to the system at every stage of its existence, from its creation to decommissioning. Technical activities are expected to assure the highest possible availability of the system through correctly carried out maintenance. Similarly, organisational activities help to maintain required system availability through adequate maintenance and providing necessary supplies. Only a correctly maintained system will keep its availability and reliability at a required level. The maintenance analysis presented hereunder facilitates setting out a maintenance strategy for the system by determining necessary organisational activities designed to maintain system's required reliability and availability.

The following operational states were defined taking into account all possible functional states as well as serial reliability structure of toll booth comprising the manual toll collection system.

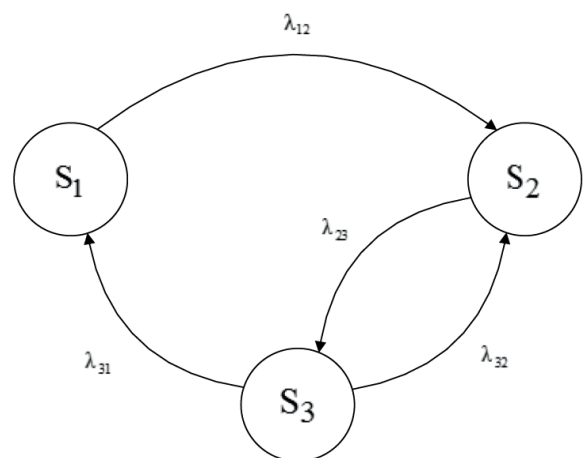
- $S_1$  - state of operation (up time),
- $S_2$  - state of pending repair (time between failures),
- $S_3$  - state of repair (repair).

States  $S_i$  create a state space described by the S set:

$$S = \{S_1, S_2, S_3\} \tag{1}$$

Figure 1 shows the maintenance process of toll booth including transition rate between operational states. Transitions between states are described by functions of process intensity rates  $\lambda_{ij}$ . Duration times of the process within the state space S are exponentially distributed.

Two fundamental phases of toll booth maintenance were distinguished in terms of its operation. Namely the operation phase, under which the  $S_1$  state falls and the servicing phase containing  $S_2$  and  $S_3$  states as illustrated in figure 1.



**Fig. 1. Maintenance process of a toll booth**

The model presented above and description of the maintenance process for the manual toll collection system does not include demand for replacement elements necessary to assure required reliability of the toll booth. Therefore a  $\Psi$  factor was introduced, which defines linear relationship between toll booth's availability rate and the number of replacement elements. It was assumed that values of the  $\Psi$  factor fall within closed interval  $\Psi \in <0,1>$ , where:

- $\Psi = 1$  the number of replacement elements assuring required reliability of toll booth,
- $\Psi = 0$  no replacement elements available.

Hence the shape of the graph in figure 2.

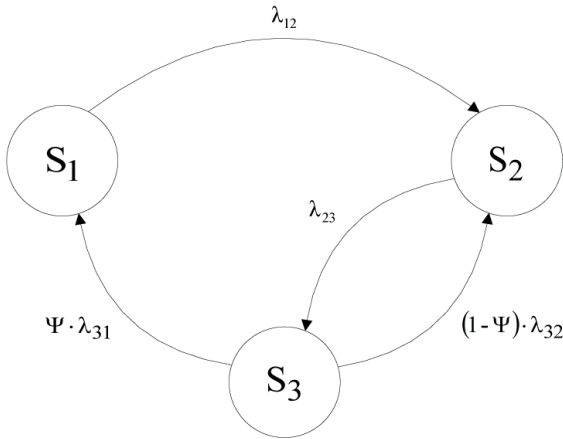


Fig. 2. The maintenance process of toll booth including transition rate between operational states and demand factor for replacement elements

The Chapman–Kolmogorov equations for the toll booth's maintenance process depicted in figure 2 are as follows:

$$\begin{cases} -\lambda_{12} \cdot P_1 + \Psi \cdot \lambda_{31} \cdot P_3 = 0 \\ -\lambda_{23} \cdot P_2 + \lambda_{12} \cdot P_1 + (1 - \Psi) \cdot \lambda_{32} \cdot P_3 = 0 \\ -\Psi \cdot \lambda_{31} \cdot P_3 + \lambda_{23} \cdot P_2 - (1 - \Psi) \cdot \lambda_{32} \cdot P_3 = 0 \end{cases} \quad (2)$$

One of the equations was substituted by normalisation condition:

$$\begin{cases} -\lambda_{12} \cdot P_1 + \Psi \cdot \lambda_{31} \cdot P_3 = 0 \\ \lambda_{12} \cdot P_1 - \lambda_{23} \cdot P_2 + (1 - \Psi) \cdot \lambda_{32} \cdot P_3 = 0 \\ P_1 + P_2 + P_3 = 1 \end{cases} \quad (3)$$

Matrix notation of the equations is the following:

$$\underbrace{\begin{bmatrix} -\lambda_{12} & 0 & \Psi \cdot \lambda_{31} \\ \lambda_{12} & -\lambda_{23} & (1 - \Psi) \cdot \lambda_{32} \\ 1 & 1 & 1 \end{bmatrix}}_A \cdot \underbrace{\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}}_P = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_B \quad (4)$$

where:

- A - matrix of factors,
- P - column matrix (steady-state probabilities of the system),
- B - row matrix of absolute terms.

The following formulae describe probabilities of manual toll collection system in boundary states given  $P = A^{-1} \cdot B$  :

$$\begin{aligned} P_1 &= \frac{\Psi \cdot \lambda_{31} \cdot \lambda_{23}}{\lambda_{12} \cdot \lambda_{23} + \lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{12} \cdot \lambda_{31} - \Psi \cdot \lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{31} \cdot \lambda_{23}} \\ P_2 &= \frac{\lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{12} \cdot \lambda_{31} - \Psi \cdot \lambda_{12} \cdot \lambda_{32}}{\lambda_{12} \cdot \lambda_{23} + \lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{12} \cdot \lambda_{31} - \Psi \cdot \lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{31} \cdot \lambda_{23}} \\ P_3 &= \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{12} \cdot \lambda_{23} + \lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{12} \cdot \lambda_{31} - \Psi \cdot \lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{31} \cdot \lambda_{23}} \end{aligned} \quad (5)$$

Availability rate of toll booth described by operational states graph in fig. 2 equals the probability of toll booth in state of operation S1:

$$A_{ST\_E\_Psi} = P_1 = \frac{\Psi \cdot \lambda_{31} \cdot \lambda_{23}}{\lambda_{12} \cdot \lambda_{23} + \lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{12} \cdot \lambda_{31} - \Psi \cdot \lambda_{12} \cdot \lambda_{32} + \Psi \cdot \lambda_{31} \cdot \lambda_{23}} \quad (6)$$

The hereunder analysis was conducted to determine demand for toll booth replacement elements necessary to assure required toll booth reliability factoring in accumulative outlays incurred to operate the booth correctly and projected failure rates of individual elements.

Prices of individual elements and accumulative outlays spent on necessary replaceable elements were determined on the basis of reports concerning maintenance and expansion of toll plazas.

A toll booth is a system whose reliability structure is serial. It was assumed that probability of two elements of the same toll booth failing simultaneously is a near zero. When a toll booth element fails it is replaced by a new one. A toll booth consists of seven elements whose projected reliability parameters were presented in paper [2]. A general Chapman–Kolmogorov system of differential equations was used to describe a booth [1], [6]:

$$\begin{cases} \frac{dR(t)}{dt} = -R(t) \cdot \sum_{i=1}^N \lambda_i(t) = -R(t) \cdot \lambda(t) \\ \frac{dQ_i(t)}{dt} = \lambda_i(t) \cdot R(t) \quad \text{dla } i = 1, 2, 3, \dots, 7. \end{cases} \quad (7)$$

and:

$$R(t) = \exp \left[ -t \cdot \left( \sum_{i=1}^N \lambda_i(t) \right) \right] = \exp \left[ -t \cdot \left( \sum_{i=1}^7 \lambda_i(t) \right) \right] \quad (8)$$

$$Q_i(t) = \frac{\lambda_i(t)}{\sum_{i=1}^N \lambda_i(t)} \cdot [1 - R(t)], \quad i = 1, 2, 3, \dots, 7. \quad (9)$$

where:

$Q_i(t)$  - probability of toll booth failure due to failure of its  $i$ -th element,  $i=1,2,3,\dots,7$ ,

$R(t)$  - probability of all booth elements in state of full ability,

$\lambda_i(t)$  - function of failure rate of  $i$ -th toll booth element.

In accordance with assumptions formulated in paper [2], durations of operational states are exponentially distributed random variables, thus:

$$Q_i = \lim_{t \rightarrow \infty} Q_i(t) \quad (10)$$

so for :  $\lambda_i(t) = \lambda_i = \text{const}$ ,  $i = 1, 2, \dots, 7$

$$Q_i = \frac{\lambda_i}{\sum_{i=1}^N \lambda_i} \quad (11)$$

Mean number of booth's replacement elements per mean element price is given by:

$$\bar{m} = \frac{N \prod_{i=1}^N m_i^{Q_i}}{\sum_{i=1}^N C_i m_i} \quad (12)$$

where:

$m_i$  - number  $i$ -th type elements,  $i=1,2,3...7$ ,

$C_i$  - price of one  $i$ -th type element,  $i=1,2,3...7$ ,

$Q_i$  - probability of toll booth failure due to failure of its  $i$ -th element,  $i=1,2,3...7$ . By simplifying  $\bar{m}$  we get:

$$\frac{d \ln \bar{m}}{d m_i} = 0 \quad (13)$$

thus:

$$\frac{Q_i}{m_i} = \frac{C_i}{C} \quad (14)$$

hence

$$\frac{Q_i}{m_i \cdot C_i} = \frac{1}{C} \quad (15)$$

$$m_i = \frac{C \cdot Q_i}{C_i} \quad \text{for } i=1,2,3 \quad (16)$$

where:

$m_i$  - number of  $i$ -th type replacement elements purchased at total outlays  $C$  in order to assure maximum reliability  $R(t)$ , given  $C_i$  price of one  $i$ -th type replacement element.

The required value of reliability of a single toll booth [2], assuming its serial reliability structure is  $R(t)=0,368$ . For instance, in paper [2], the number of  $i$ -th type elements (induction loop) purchased at total outlays  $C$  to assure required reliability of toll booth was:

$$m_1 = \frac{C \cdot Q_1}{C_1} = \frac{C \cdot \left( \frac{\lambda_1}{2,544 \cdot 10^{-4}} \cdot 0,632 \right)}{C_1} = \frac{C \cdot \left( \frac{10,07 \cdot 10^{-6}}{2,544 \cdot 10^{-4}} \cdot 0,632 \right)}{C_1} = \frac{5 \cdot 10^5 \cdot \left( \frac{10,07 \cdot 10^{-6}}{2,544 \cdot 10^{-4}} \cdot 0,632 \right)}{1 \cdot 10^5} = 0,125 \approx 0 \quad (17)$$

where:

- price of induction loop element  $C_1 = 100\ 000$  PLN ,
- accumulative outlays necessary for maintenance of toll booth  $C = 500\ 000$  PLN.

Table 2 collates toll booth replacement elements necessary to assure required toll booth reliability  $R(t) = 0,368$  given accumulative outlays incurred to operate the booth correctly and projected failure rates of individual elements presented in paper [2].

**Table 2. Number of replacement elements for MSP toll booth necessary to assure required reliability**

Element description	Quantity [items]	Price [PLN]
Induction loop	0	100 000
Traffic gate	6	10 000
Traffic lights	11	5 000
Information display	1	3 000
Cash register	6	1 000
Power supply	1	100 000
Traffic lane monitoring	1	100 000

Maintenance strategy maximising the availability rate factoring in demand for replacement elements and actual financial outlays incurred for maintenance was developed thanks to presented herein rationalisation of manual toll collection system maintenance process based on example of a toll booth.

## 4. Conclusion

Issues related to chosen operational states of analysed manual toll collection system were presented through rationalisation of a maintenance process. Demand for toll booth replacement parts was analysed and a factor taking into account financial outlays incurred was introduced. Maintenance strategy maximising the availability rate factoring in demand for replacement elements and actual financial outlays incurred by maintenance was developed. Thanks to that strategy decision makers (toll road management) responsible for toll collection plazas may rationalise their maintenance spending and optimise availability rates depending on budget for replacement parts.

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