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Applying Graph Theory Terms to Description of VTS

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ABSTRACT: The paper presents an example of applying graph theory notation to description of a VTS; it also contains some remarks on applicability of such notation for marine traffic systems

1 INTRODUCTION

Nowadays - as VTSes are growing in number and their area is still expanding - it may be advisable to take into account description of marine traffic systems in terms of graph theory, with the purpose of finding its advantages or disadvantages. A model of a VTS, assumed to illustrate an application of graph theory formalism, need not be very complex (though it is the formalism invented for depicting and analysing various traffic systems of great complexity).

2 EXAMPLE OF DESCRIPTION

Let system in consideration comprises two fairways leading to pilot station, two anchorages and one fairway from pilot station to port entrance. Its graph representation is shown in Fig.1

Graph arcs can also have a numerical notation, for instance: a $\rightarrow$ (1,2), b $\rightarrow$ (2,1), c $\rightarrow$ (2,4), d $\rightarrow$ (4,2), e $\rightarrow$ (3,2), f $\rightarrow$ (2,3), g $\rightarrow$ (3,5), i $\rightarrow$ (5,2), j $\rightarrow$ (4,6), m $\rightarrow$ (6,2).

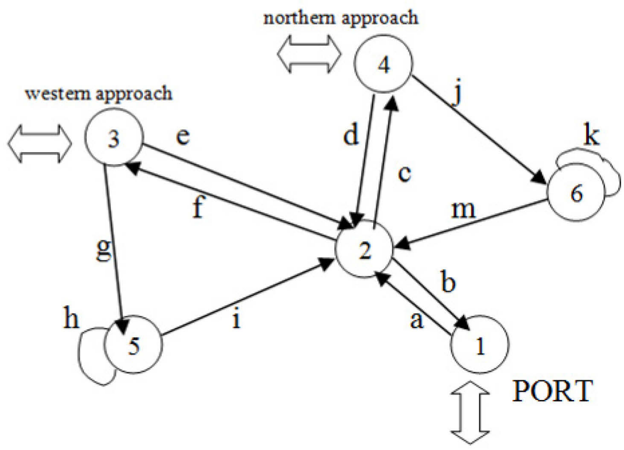

Figure 1. VTS graph:
vertex 1 - port entrance
vertex 2 - pilot station
vertex 3 - western reporting point
vertex 4 - northern reporting point
vertex 5 - western anchorage
vertex 6 - north-eastern anchorage
arcs (directed branches) a,b,c,d,e,f,g,i,j,m
- stand for traffic lanes
loops h,k - for denoting anchored vessels

Incidence matrix **J** of the graph and its binary matrix of adjacency **A** are as follows:

$$J = \begin{bmatrix} +1 & -1 & 0 & 0 & 0 & 0 \\ -1 & +1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & +1 & 0 & 0 \\ 0 & +1 & 0 & -1 & 0 & 0 \\ 0 & -1 & +1 & 0 & 0 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & +1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & +1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

(In incidence matrix: +1 denotes arc directed towards the vertex, 2 symbolizes the loop).

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

The graph has no edges, which means that along each traffic lane vessels may proceed in one direction only.

With the allowance for the established direction of traffic flow, the adjacency matrix **A** transforms into matrix **B**:

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

At any moment a traffic in the system can generally be characterized by flow matrix **F** determining the number of vessels which have departed from a point (matrix $\mathbf{F}_O$ of outcoming traffic, with graph vertices as its sources) or vessels making for a point (matrix $\mathbf{F}_I$ of incoming traffic, vertices as outlets).

Let an example traffic distribution (for the system in consideration) be given by flow matrix **F** in one of the following forms:

$$F_O = \begin{bmatrix} 0 & 6 & 0 & 0 & 0 & 0 \\ 4 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 & 0 & (0) \\ 0 & 1 & 0 & 0 & (3) & 0 \\ 0 & 2 & 0 & 0 & 0 & (4) \end{bmatrix}$$

or

$$F_I = \begin{bmatrix} 0 & 4 & 0 & 0 & 0 & 0 \\ 6 & 0 & 1 & 3 & 1 & 2 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & (3) & 0 \\ 0 & 0 & 0 & (0) & 0 & (4) \end{bmatrix}$$

where numerals in brackets (3),(4) denote vessels awaiting at anchor and symbol (0) indicates that there is no traffic in the lane.

(It is easy to notice, that $\mathbf{F}_O^T = \mathbf{F}_I$, that is each matrix is transposition of the other.)

Instead of matrices $\mathbf{F}_O$, $\mathbf{F}_I$ (related to vertices) there can be used matrix $\mathbf{F}_B$ for all graph branches:

$$F_B = \begin{bmatrix} -6 & +6 & 0 & 0 & 0 & 0 \\ +4 & -4 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & +1 & 0 & 0 \\ 0 & +3 & 0 & -3 & 0 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 \\ 0 & -2 & +2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & (3) & 0 \\ 0 & +1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (4) \\ 0 & +2 & 0 & 0 & 0 & -2 \end{bmatrix} \begin{matrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \\ j \\ k \\ m \end{matrix}$$

where algebraic signs mark the direction of an arc (minus, if directed from a vertex, plus – if towards it); the numbers of vessels at anchor in brackets (as above).

The traffic network defined by matrices $\mathbf{F}_O$, $\mathbf{F}_I$ or $\mathbf{F}_B$ is illustrated in Fig.2, (next page), where:

vertices – 1,2,3,4,5,6;

arcs (traffic lanes) – a,b,c,d,e,f,g,i,m;

(in brackets the number of ships underway);

arc (lane) – j (with no traffic);

loops – h & k (in brackets the number of vessels at anchor).

Matrices $\mathbf{F}_O$, $\mathbf{F}_I$ or $\mathbf{F}_B$ and its graph representation constitute a very general description, however. To give more detailed information, each arc of the graph (i.e. each traffic lane in the system) should have its ascribed vector of state which, at any given moment, characterizes the traffic flow in the lane. (And similarly, each vertex can be described by its state vector as well.)

For instance, state vectors describing port approach fairway at a chosen moment could be as follows:

a (lane 1,2):
$[\ 0^{H}15^{M}, 0^{H}40^{M}, 0^{H}55^{M}, 1^{H}25^{M}, 1^{H}55^{M}, 2^{H}10^{M}\ ]$,

b (lane 2,1):
$[\ 0^{H}20^{M}, 0^{H}45^{M}, 1^{H}10^{M}, 2^{H}15^{M}\ ]$,

where vector **a** (1,2), for 6 vessels proceeding to pilot station, gives remaining time to go for each of them and vector **b** (2,1), for 4 vessels approaching port entrance – remaining time to enter the harbour.

Exemplary state vectors for anchorages,

h (5,5):
$[\ 3^{H}15^{M}, 6^{H}30^{M}, 10^{H}00^{M}\ ]$

and **k** (6,6):
$[\ 0^{H}30^{M}, 2^{H}45^{M}, 8^{H}00^{M}, 12^{H}00^{M}\ ]$,

define time to wait at anchor, for each vessel. (For vertices, which are junction nodes of the traffic system, the notion "state" may mean whether the node is accessible and passable, or not.)

Of course, the examples given are the simplest ones. The vectors of state, if necessary, may include many more particulars, such as next destination point or allotted berthing place, kind and amount of cargo, some ship's data, existing restrictions and constrains etc. (And for such "vertex", as p o r t , the state of the "point" may depend, in very complex and sophisticated way, on internal port traffic, cargo handling operations and other technical and economical factors.)

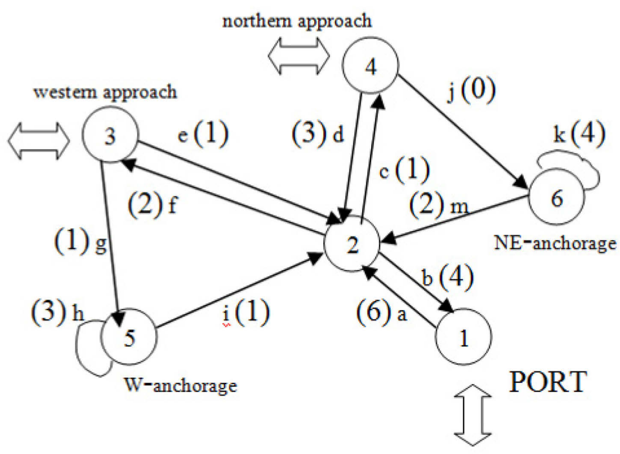

Figure 2. Graph of traffic

Vectors of state of every traffic lane and waiting area (**a**, **b**, **c**, **d**, **e**, **f**, **g**, **h**, **i**, **j**, **k**, **m**) together with state vectors of vertices ($\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, ..., $\mathbf{v}_6$) and flow matrix **F** define the state of the whole system.

Transformation of state may be determined by two sets of functions:

$\{\mathbf{v}_x(t)\}$ for vertices and $\{\mathbf{w}_u(t)\}$ for arcs,

where t denotes time;

(in considered example of traffic system, index x: 1,2,3,4,5,6 and u: a,b,c.d,e,f,g,h,i,j,k,m);

these functions also implicate transformation of flow matrix: $\mathbf{F}(t) = \mathbf{F}(\{\mathbf{v}_x(t)\}, \{\mathbf{w}_u(t)\})$.

In general, transformation functions are deterministic, but they may include statistical parameters and random variables as well, or be stochastically modified. It would be useful to reckon and apply such transformation operators **W**, **V**, **T**, that:

$\mathbf{w}_u(t) = \mathbf{W}(t, t_o)\ \mathbf{w}_u(t_o)$,

$\mathbf{v}_x(t) = \mathbf{V}(t, t_o)\ \mathbf{w}_u(t_o)$,

$\mathbf{F}(t) = \mathbf{T}(t, t_o)\ \mathbf{F}(t_o)$ or $\mathbf{F}(t) = \mathbf{T}(t)\ \mathbf{B}$

Finding affine forms

W, **V**, **T**,

however, is not an easy task, as usually the problem is non-linear, or the attempts to solve it may entail the necessity of inversion of a singular matrix.

3 FINAL REMARKS

Graph description of traffic systems is inseparably associated with matrix algebra formalism. A major practical difficulty with application of this description, as it seems now, is the problem of finding linear (matrix) operators for transformation of state of the depicted system. Searching for a solution may be done in the way of decomposing the transformation into a few stages, doing indispensable simplifications and finally introducing such variables and parameters (resulting from the intermediate stages of transformation), which – albeit somewhat artificial – make possible to express transformation of state by required matrix operators. It is clear, that such decomposition can not be excessive (too many stages of transformation may turn one complex problem into another) and also that undue simplifications may affect negatively the result of transformation.

All of these may hinder the application of graph description to marine traffic systems.

On the other hand, however, its expected advantages are obvious. Matrix notation is especially suitable for real-time automatic data processing and ensure obtaining requested information quickly and easily.

As to problems with creation of dynamic graph models of traffic systems, which may arise in case of very complex and extensive systems – they can be overcome gradually: by proceeding from simplest version of the description towards more sophisticated ones.

The existing possibilities of simulation experiments and examining the effects of theoretical investigations by simulator tests shall make it manageable.

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