INFORMATION SYSTEMS IN MANAGEMENT

Information Systems in Management (2018) Vol. 7 (1) 35−46
DOI: 10.22630/ISIM.2018.7.1.4

FUZZY RECURSIVE RELATIONSHIPS IN RELATIONAL DATABASES

KRZYSZTOF MYSZKOROWSKI

Institute of Information Technology, Lodz University of Technology

Recursive relationships are used for modelling problems coming from the real life, such as, for example, a relationship describing formal dependencies between employees of an enterprise, where creation of work groups and teams requires analysis of many elements. In conventional database systems, the precision of data is assumed. If our knowledge of the fragment of reality to be modelled is imperfect one should apply tools for describing uncertain or imprecise information. One of them is the fuzzy set theory. The paper deals with recursive relationships in fuzzy databases. The analysis is performed with the use of the theory of interval-valued fuzzy sets. A definition of a fuzzy interval recursive relationship has been presented. The paper defines different connections of entities which participate in such relationships. Operations of the extended relational algebra are also discussed.

Keywords: Fuzzy databases, interval-valued fuzzy sets, recursive relationships, relational algebra, fixpoint operator

1. Introduction

Conventional database systems are designed with the assumption of precision of information collected in them. The problem becomes more complex if our knowledge of the fragment of reality to be modelled is imperfect. In such cases one has to apply tools for describing uncertain or imprecise information [4, 10]. One of them is the fuzzy set theory. A fuzzy set (FS) is a generalization of an ordinary set. Its definition contains a membership function which is a mapping $X \rightarrow [0,1]$, where

X denotes the universe of discourse. The precise determination of the membership degree is not always possible. In such cases it can be expressed by means of the closed subinterval of the unit interval [0,1]. This leads to the concept of the interval-valued fuzzy set (IVFS). The idea was proposed in 1975 [14, 15]. Alcade et al. defined interval-valued linguistic variables and studied their properties [1]. Validity of the principle of inclusion and exclusion for IVFSs has been investigated in [12].

So far, a great deal of effort has been devoted to the development of fuzzy data models. Numerous works discuss how uncertainty existing in databases should be handled. Some authors proposed incorporating fuzzy logic into data modeling techniques. In particular an extension of the entity-relationship model by fuzzy logic has been proposed. The main elements of the model were presented in the context of fuzzy sets.

In the relational model entity sets and relationships between them are represented in the same way – i.e. by means of relations. A relation is defined as a subset of Cartesian product of attribute domains. A set of operations in relational model is referred to as relational algebra. Inclusion of fuzzy logic requires extension of the main notions occurring in the relational model. In particular, classical operations of relational algebra must be appropriately modified.

In database models, usually binary relationships occur between entity sets. In such relationships each entity plays a unique role. The roles of entities are clear and so they are not defined explicitly. When designing, it may be necessary to define relationships which involve only one entity set i.e. relationships between entities belonging to the same set. They are referred to as recursive relationships (RRs). In this case roles of entities must be defined explicitly.

The problem of RRs does not occur often in the professional literature. They are usually discussed in papers dealing with the entity-relationship model. Different approaches to design RRs have been presented in [2, 3]. A taxonomy of RRs has been described in [8]. The author presented different relationship structures that that can exist in RRs and defined four types of them. In [7] theorems dealing with cardinalities of RRs have been formulated.

Although the semantics of RRs is to some extent difficult to capture they are useful for modelling problems coming from the real life. In particular one can apply them in management. An example of a RR is a relationship describing formal dependencies between employees of an enterprise. If each employee, apart from the boss, is supervised by exactly one employee there exists a multilevel hierarchy. At the bottom there are employees who do not supervise anybody. Participation of employees in work groups can be limited by different constraints. For example, one can impose a constraint on the number of teams supervised by one employee. These constraints determine the type of the recursive relationship. Creation of work groups and teams in organizations requires analysis of many elements e.g. qualifications of employees and possibilities of using them.

The fuzzy set theory can be also applied for modelling of cooperation between enterprises [6,11,13]. Enterprises which cooperate with one another are elements of a business network. The strength of a direct connection between two enterprises can be estimated by a subinterval of [0,1]. This is a case of the interval-valued fuzzy relationship (IVFRR). Analysis of the network enables to determine relationships between any enterprises.

In conventional procedures there are made unique decisions „accept or reject" which may lead to improper solutions. The estimation of the mentioned elements is not always unique. Applying of the fuzzy set theory allows for analysis and comparison of different versions and for adjusting them to new information.

The paper is organized as follows. The next section contains the basic notions dealing with interval-valued fuzzy sets. Section 3 presents fuzzy relations and operations of the extended relational algebra. Different kinds of structures created by entities occurring in interval-valued fuzzy relationships are defined in Section 4. Section 5 discusses algebraic operations dealing with relational representation of IVRRs.

2. Interval-valued fuzzy sets

In classical set theory one can define a characteristic function which indicates membership of elements in sets. It is a mapping $X \to \{0,1\}$, where X denotes the universe of discourse. The characteristic function of the set A takes the value 1 for the element e if $e \in A$ and 0 in the opposite case. However, if there are no sharp boundaries of membership the unique qualification of elements is not always obvious. In order to express this uncertainty the set {0,1} has been replaced with the interval [0,1].

Definition 1. Let X be a universe of discourse. A fuzzy set A in X is defined as a set of ordered pairs:

$$A = \{< x, \mu_A(x) >: x \in X, \mu_A: X \to [0,1]\} , \tag{1}$$

where $\mu_A(x)$ is a membership function.

Replacing the mapping $X \to [0, 1]$ with $X \to F([0, 1])$, where $F([0, 1])$ denotes the set of all FSs in the interval [0, 1], leads to the definition of type-2 fuzzy sets. A particular case of this notion is the concept of an interval-valued fuzzy set [9]. The elements of the IVFS are assigned with closed subintervals of [0,1]. The idea of the IVFS extends the notion of the ordinary fuzzy set. The assigned intervals approximate the correct value of membership degrees.

Definition 2. Let X be a universe of discourse. An interval-valued fuzzy set A in X is defined as:

$$A = \{< x, \mu_A(x) >: x \in X, \mu_A: X \to Int([0,1])\} \quad , \tag{2}$$

where $\mu_A(x) = [\mu_{A_L}(x), \mu_{A_U}(x)]$ is an interval-valued membership function and $Int([0,1])$ stands for the set of all closed subintervals of [0,1]: $Int([0,1]) = \{[a, b]: a, b \in [0,1]\}$.

Values of $\mu_{A_L}(x)$ and $\mu_{A_U}(x)$ are interpreted as the lower and upper membership functions, respectively, and satisfy the following condition:

$$0 \le \mu_{A_L}(x) \le \mu_{A_U}(x) \le 1 \ . \tag{3}$$

In order to compare interval-valued fuzzy sets one has to establish an order relations for subintervals of [0,1]. In further considerations we will consider the following ordering

$$[a_L, a_U] \le [b_L, b_U] \ \text{ if } \ a_L \le b_L \text{ and } a_U \le b_U \quad . \tag{4}$$

If $\mu_{A_L}(x) = \mu_{A_U}(x)$ for every x then A is an ordinary fuzzy set. Membership functions $\mu_{A_L}(x)$ and $\mu_{A_U}(x)$ determine two ordinary fuzzy sets:

$$\begin{aligned} A_L &= \{< x, \mu_{A_L}(x) >: x \in X, \mu_{A_L}: X \to [0,1]\} \quad , \\ A_U &= \{< x, \mu_{A_U}(x) >: x \in X, \mu_{A_U}: X \to [0,1]\} \quad . \end{aligned} \tag{5}$$

The support of A - the classical set of elements that belong to A with non-zero membership degrees - is determined by supports of A_L and A_U :

$$\begin{aligned} \mathrm{supp}(A)_L &= \mathrm{supp}(A_L) = \{x : \mu_{A_L}(x) > 0\} \quad , \\ \mathrm{supp}(A)_U &= \mathrm{supp}(A_U) = \{x : \mu_{A_U}(x) > 0\} \quad . \end{aligned} \tag{6}$$

From (5) and (6) we can conclude that $\mathrm{supp}(A)_L \subseteq \mathrm{supp}(A)_U$. Other characteristics can be determined in the similar way.

3. Fuzzy relations

In the fuzzy entity-relationship model fuzziness can occur at three levels [5]: a) the level of types reflecting partial belongingness of the given type to the model, b) the level of entities and relationships reflecting partial belongingness of elements to their types, c) the level of attributes reflecting fuzzy values of them. The presented considerations deal with the second level.

In the relational model data is represented by tuples, which are elements of relations. Allowing for subintervals of [0,1] to appear as membership degrees of tuples one arrives at interval-valued fuzzy relations (IVFR). An IVFR is an interval-valued fuzzy set on the Cartesian product of attribute domains. For example an IVFR regarding projects may look like relation LARGE_PROJECTS in Table 1.

Table 1. Relation LARGE_PROJECTS

P#	NAME	BUDGET	TYPE	μ
P1	Credit	110 000	Banking	[1,1]
P2	Finn	60 000	Accounting	[0.8, 0.9]
P3	Policy	40 000	Insurance	[0.4, 0.6]
P4	Broker	30 000	Stock_Exchange	[0.3, 0.5]
P5	Visa	200 000	Banking	[1,1]
P6	Balance	20 000	Accounting	[0.2, 0.3]
P7	Agent	50 000	Insurance	[0.7, 0.8]

Classical operations of relational algebra must be extended by defining membership degrees for final relations. Some of the operations are defined below. Let R and S be interval-valued fuzzy relations with membership degrees $[\mu_{R_L}(r), \mu_{R_U}(r)] \subseteq [0,1]$ and $[\mu_{S_L}(s), \mu_{S_U}(s)] \subseteq [0,1]$, where r and s denote tuples of R and S, respectively.

Union: $R \cup S$ is an IVFR containing tuples which belong to R or S. The membership degree of a tuple t of $R \cup S$ equals $\mu_{R\cup S}(t) = [\mu_{R\cup S_L}(t), \mu_{R\cup S_U}(t)]$, where

$$\mu_{R\cup S_L}(t) = \max(\mu_{R_L}(t), \mu_{S_L}(t)) \ , \ \mu_{R\cup S_U}(t) = \max(\mu_{R_U}(t), \mu_{S_U}(t)) \ . \tag{7}$$

Intersection: $R \cap S$ is an IVFR containing tuples which belong to R and S. The membership degree of a tuple t of $R \cap S$ equals $\mu_{R\cap S}(t) = [\mu_{R\cap S_L}(t), \mu_{R\cap S_U}(t)]$, where

$$\mu_{R\cap S_L}(t) = \min(\mu_{R_L}(t), \mu_{S_L}(t)) \ , \ \mu_{R\cap S_U}(t) = \min(\mu_{R_U}(t), \mu_{S_U}(t)) \ . \tag{8}$$

Difference: $R - S$ is an IVFR containing tuples which belong to R and do not belong to S . The membership degree of a tuple t of $R - S$ equals $\mu_{R\text{-}S}(t) = [\mu_{R\text{-}S_L}(t), \mu_{R\text{-}S_U}(t)]$, where

$$\mu_{R-S_L}(t) = \min(\mu_{R_L}(t),\ 1-\mu_{S_U}(t)) \ , \ \mu_{R-S_U}(t) = \min(\mu_{R_U}(t),\ 1-\mu_{S_L}(t)) . \tag{9}$$

Selection: $\sigma_W(R)$ is an IVFR containing tuples of R which satisfy a fuzzy condition W. Let $\mu_W(t) = [\mu_{W_L}(t), \mu_{W_U}(t)]$ be an interval expressing the fulfilment degree of W. Thus the membership degree of a tuple t of $\sigma_W(R)$ equals $\mu_{\sigma_W(R)}(t) = [\mu_{\sigma_W(R)_L}(t), \mu_{\sigma_W(R)_U}(t)]$, where

$$\mu_{\sigma_W(R)_L}(t) = \min(\mu_{R_L}(t),\ \mu_{W_L}(t)) \ , \ \mu_{\sigma_W(R)_U}(t) = \min(\mu_{R_U}(t),\ \mu_{W_U}(t)) \ . \tag{10}$$

Projection: $\prod_X(R)$ is an IVFR over X, where X is a set of attributes, with $\mu_{\prod_X}(t) = [\mu_{\prod_X(R)_L}(t), \mu_{\prod_X(R)_U}(t)]$, where

$$\mu_{\Pi_X(R)_L}(t) = \sup_{t(X)=r(X)} \mu_{R_L}(r) \ , \ \ \mu_{\Pi_X(R)_U}(t) = \sup_{t(X)=r(X)} \mu_{R_U}(r) \ . \tag{11}$$

Natural join: $R(X, Y) * S(Y, Z)$ is an IVFR of the scheme $SCH = \{X, Y, Z\}$. The membership degree of a tuple t of $R * S$ equals $\mu_{R*S}(t) = [\mu_{R*S_L}(t), \mu_{R*S_U}(t)]$, where

$$\begin{aligned} & r(Y) = s(Y), \\ & \mu_{R*S_L}(t) = \min(\mu_{R_L}(r), \mu_{S_L}(s)) \ , \quad \mu_{R*S_U}(t) = \min(\mu_{R_U}(r), \mu_{S_U}(s)) \ . \end{aligned} \tag{12}$$

4. Fuzzy interval-valued recursive relationships

A graphical representation of the recursive relationship in the entity-relationship model is shown in Figure 1. In order to distinguish entities of the entity set E which participate in the relationship R there have been defined roles R_A and R_B. Numbers in brackets denote cardinalities i.e. a (b) is a minimal (maximal) number of entities of E which fulfill the role R_A with respect to one entity of E.

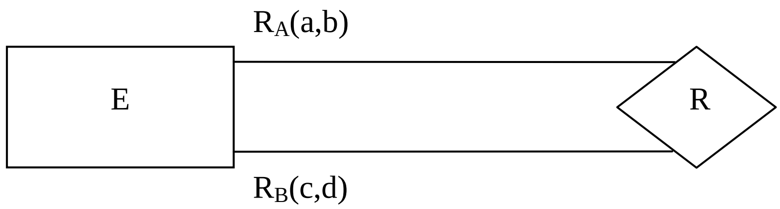

Figure 1. Recursive relationship

An interval-valued fuzzy recursive relationship (IVFRR) can be defined as follows:

$$R = \{< e_1, e_2, \mu_R(e_1, e_2) >: e_1, e_2 \in E, e_1 \neq e_2, \mu_R : E \times E \to Int([0,1])\} \ . \tag{13}$$

The order of entities defines their roles in R. In the relational model an IVFRR is represented by relation $R(R_A.K, R_B.K, \mu_R)$, where K denotes the key of E. The membership function $\mu_R(e_1, e_2)$ can express the strength of the relationship or estimates the degree of uncertainty. Tuples of R represent direct connections between entities. From (13) one can derive pairs of entities which are connected indirectly. If there exist tuples $<e_1, e_2, \mu_R(e_1, e_2)>$ and $<e_2, e_3, \mu_R(e_2, e_3)>$ then there exist a relationship between entities e_1 and e_3 with $\mu_{R_L}(e_1, e_3) = \min(\mu_{R_L}(e_1, e_2), \mu_{R_L}(e_2, e_3))$ and $\mu_{R_U}(e_1, e_3) = \min(\mu_{R_U}(e_1, e_2), \mu_{R_U}(e_2, e_3))$.

According to cardinalities one can distinguish the following types of recursive relationships:

a) 1:1 with possible cardinalities: (0,1)(0,1), (0,1)(1,1), (1,1)(0,1), (1,1)(1,1),

b) 1:N with possible cardinalities: (0,N)(0,1), (0,N)(1,1), (1,N)(0,1), (1,N)(1,1), (0,1)(0,N), (0,1)(1,N), (1,1)(0,N), (1,1)(1,N),
c) M:N with possible cardinalities: (0,N)(0,M), (0,N)(1,M), (1,N)(0,M), (1,N)(1,M).

The structure of recursive relationships can be presented graphically as single lines for 1:1 type, hierarchies for 1:N type and networks for M:N type. If entities cannot fulfill both roles the structure of the relationship contains two levels.

Definition 3. Let *f* and *g* be entities of the set *E*. Let α be a subinterval of [0,1]. Entities *f* and *g* are α-connected with respect to the relationship *R*, denoted by $fC_{R\alpha}g$, if $\mu_R(f, g) \geq \alpha$ or there exists a finite sequence $e_1, e_2,..., e_n \in E$ such that

$$\mu_R(f, e_1) \geq \alpha, \mu_R(e_1, e_2) \geq \alpha, ..., \mu_R(e_{n-1}, e_n) \geq \alpha, \mu_R(e_n, g) \geq \alpha \ . \tag{14}$$

Definition 3 deals with entities occurring in the same path. In hierarchical structures between two elements there exists exactly one route containing one or two paths. Let us consider two pairs (*e*, *f*) and (*e*, *g*) of α-connected entities. Thus, $eC_{R\alpha}f$ and $eC_{R\alpha}g$. Entities *f* and *g* occur in the same hierarchy. For this case one can define an α-hierarchical connection.

Definition 4. Let *f* and *g* be entities of the set *E*. Let α be a subinterval of [0,1]. Entities *f* and *g* are α-hierarchically connected with respect to the relationship *R*, denoted by $fH_{R\alpha}g$, if $fC_{R\alpha}g$ or there exists an entity *e* such that , $eC_{R\alpha}f$ and $eC_{R\alpha}g$.

In more complex structures there can exist more routes between two elements. A route can contain more than one hierarchical connection. For this case one can define an α-network connection.

Definition 5. Let *f* and *g* be entities of the set *E*. Let α be a subinterval of [0,1]. Entities *f* and *g* are α-network connected with respect to the relationship *R*, denoted by $fN_{R\alpha}g$, if $fH_{R\alpha}g$ or there exists a finite sequence $e_1, e_2,..., e_n \in E$ such that $fH_{R\alpha}e_1$, $e_1H_{R\alpha}e_2$,..., $e_1H_{R\alpha}e_n$ and $e_nH_{R\alpha}g$ or $gC_{R\alpha}e_n$.

5. Recursive operations

Let us consider an IVFR *R*(*A*,*B*) which represents an interval-valued recursive relationship. Values of attributes *A* and *B* are identifiers of related entities.

Let $t_1(a, x)$ and $t_2(a, y)$ be two tuples of *R*. This means that entities identified by *x* and *y* fulfill the same role with respect to the other entity identified by *a*.

Identifiers *x* and *y* can be obtained by means of the following operation:

$$T(X,Y) = \Pi_{X,Y}(\rho_{R(A,X)}(R) *_{X \neq Y} \rho_{R(A,Y)}(R)) \quad , \tag{15}$$

where ρ is a rename operator. The membership degree $\mu_T(s) = [\mu_{T_L}(s), \mu_{T_U}(s)]$ of a tuple t $(x,y) \in T$ equals:

$$\mu_{T_L}(x,y) = \max_a \min(\mu_{R_L}(a,x), \mu_{R_L}(a,y)) \ , \\ \mu_{T_U}(x,y) = \max_a \min(\mu_{R_U}(a,x), \mu_{R_U}(a,y)) \ . \tag{16}$$

Let $t_1(x, z)$ and $t_2(z, y)$ be two tuples of R. This means that entity x fulfills a certain role with respect to z and z fulfills the same role with respect to y. Thus entities x and y are connected indirectly. In order to get pairs of entities which are connected in this way one should also apply the join and projection operators. The following expression allows to find pairs of entities x and y which are connected by one intermediate entity z:

$$T(X,Y) = \Pi_{X,Y}(\rho_{R(X,Z)}(R) * \rho_{R(Z,Y)}(R)) \ . \tag{17}$$

The route between entities x and y contains two edges : $<x,z>$ and $<z,y>$. In network connections there can exist more than one route between x and y. Thus, the membership degree of a tuple t $(x,y) \in T$ equals:

$$\mu_{T_L}(x,y) = \max_z \min(\mu_{R_L}(x,z), \mu_{R_L}(z,y)) \ , \\ \mu_{T_U}(x,y) = \max_z \min(\mu_{R_U}(x,z), \mu_{R_U}(z,y)) \ . \tag{18}$$

In order to receive all pairs of entities $<u,v>$ which are connected by more intermediate entities one should solve the following equation:

$$S(U,V) = \rho_{R(U,V)}(\Pi_{A,V}(\rho_{R(A,U)}(R) * S(U,V))) \cup \rho_{R(U,V)}(R) \ . \tag{19}$$

The smallest relation which satisfies (19) is determined by the fixpoint operator (FP) :

$$S(U,V) = \text{FP}(S = \rho_{R(U,V)}(\Pi_{A,V}(\rho_{R(A,U)}(R) * S(U,V))) \cup \rho_{R(U,V)}(R)) \ . \tag{20}$$

The result is obtained by means of the iterative fixpoint process. In each iteration new tuples s of S are created. They denote pairs of connected entities. The algorithm of FP terminates when no new tuples of S are created as a result of the subsequent iteration. If R is a fuzzy relation for each iteration one must determine the membership degree. Let $C(u,v)$ be a set of routes s between entities u and v. For each route $s \in C(u,v)$ one must determine values of $\mu_S(s) = [\mu_{S_L}(s), \mu_{S_U}(s)]$. The final membership degree $\mu_S(u,v) = [\mu_{S_L}(u,v), \mu_{S_U}(u,v)]$ equals the maximal value:

$$\mu_{S_L}(u,v) = \max_{s \in C(u,v)} \mu_{S_L}(s) \ , \ \mu_{S_U}(u,v) = \max_{s \in C(u,v)} \mu_{S_U}(s) \ . \tag{21}$$

Example 1. Relation R (table 2) presents an IVFRR. Entities which participate in the relationship form a structure shown in Figure 2.

Table 2. Relation R

A	B	μ
a	b	μ_1
a	c	μ_2
b	c	μ_3
b	d	μ_4
c	d	μ_5
c	e	μ_6
d	e	μ_7

The membership degrees μ_i are expressed by subintervals of [0,1]. Let $\mu_{i_1,i_2,\ldots,i_n}$ denote the following interval:

$$\mu_{i_1,i_2,\ldots,i_n} = [\mu_{(i_1,i_2,\ldots,i_n)_L}, \mu_{(i_1,i_2,\ldots,i_n)_U}] \text{ , where}$$
$$\mu_{(i_1,i_2,\ldots,i_n)_L} = \min(\mu_{1_L}, \mu_{2_L}, \ldots, \mu_{n_L}) \text{ , } \mu_{(i_1,i_2,\ldots,i_n)_U} = \min(\mu_{1_U}, \mu_{2_U}, \ldots, \mu_{n_U}) \text{ .} \quad (22)$$

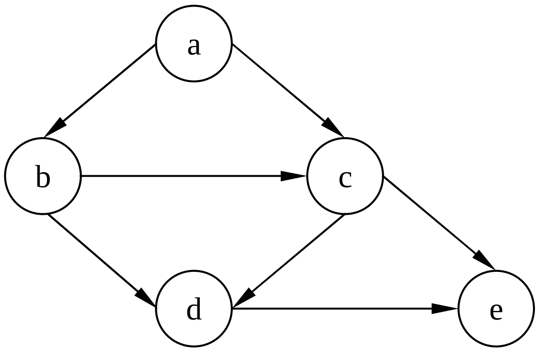

Figure 2. Structure of the IVFRR – Example 1

Table 3 presents results of the operation (15), (16).

Table 3. Relation T

X	Y	μ
b	c	μ_{12}
c	d	μ_{34}
d	e	μ_{56}

For the operation (20), (21) three iterations should be performed. The first iteration finds pairs of entities connected by one intermediate entity. There exist the following routes: <*abc*,μ_{13}>, <*abd*,μ_{14}>, <*acd*,μ_{25}>, <*ace*,μ_{26}>, <*bcd*,μ_{35}>, <*bce*,μ_{36}>, <*bde*,μ_{47}>, <*cde*,μ_{67}>. The result is shown in table 4.

Table 4. Relation S after the first iteration

U	V	μ
a	b	μ_1
a	c	$[\max(\mu_{2_L}, \mu_{13_L}), \max(\mu_{2_U}, \mu_{13_U})]$
b	c	μ_3
b	d	$[\max(\mu_{4_L}, \mu_{35_L}), \max(\mu_{4_U}, \mu_{35_U})]$
c	d	μ_5
c	e	$[\max(\mu_{6_L}, \mu_{57_L}), \max(\mu_{6_U}, \mu_{57_U})]$
d	e	μ_7
a	d	$[\max(\mu_{14_L}, \mu_{25_L}), \max(\mu_{14_U}, \mu_{25_U})]$
a	e	μ_{26}
b	e	$[\max(\mu_{36_L}, \mu_{47_L}), \max(\mu_{36_U}, \mu_{47_U})]$

The following routes with two intermediate entities are obtained by the second iteration: <*abcd*,μ_{135}>, <*abce*,μ_{136}>, <*abde*,μ_{147}>, <*acde*,μ_{257}>, <*bcde*,μ_{357}>. The third iteration gives one route: <*abcde*,μ_{1357}>. Table 5 presents the final result.

Table 5. Relation S

U	V	μ
a	b	μ_1
a	c	$[\max(\mu_{2_L}, \mu_{13_L}), \max(\mu_{2_U}, \mu_{13_U})]$
b	c	μ_3
b	d	$[\max(\mu_{4_L}, \mu_{35_L}), \max(\mu_{4_U}, \mu_{35_U})]$
c	d	μ_5
c	e	$[\max(\mu_{6_L}, \mu_{57_L}), \max(\mu_{6_U}, \mu_{57_U})]$
d	e	μ_7
a	d	$[\max(\mu_{14_L}, \mu_{25_L}, \mu_{135_L}), \max(\mu_{14_U}, \mu_{25_U}, \mu_{135_U})]$
a	e	$[\max(\mu_{26_L}, \mu_{136_L}, \mu_{147_L}, \mu_{257_L}, \mu_{1357_L}), \max(\mu_{26_U}, \mu_{136_U}, \mu_{147_U}, \mu_{257_U}, \mu_{1357_U})]$
b	e	$[\max(\mu_{36_L}, \mu_{47_L}, \mu_{357_L}), \max(\mu_{36_U}, \mu_{47_U}, \mu_{357_U})]$

4. Conclusion

In the paper properties of interval-valued fuzzy recursive relationships, their representation in fuzzy relational databases and respective operations of the extended relational algebra have been considered. Each pair of related entities is assigned with a subinterval of [0,1]. For processing of recursive data an extended fixpoint operator has been applied. According to roles of entities there have been defined different kinds of fuzzy connections occurring in IVFRRs. The presented theory is planned to be applied in real life tasks like supporting collaboration in enterprise networks, for example. Moreover, one can extend the considerations by application of type-2 fuzzy sets. Another line of future work is taking into account more than two roles fulfilled by entities in the recursive relationship.

REFERENCES

[1] Alcade C., Burusco A., Fuentes-Gonzales, R. (2010) *Interval-valued linguistic variables: an application to the L-fuzzy contexts with absent values*. International Journal of General Systems, 39(3), 255–270.

[2] ter Bekke J.H., Bakker J.A. (2003) *Modeling and Querying Recursive Data Structures Introduction*, Proceedings of the Seventh International Conference on Artificial Intelligence and Soft Computing, Banff, Canada.

[3] ter Bekke J.H., Bakker J.A. (2003) *Modeling and Querying Recursive Data Structures A Semantic Approach*, Proceedings of the Seventh International Conference on Artificial Intelligence and Soft Computing, Banff, Canada.

[4] Campana J., Medina J., Vila M. (2014) *Semantic Data Management Using Fuzzy Relational Databases*. In: Pivert O., Zadrożny S. (eds.) Flexible Approaches in Data, Information and Knowledge Management. Studies in Computational Intelligence, Springer International Publishing, vol. 497, 115-140.

[5] Chen G. (1998) *Fuzzy Logic in Data Modeling. Semantics, Constraints and Database Design*. Kluwer Academic Publishers, Boston.

[6] Chen C., Huang S. (2006) *Order fulfillment ability analysis in the supply-chain system with fuzzy operation times.* International Journal of Production Economics, 101(1), 185-193.

[7] Dullea J., Song I., Lamprou I. (2003) *An analysis of structural validity in entity-relationship modeling,* Data and Knowledge Engineering, 47(2), 167–205.

[8] Lee H. (1999) *Semantics of recursive relationships in entity-relationship model*, Infromation and Software Technology, 41(13), 877-886.

[9] Liang, Q., Mendel, J. M. (2000) *Interval Type-2 Fuzzy Logic Systems*, Theory and Design. IEEE Transactions on Fuzzy Systems, 8(5), 535–550.

[10] Link S., Prade H. (2014) *Relational database schema design for uncertain data*. CDMTCS-469 Research Report, Centre for Discrete Mathematics and Theoretical Computer Science.

[11] Mun J., Shin M., Lee K., Jung M. (2009) *Manufacturing enterprise collaboration on a goal-oriented fuzzy trust evaluation model in virtual enterprise*. Computers & Industrial Engineering, 56(3), 888-901.

[12] Navara M., Navarova M. (2017) *Principles of inclusion and exclusion for interval-valued fuzzy sets and IF-sets.* Fuzzy Sets and Systems, 324, 60-73.

[13] Ray P., Shahrestani S., Daneshgar F. (2005*) The Role of Fuzzy Awareness Modelling in Cooperative Management*. Information Systems Frontiers, 7(3), 299-316.

[14] Sambuc, R. (1975) *Founctions Φ-floues. Application ´a l'aide au diagnostic en pathologie thyroidienne*. PhD thesis, University de Marseill´e, France.

[15] Zadeh, L.A. (1975) *The concept of a linguistic variable and its application to approximate reasoning*. Information Sciences, 8(3), 199–249.