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Fairway traffic intersection processes models

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Abstract
This author presents models of processes taking place at a waterway intersection. The measures of traffic processes under consideration are the number of vessels waiting to enter an intersection and delay time of these vessels. The applicability of specific models is discussed along with their usefulness as a function if input intensity of vessel stream flowing into a waterway intersection.

Introduction

An intersection of waterways is a major element of waterway infrastructure. The area of intersection may be affected by such phenomena as excessive approach of vessels or delayed traffic. Two fairways crossing each other is the most common type of intersection. Its specific type is the one where a fairway is cut by a ferry shuttle route with ferry traffic moving across the fairway. Research problems related to the safety at an intersection are discussed in studies [1, 2, 3, 4, 5, 6, 7], at the time of delay that occurs in the traffic process has been analysed in the works [6, 8] and others.

Problems of delays in water intersection traffic directly affect safety. Vessels which have to give way at an intersection must either reduce their speed or stop before an intersection. Both manoeuvres are not safe, because ships proceeding at slow speed below a certain value typical of the given type of vessel may lose their manoeuvrability. Besides, a stopping vessel must be careful because it has to maintain its track within the certain area of approach to a given waterway intersection. Therefore, we should pay particular attention to phenomena related to:

- times of delay that occur in the traffic process;
- number of vessels waiting to enter the intersection.

Measures describing the above phenomena may be defined analytically [7, 8, 9] using the method of deterministic analysis or by the determination of characteristics using computer simulation. Due to the fact that in publications to date either parameters such as delay times or the number of waiting vessels have been determined, this article attempts to synthesize solutions used so far and verifies the applicability of individual methods.

Formulation of the problem

An example diagram of an intersection traffic process has a form presented in figure 1.

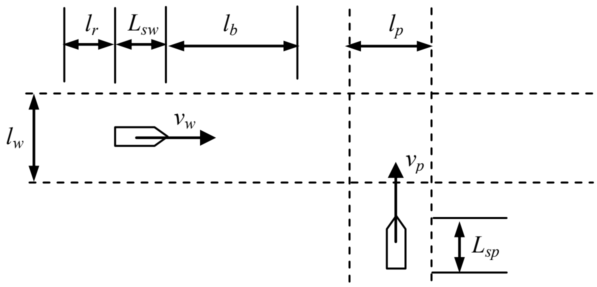

Fig. 1. Process of intersection traffic [6]

Vessels proceeding in the fairway make up a stream with an intensity $\lambda_w(t)$, herein referred to as the longitudinal stream. Vessels crossing the fairway make up a stream with an intensity $\lambda_p(t)$, called here the cross traffic. Vessels move at a speed v_w in the longitudinal stream and v_p in the cross stream.

Analysing traffic processes in real systems we can observe that generally one of the fairways is constantly privileged at an intersection. In the examined case we assume that vessels of the longitudinal stream have the right of way relative to the cross stream vessels. The intensity of longitudinal traffic at the intersection subsystem input is denoted as $\lambda_{win}(t)$, and at its output – $\lambda_{wout}(t)$. As vessels in the longitudinal traffic – along the fairway – are privileged, this relationship takes place:

$$\lambda_{\text{win}}(t) = \lambda_{\text{wout}}(t) \text{ [1/h]} \tag{1}$$

In the cross traffic a stream of vessels with the intensity $\lambda_{pin}(t)$ enters the intersection, while a stream of vessels with the intensity $\lambda_{pout}(t)$ leaves the intersection. As these vessels cannot enter the intersection at any time as they are obliged to give way to vessels moving along the fairway (longitudinal stream), the intensities $\lambda_{pin}(t)$ and $\lambda_{pout}(t)$ do not have to be equal. Due to the fact cross traffic vessels have to wait for a fairway vessel to clear the intersection, there may be a number of cross traffic vessels waiting – $N_o(t)$.

The distance covered by a fairway vessel along the intersection is denoted l_p, while l_w is the distance covered by a crossing vessel. It is worth noting that the section l_p for the privileged vessel plays a warning role only. The section l_w for the subordinate vessel is of major importance. A vessel can find itself within this section only when adopted safety requirements have been satisfied.

Let us assume that a crossing vessel may enter the intersection (section l_w), if all the conditions given below are satisfied:

– the stern of a fairway vessel is at least at a distance l_r away from the point of intersection of routes;
– a crossing vessel will be able to leave the section l_w before a fairway vessel reaches a point defining the distance l_b from the point of route intersection;
– preceding vessel of the same traffic stream is at a minimum admissible distance $\Delta l_{\min}$.

Figure 1 presents a graph of the traffic process at an intersection.

Vessels in the cross stream that may enter the intersection without delay form a stream with the intensity $\lambda_{bo}(t)$. When a longitudinal stream vessel proceeding along the fairway is at a distance shorter than l_b, or a cross stream vessel is already waiting, another cross stream vessel approaching the intersection area stops and acquires a waiting status. Vessels changing their status to waiting make up a stream with intensity $\lambda_{do}(t)$. At the same time, vessels waiting start entering the intersection forming a stream with intensity $\lambda_{zo}(t)$. There is still a number of vessels $N_o(t)$ with the waiting status.

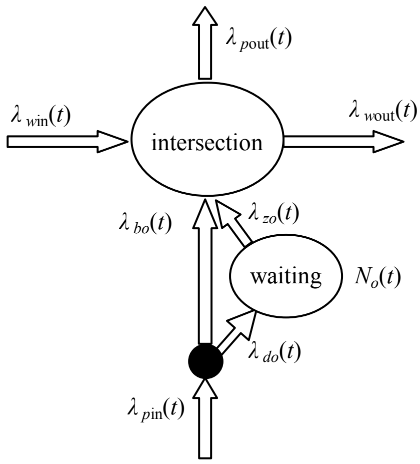

Fig. 2. Graph of the traffic process at a waterway intersection [5]

Analytical research

In studies [6, 7] to solve same traffic processes apply method used vessels traffic streams kinematics equations. Worked out method is deterministic in such sense, that the random variable are represent by expected value. In the analysis and construction of some of the measures used some probability calculus. Simplification is assumed only uniform distribution as stochastic model.

Kinematics equation traffic streams description is one of the main assumption of the method, and observation that vessels on fairway occupy some segment (domain) limited by his dimension and also by distance before bow and after stern where no other vessel should not be. The length of the segment is worth to refer to repeated vessels in the stream expected distance. Observation may be taken in the variable of time. In this consideration, the vessel occupy fairway (intersection) by some time considered as mean recurrence vessels in the stream period, Entered on fairway (intersection) vessel with his occupancy time, encounter there situation (some segment location, repeated vessel period). If occupancy segment may be although partially covered what is unacceptable entered ship must wait. Probability p_{op} to appear situation, in which ship will be has to wait, is described by formula [6]:

$$p_{op} = \frac{T_z}{T_{\text{in}}} \tag{2}$$

where:

T_z – time of fairway occupation by a vessel (at the intersection, it is a sum of occupation times for both vessels, that is the privileged and subordinate vessels);

T_{in} – period of vessels appearing in an exciting stream with an intensity λ_{in} (for the intersection it is a privileged stream).

The mean intensity of delays will be:

$$\lambda_{op} = p_{op}\lambda_b = \frac{\lambda_b T_z}{T_{\text{in}}} \tag{3}$$

where λ_b is an intensity of the examined stream (for the intersection it is the subordinate stream) (give-way vessel).

If a delay occurs it will oscillate in the interval:

$$\Delta t_{op} \in \left\langle 0\,, T_z \right\rangle \tag{4}$$

This author assumes that each value of the above interval is equally possible. This means that we assume a model of this phenomenon in the form of a uniform distribution of vessel arrival interval, which is an essential constraint for the applicability of the model formula. Based on the above constraint, we can claim that if a delay occurs, then the mean delay time Δt_{op} will be:

$$\Delta t_{op} = 0.5 T_z \tag{5}$$

The total delay time of vessels moving in the examined stream in a certain interval Δt equals:

$$t_{opc} = \lambda_{op} \cdot \Delta t \cdot \Delta t_{op} \tag{6}$$

The resultant assessment measure is the mean delay time falling on one passage of a vessel in the examined stream, expressed as a quotient of total delay time t_{opc} divided by the number of vessels in the examined stream, appearing at time interval Δt. After transformations we obtain:

$$t_{op} = \frac{1}{2}\lambda_{\text{in}} T_z^2 = \frac{\lambda_{\text{in}}}{2\mu^2} \ \text{[s]} \tag{7}$$

On the other hand, the delay time of an ingoing vessel is equal to the number of vessels waiting N_o multiplied by the time T_z, because each of those vessels will be occupying the subsystem within that time. The inverse of time T_z is subsystem capacity μ for the examined stream. After transformations, the number of vessels N_o waiting for entering the fairway has this form:

$$N_o = \frac{t_{op}}{T_z} = \frac{\lambda_{\text{in}}}{2\mu} \ \text{[vessels]} \tag{8}$$

Of course both formulas have a physical sense for $\lambda_{\text{in}} < \mu$.

To sum up, we can claim that a model developed by the above method is characterized by the deterministic interval between vessels in the stream, deterministic time of subsystem occupation by a vessel and random character (limited uniform distribution) of positions of the two time intervals on the time axis. After the name of the model's author let us call it Piszczek's model.

The study [8] presents another analytical method which takes into account the random character of time intervals between vessels (limited exponential distribution), assuming the determinism of time of subsystem occupation by a vessel and random relations of positions of the two sections on the time axis. The developed model, also named after its author: Olszamowski's model has been used for determining the mean waiting time for river vessels on crossing routes.

The model assumes that vessels in the exciting stream are moving at time intervals τ, realizing Poisson's process with the mean intensity λ_{in}. Vessels are treated as material points. Let T_z denote a time of subsystem occupation by a vessel of the examined stream for a fairway, or two vessels for an intersection. A vessel to enter the fairway (intersection) has to wait until the time interval τ is greater than T_z. Olszamowski has assumed that if by τ_1, τ_2, τ_3 we denote subsequent values of random variable of an interval between vessels of the exciting stream, then we require not more than three subsequent samplings to draw a number τ greater than T_z. This is an essential constraint and comes down to the conclusion that the following simplifications will create model adequacy to reality only for a deeply subcritical states. In such cases the periods of vessels appearance will be sufficiently large. Delay time is then a random variable of three random variables with the same exponential distributions, provided that each of them separately is less than T_z. To determine the mean delay time we have to consider the distributions of random variables z_2 and z_3:

$$\begin{aligned} z_1 &= \tau_1 \\ z_2 &= \tau_1 + \tau_2 \\ z_3 &= \tau_1 + \tau_2 + \tau_3 \end{aligned} \tag{9}$$

The density function of random variable z_2 probability can be determined from the resultant of two density functions f_1 and f_2, which for non-negative τ, after convenient transformations due to the exponential distribution form, gets this form:

$$\begin{aligned} g(z_2) &= \int_{-\infty}^{+\infty} f_1(\tau_1) f_2(z_2 - \tau_1)\,\mathrm{d}\tau_1 = \\ &= \int_0^{z_2} \lambda_{\text{in}} e^{-\lambda_{\text{in}}\tau_1} \lambda_{\text{in}} e^{-\lambda_{\text{in}}(z_2-\tau_1)}\,\mathrm{d}\tau_1 \end{aligned} \tag{10}$$

$$g(z_2) = \lambda_{\text{in}}^2 z_2 e^{-\lambda_{\text{in}} z_2}$$

Similarly:

$$g(z_3) = \int_0^{z_3} g(z_2) f_3(z_3 - z_2) \mathrm{d}z_2 = \frac{1}{2}\lambda_{\text{in}}^3 z_3^2 e^{-\lambda_{\text{in}} z_3} \quad (11)$$

The expected value of density function $w(z)$ of waiting time distribution for a case when $\tau_1 > T_z$ has this form:

$$\begin{aligned} EX(w_1(z)) &= \lambda_{\text{in}} \int_0^{T_z} \tau e^{-\tau\lambda_{\text{in}}} \,\mathrm{d}\tau = \\ &= \frac{1}{\lambda_{\text{in}}}\left(1 - e^{-T_z\lambda_{\text{in}}}\left(T_z\lambda_{\text{in}} + 1\right)\right) \end{aligned} \quad (12)$$

For the condition when $\tau_2 > T_z$, has this form:

$$\begin{aligned} EX(w_2(z)) &= \lambda_{\text{in}}^2 \int_0^{2T_z} \tau^2 e^{-\tau\lambda_{\text{in}}} \,\mathrm{d}\tau = \\ &= \frac{2 - e^{-2\lambda_{\text{in}}T_z}\left[(2\lambda_{\text{in}}T_z + 1)^2 + 1\right]}{\lambda_{\text{in}}} \end{aligned} \quad (13)$$

For the condition gdy $\tau_3 > T_z$, has the form:

$$\begin{aligned} EX(w_3(z)) &= \frac{1}{2}\lambda_{\text{in}}^3 \int_0^{3T_z} \tau^3 e^{-\tau\lambda_{\text{in}}} \,\mathrm{d}\tau = \\ &= \frac{1}{2\lambda_{\text{in}}}\left\{6 - e^{3T_z\lambda_{\text{in}}}\left[(3T_z\lambda_{\text{in}} + 1)^3 + 3(3T_z\lambda_{\text{in}} + 2)\right]\right\} \end{aligned} \quad (14)$$

Further computational description would take a lot of space while we can already see that the model requires troublesome calculations and has many simplifications.

Ultimately, if we assume the previously mentioned conditions and replace T_z by the inverse subsystem capacity μ, the mean time t_{op} of delay of a vessel in the examined stream has the form:

$$t_{op} = \frac{1}{\lambda_{\text{in}}} \cdot \left(e^{\frac{\lambda_{\text{in}}}{\mu}} - 1 - \frac{\lambda_{\text{in}}}{\mu} \right) \text{ [h]} \quad (15)$$

where:

λ_{in} – input intensity of the exciting stream [1/h];

μ – subsystem capacity for the examined stream λ_b [1/h].

In publication [8] there are no more derivations of relations of interest to us. Therefore we have to independently derive the relation for the mean number of waiting vessels N_o. The number can be determined from the relation stating that the delayed time of moving vessel is equal to the number of waiting vessels N_o multiplied by time T_z, because each of them will be occupying the subsystem over that time. The inverse of time T_z is the subsystem capacity μ for the examined stream. After substitutions and transformations, the relation expressing the mean number of waiting status vessels N_o has this form:

$$N_o = \frac{\mu}{\lambda_{\text{in}}} \left(e^{\frac{\lambda_{\text{in}}}{\mu}} - 1 - \frac{\lambda_{\text{in}}}{\mu} \right) \text{ [vessels]} \quad (16)$$

To sum up, we can state that the model developed by the above method has a random character (exponential distribution) of the time interval between vessels, deterministic time of subsystem occupation by a vessel and random positions of the two intervals on the time axis.

Another analytical method is the method of mass service applied to a classical model presented in publication [9]. The model known as Gutenbaum's model named after its creator, takes into account the random character of time interval between vessels (exponential distribution), random time of subsystem occupation by a vessel (exponential distribution) and random relations of the positions of two time intervals on the time axis. The degree to which the processes are random is in this case the greatest. Exponential distributions are without a memory, because random realizations are independent. We have transformed the model from [9] to the notation and interpretation of the examined intersection subsystem. The mean number of vessels waiting for subsystem entry has been determined. This model distinguishes a set of four basic events and assumes a short time interval Δt, sufficient for only one vessel to arrive or leave. The probability of an event such that, at instant $(t + \Delta t)$ at the subsystem entrance there will be n $(n > 0)$ vessels waiting is equal to the sum of probabilities of four independent compound events:

1. An event in which at instant t the number of waiting vessels was n, and at time interval Δt no new vessel went into or out of the subsystem.
2. An event in which at instant t the number of waiting vessels was $n - 1$, and at time interval Δt one vessel went into, but none went out of the subsystem.
3. An event in which at instant t the number of waiting vessels was $n + 1$, and at time interval Δt one vessel went out, but none went into the subsystem.
4. An event in which at instant t the number of waiting vessels was n, and at time interval Δt one vessel went into, and one went out of the subsystem.

In the next step, probabilities of these events are computed, components with Δt^2 are rejected as insignificant, and two basic differential equations are formulated, models of the phenomenon under consideration. The equations relate probabilities of n waiting vessels occurrence and have these forms:

$$\frac{\mathrm{d}P_n(t)}{\mathrm{d}t} = \lambda_{\text{in}} \cdot P_{n-1}(t) + \mu \cdot P_{n+1}(t) - (\lambda_{\text{in}} + \mu)P_n(t)$$

$$\frac{\mathrm{d}P_o(t)}{\mathrm{d}t} = -\lambda_{\text{in}} P_0(t) + \mu P_1(t) \tag{17}$$

Solving these equations in respect to time would allow to analyze quasi-stationary and dynamic processes. However, this author assumes a constant number of vessels in time, that is the stationary character of the process in this form $P_n(t) = \text{const.}$ Then, naturally, all derivatives of the time function $P_n(t)$ assume zero value, differential equations turn into difference equations and the solution becomes simpler.

Finally, expected value of the number of waiting status vessels N_o assume value according formula [9]:

$$N_o = \frac{\lambda_{\text{in}}}{\mu - \lambda_{\text{in}}} \quad \text{[vesseles]} \tag{18}$$

where:

λ_{in} – input intensity of the exciting stream [1/h];

μ – subsystem capacity [1/h].

On this, J. Gutenbaum [1] finished his considerations. It must be continued his idea to derive a formula of the mean time t_{op} of delay of a vessel in the examined stream cause to waiting to subsystem. The basic is remark, that each waiting vessels after finish waiting cause occupy subsystem on time T_z, which are capacity inverse μ on examine subsystem. On this base the mean time t_{op} of delay of a vessel cause to waiting to subsystem, on vessel has this form:

$$t_{op} = \frac{N_o}{\mu} = \frac{\lambda_{\text{in}}}{\mu(\mu - \lambda_{\text{in}})} \quad \text{[h/vesseles]} \tag{19}$$

where:

λ_{in} – input intensity of the exciting stream [1/h];

μ – subsystem capacity [1/h];

N_o – the mean number of waiting status vessels.

Phenomena existed in transport systems are random phenomena Nevertheless often expected value of random variable (mean value) is used in analysis of completed processes For input intensity value λ_{in} less when capacity μ traffic on intersection are without delay:

$$\begin{aligned} t_{op} &= 0 \qquad \text{[s]} \\ N_o &= 0 \qquad \text{[1]} \end{aligned} \tag{20}$$

When λ_{in} achieve capacity value μ, both the mean time t_{op} of delay of the vessels and the mean number of waiting status vessels strive to infinity:

$$\begin{aligned} t_{op} &\to \infty \qquad \text{[s]} \\ N_o &\to \infty \qquad \text{[1]} \end{aligned} \tag{21}$$

Of course, above relations are true if infinity time horizon of working system are considered.

Examples

We have chosen numerical values of simulation parameters to render possibly accurate image of the situation described by Olszamowski's model, because only this analytical model at the same time considers the intersection of (river) vessels tracks / routes, and Poisson's process of the exciting stream vessel entries. This has led to certain simplifications compatible with this model, but not with reality, for example privileged vessels are regarded as material points whose track can be crossed closely ahead or astern. Does not change the curves qualitatively, but overestimates the value of capacity μ. This happens because the intersection occupation time T_z is additive, therefore it is a sum of components of the vessels proceeding in both directions.

The following assumptions are made for the investigation:

- a homogeneous stream of vessels is moving as a privileged traffic on the fairway;
- a period of vessel appearance in that stream is described by an exponential distribution with a mean value T_{in} [s], that is intensity $\lambda_{\text{in}} = 1/T_{\text{in}}$;
- speed of vessels in the privileged stream is constant and equal to $v_w = 6$ kn (3.09 m/s);
- fairway width is $l_w = 100$ m;
- length of the vessel in the subordinate traffic is $L_p = 50$ m;
- speed of the vessel in the subordinate traffic is $v_p = 5$ kn (2.57 m/s);
- period of appearance of vessels in the subordinate stream is constant $T_p = 1800$ s (corresponding to the intensity $\lambda_p = 2$ vessels/h).

We have performed a series of simulation experiments for the above determinants, aimed at the verification of the results in comparison to the presented methods, adapted and used for example calculations. In the tests the input intensity of exciting streams was being increased from zero to the value of capacity μ.

Figure 3 presents the mean delay time t_{op} falling on a vessel of the subordinate traffic as a function of the period T_{in} of vessels present in the privileged stream, obtained from the tests of these models:

– Gutenbaum's;
– simulation;
– Olszamowski's;
– Piszczek's;
– deterministic.

You will see in the diagram a considerable similarity of the results obtained by all methods for deeply subcritical states (privileged vessels proceeding every hour). Olszamowski's and Piszczek's models give fully convergent results for a period as long as ten minutes.

For untypical conditions, where the period T_{in} of vessels appearance tends to $1/\mu$, the curves of time delay obtained from tests performed by all methods have a similar shape. As T_{in} decreases, t_{op} tends to infinity in the simulation and Gutenbaum's models, in the deterministic model assumes the infinite value for $T_{in} = 1/\mu$ (when $T_{in} > 1/\mu - t_{op} = 0$), while for

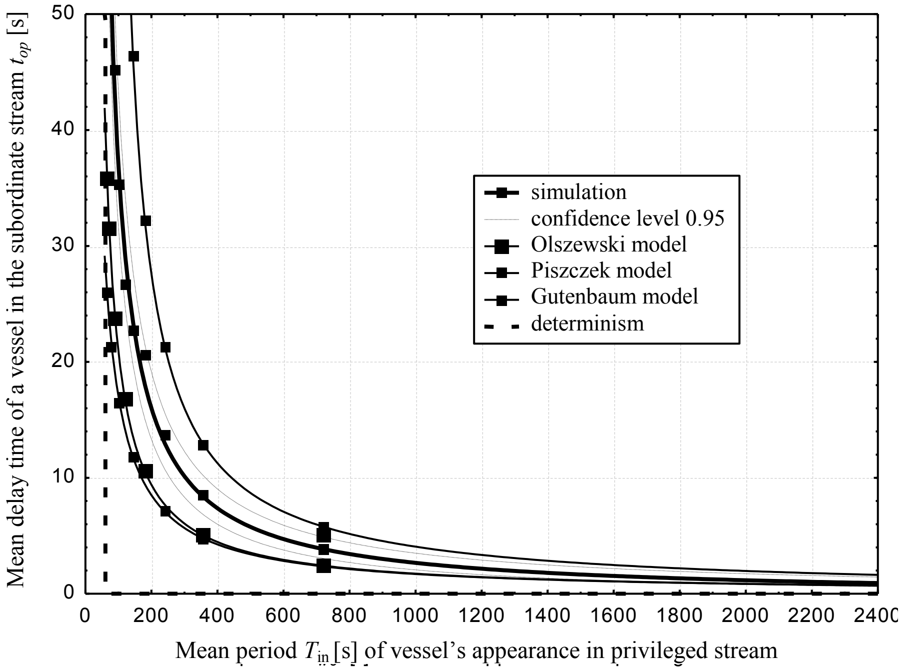

Fig. 3. Mean delay time t_{op} [s] of a vessel in the subordinate stream as a function of the mean period T_{in} [s]of vessels' appearance in the privileged stream [own study]

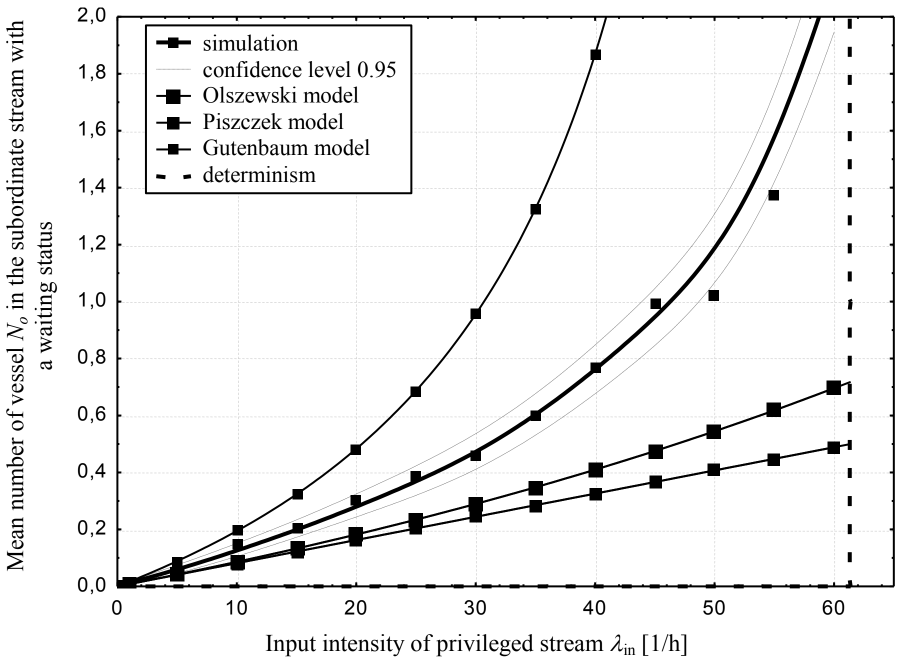

Fig. 4. Mean number of vessels N_o in the subordinate stream with a waiting status as a function of input intensity of the privileged stream λ_{in} [own study]

the remaining models, time delay has finite values. The reason is that in critical and supercritical states the delay time tends to infinity. Delay time are calculated from analytical Olszamowski's and Piszczek's models for $T_{\text{in}} = 1/\mu$ assume finite values, which is naturally an effect of simplifications adopted while at the stage deriving the relationships.

Figure 4 presents a mean number N_o of vessels in the subordinate stream, waiting to enter the intersection as a function of privileged traffic intensity λ_{in}, obtained from tests of the same models.

In reference to the results generated by the simulation model, 95% confidence intervals have also been marked. The simulation model results are contained within the entire variation interval above the curves representing the results from analytical Olszamowski and Piszczek models. We can make a hypothesis that it is due to the above mentioned assumptions for the situation modelled and the adoption of an exponential distribution without displacement in the privileged stream model. This confirms a thesis that there is a wide range of possible curves in an area of feasible solutions (acceptable in reality), that is in an area enclosed by Gutenbaum's model curve and a polyline of the deterministic model. The establishment of accurate curves of the tested functions, including a possible choice of an analytical model, is possible provided that we build a precise identification model.

Conclusions

From the research performed this author can formulate the following conclusions:

- results of Olszamowski's and Piszczek's models are similar to the results of the simulation method for deeply subcritical states;
- simulation method for quasi-critical and critical states brings acceptable results, contrary to the results obtained by analytical methods;
- for supercritical states analytical models are declaratively not applicable, while simulation methods cope well with such states;
- Gutenbaum's model and results of a deterministic analysis define an area of acceptable values for a simulation model, so they may act as preliminary verifier of the logical correctness of simulation tests.

The choice of a model depends on the objective and the input intensity-capacity ratio, but from a wide perspective we can state that the simulation method is the most universal one, performing well in subcritical and quasi-critical states.

References

1. KASYK L.: Analiza Bezpieczeństwa ruchu statków na przeprawach promowych dla różnych systemów regulacji ruchu. Rozprawa doktorska, Szczecin 2001.
2. KASYK L.: Prawdopodobieństwo uniknięcia sytuacji kolizyjnej na projektowanej przeprawie promowej Police Święta w różnych modelach probabilistycznych. Zeszyty Naukowe Nr 70 Wyższej Szkoły Morskiej w Szczecinie, Inżynieria Ruchu Morskiego, Szczecin 2003.
3. MAJZNER P.: Metoda oceny akwenów ograniczonych z wykorzystaniem symulacji ruchu strumieni jednostek. Rozprawa doktorska, Szczecin 2008.
4. MAJZNER P., PISZCZEK W.: Badanie przepustowości skrzyżowania torów wodnych. Materiały konferencyjne 12th International Scientific and Technical Conference on Marine Traffic Engineering, 2007.
5. MAJZNER P., PISZCZEK W.: Badanie procesów ruchu na skrzyżowaniach torów wodnych. Zeszyty Naukowe AM w Szczecinie 21(93), 2010, 62–66.
6. PISZCZEK W.: Modele miar systemu inżynierii ruchu morskiego. Studia nr 14, WSM, Szczecin 1990,
7. PISZCZEK W.: Model bezpieczeństwa ruchu promów na skrzyżowaniu toru wodnego i przeprawy promowej. Zeszyt Naukowy WSM nr 59, Szczecin 2000.
8. OLSZAMOWSKIJ S.B., ZEMLAKOWSKIJ D.K., SCEPETOW I.A.: Organizacja bezopasnosti plawanija sudow. Transport, Moskwa 1979.
9. GUTENBAUM J.: Modelowanie matematyczne systemów. Akademicka Oficyna Wydawnicza EXIT, Warszawa 2003.