Computer Science • 14 (4) 2013 http://dx.doi.org/10.7494/csci.2013.14.4.547

Tomasz Jurczyk
Barbara Głut

PREPARATION OF CONTROL SPACE FOR REMESHING POLYGONAL SURFACES

Abstract *The subject of the article concerns the issues of remeshing, transforming a polygonal mesh into a triangular mesh adapted to its surface. From the initial polygonal mesh, the curvature of surface and boundary is retrieved and used to calculate a metric tensor varying in three-dimensional space. In the proposed approach, the curvature is computed using local approximation of surfaces and curves on the basis of vertices of the polygonal mesh. An essential part of the presented remeshing procedure is a creation of a control space structure based on the retrieved discrete data. The subsequent process of remeshing is then supervised by the contents of this auxiliary structure. The article presents various aspects related to the procedure of initialization, creation, and adjustment of the control space structure.*

Keywords remeshing, anisotropic metric, control space, mesh adaptation, polygonal mesh

1. Introduction

Geometric modeling of complex three-dimensional objects is now widely used in many fields of science and technology. The main areas of application are computational geometry, computer graphics, scientific visualization, and the creation of virtual reality. This topic is also present in domains like computer simulations of physical processes, design of machinery and equipment, medical imaging, biomedical research, and cartography.

Many models considered in these domains are obtained by means of 3-D scanning and other techniques of digitizing the surface of 3-D models (e.g. using magnetic resonance imaging or computed tomography).

The result of such techniques is a discrete set of points from which the surface is reconstructed. A survey of methods of surface reconstruction can be found in [7, 3]. The result of this step is usually a polygonal mesh. Unfortunately, the quality of meshes obtained in such a manner is often far from satisfactory. They contain a lot of defects, including artifacts, inverted elements, missing elements (creating holes), redundant vertices, and noise. The first step, therefore, is to apply some methods which help solve these problems and allow a surface to be described through a topologically-correct mesh. Then, the subsequent step can be executed; namely, remeshing.

In [2], the following definition of remeshing is proposed: "Given 3-D mesh, compute another mesh, whose elements satisfy some quality requirements, while approximating the input acceptably". Such a definition is very general and does not precisely specify the concept of the quality of approximation. It may be mesh density, its regularity, or the size and shape of the elements. Often, a combination of these requirements is necessary in practical applications. The quality criterion needs to be adjusted for a particular application. An overview of the most commonly used methods of remeshing can be found in [2, 5].

Remeshing techniques can be divided into two categories – working directly in 3-D space (e.g. [20, 8]) or utilizing the parameterization [13, 19]. The goal of parameterization is to project surface meshes onto a 2-D plane. It is necessary to find the correct mapping function, translating boundary vertices of the mesh onto the boundary of the plane, and the inner mesh vertices into the inside of the 2-D domain, all the while respecting the correct connection of the vertices. The procedures of adaptation are carried out for the mesh projected into 2-D parametric space. The improved mesh is then transformed back into three-dimensional space.

The global parameterization, which parameterizes the whole mesh, is practically feasible only for specific models, homeomorphic to a disc. Otherwise, it is necessary to properly divide the input mesh into sub-domains which, only then, can be successfully parametrized. Unfortunately, finding the proper division is a separate and difficult task. Consequently, many remeshing techniques use local parameterization, where the surface is parametrized for a suitably selected neighborhood of elements (e.g. [21]). The procedure of remeshing is run locally as well. Both of these approaches have the unfortunate disadvantages resulting from the projection, which introduces various types of distortion. This problem does not seem to be completely solved for increasingly-complicated shapes of objects encountered in practice.

2. The proposed solution

The algorithm proposed in this article is essentially running in 3-D space, using 2-D parametric space locally for few operations. The input data is a polygonal mesh. Such a mesh often contains a number of various errors and inconsistencies, which are corrected and solved in subsequent procedures of the presented algorithm. Then, the actual remeshing of the polygonal mesh is carried out. The final result of this procedure is a mesh adapted to automatically-determined properties of the surface. The essential part of this adaptation technique is the curvature of surfaces and curves recovered from the discrete input mesh. These values are converted into appropriate metric tensor calculated in discrete points of the 3-D domain. Methods applying the concept of a non-Euclidean metric are now widely used for the generation and adaptation of unstructured anisotropic meshes in two and three dimensions [1, 17, 6, 9].

The element of novelty in the proposed algorithm in relation to known techniques is the application of a control space concept, which is an additional structure guiding the process of mesh generation and adaptation [12]. The idea of utilizing an auxiliary background data structure for facilitating mesh generation process is not new [22, 18]. However, the concept of the proposed adaptive control space structure and its application for remeshing is further extended, which allows us to take advantage of it more efficiently and robustly in a number of meshing applications. A specific feature of our technique is also the method of determining the local metric. We use local approximation of curves and surfaces with quadrics based on local set of mesh vertices.

The subsequent phases of remeshing a polygonal mesh are as follows:

1. Initial preprocessing of the polygonal mesh – recovery of the topological boundary (if it exists – for open or multi-domain models), determination of normals for all faces in the mesh, correction of inverted elements, identification of sharp edges.
2. Construction of the control space structure.
3. Determination of a set of local surfaces and local curves.
4. Division of polygons in order to obtain a uniform mesh composed of triangular faces.
5. Actual remeshing procedure consisting of a mixture of local techniques, including collapsing of edges, edge splitting, geometrical and topological improvement of the mesh.

The control space is used in all steps after its creation – from (3) to (5). The required representations of the metric are retrieved from the control space in order to facilitate evaluation of the fitting, validity and quality of remeshing operations. This article is devoted to the step (2) of the remeshing process, where the control space structure for remeshing the polygonal grid is created, adjusted, and initialized.

3. Control space structure

The typical control space structure has been extended to store additional information supervising the size and shape of mesh elements, like the local metric gradation rate or the maximum anisotropy ratio. Depending on the application, the control space may have a different structure; e.g., quadtree/octree grid or background mesh, with sizing data in the form of a metric tensor stored in nodes of those grids and metric within the leaves being calculated using appropriate shape functions. In our approach, the metric tensor stored in the nodes of control space is represented by a metric transformation matrix [10, 14].

In order to enable the possibility of preparation and utilization of various control space structures, a common interface of adaptive control space (ACS) was created [15]. This allows us to develop the automated construction of the sizing field independently from the selected type of control space. Thus, further extension of the CS structure types is easily possible without influencing the procedures of remeshing.

Additionally, the ACS interface includes operations which allow us to update or correct the metric already stored in the control space, which is especially useful for adapting the mesh to the shape of the domain.

4. Construction of the control space for remeshing a polygonal mesh

The control space is formed based on a set of metric sources defined in points. The procedure of control space creation can be divided into two tasks (Fig. 1): determining the set of metric sources, and using them to create a sizing field in the control space.

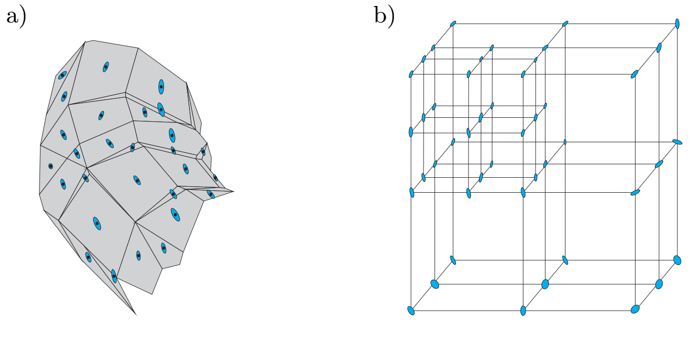


Figure 1. Construction of control space: a) polygonal mesh with computed set of metric sources, b) control space with metric tensor set at the nodes of the octree structure.

4.1. Determining the set of metric sources

The main sizing information for remeshing is gathered by assessing the local curvature of the surface mesh. For each face of the polygonal mesh, a local surface is approximated using the local vicinity of mesh entities. If the approximation quality is satisfactory, the curvature of the surface is computed for the point at the middle of the face. From this curvature, the metric tensor is calculated, and a new discrete metric source is created and ready to be inserted into the control space. Additionally, if such a face is adjacent to boundary edges, a local curve approximation and calculation of contour curvature is attempted in order to further adjust the control space information. The computation and insertion of metric sources is performed according to the methodology presented in [11, 16].

4.1.1. Local surface approximation for faces

For each face of the polygonal mesh, the procedure begins with collecting a local set of points, which are the vertices of that face and of all of the elements from the prescribed number of topological layers of elements around the face (Fig. 2). While gathering these layers of elements, the edges marked as boundaries are never crossed. The number of layers is typically set to one or two layers.

The minimum number of points for surface approximation is 6. If the number of vertices in the set is lower, additional layers are included until this requirement is fulfilled. If the set of vertices is still too small (despite the extended number of layers), the local surface approximation for such a face is canceled. Otherwise, a least-square approximation with quadric ($z = f(1, x, y, xy, xx, yy)$) is computed using a reference plane approximated earlier for this set of points.

Since the control space structure is not yet ready, the metric information stored therein can not be used to evaluate the quality of approximation. Instead, the acceptability of the approximation is determined by using the overall size of the set of points used for approximation as a measure of local mesh density. In order for the approximation surface to be accepted, the maximum approximation error (maximum distance between the points of the set and the approximating quadric surface) needs to be lower than the average radius of the layer of elements multiplied by the coefficient of surface curvature adaptation.

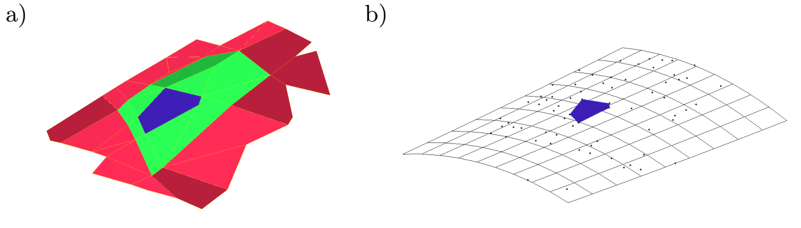


Figure 2. Local surface approximation for face: a) topological layers, b) set of vertices and the approximated quadric.

4.1.2. Determining of the metric tensor

The metric tensor is formed basing on the main curvatures χ_1, χ_2 and main directions e_1, e_2 of the approximation surface.

The three-dimensional metric tensor $\mathbf{M}$ is represented as

$$\mathbf{M} = \mathbf{R}\boldsymbol{\Lambda}\mathbf{R}^T. \quad (1)$$

The columns of $\mathbf{R}$ are the eigenvectors e_1, e_2 calculated from the approximation surface and e_3 – orthogonal to e_1 and e_2. $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_i)$ is created as the diagonal eigenvalue matrix ($\lambda_i = 1/h_i^2$).

The requested length h_i of edges along the main directions is calculated as

$$h_i = \max\left(\min\left(\frac{\gamma_c}{\chi_i}, h_{\max}\right), h_{\min}\right), \quad i = 1, 2, \quad (2)$$

where γ_c is the constant ratio of proportionality of the edge length and curvature radius. $h_{\min}$ and $h_{\max}$ are bounds necessary to avoid creation of too extreme elements and also to ensure that the metric tensor is a non-singular matrix.

The length h_3 is calculated as

$$h_3 = \gamma_a \ \min(h_1, h_2), \quad (3)$$

where the anisotropy ratio γ_a is introduced in order to facilitate controlling of the maximum stretch ratio of elements

$$\frac{\max(h_i)}{\min(h_i)} \leq \gamma_a, \quad i = 1, \ldots, d. \quad (4)$$

The meshing parameters $h_{\max}$, $h_{\min}$, γ_c and γ_a can be adjusted by the user depending on the considered problem.

4.1.3. Reusing approximated surface for generation of multiple metric sources

For each inspected face of the polygonal mesh, the approximated surface is being fitted not only to this face, but also for a two layers of elements in its neighborhood. The possible optimization of the procedure of retrieving the metric sources could infer from extending the utilization of the computed approximation surface. Instead of using it only for calculation of the curvature for a single point within the selected face, is can be also applied to compute the curvature for some additional faces in one or two surrounding layers of elements. Such an approach would reduce the number of necessary surface approximations. Unfortunately, at this point of research, some additional checks seem to be required in order to avoid a degradation of the quality of thus-calculated metric sources.

4.1.4. Local curve approximation for edges

In the case of a successful approximation of a surface for a face, a potential curve approximation for the boundary edges of such face is considered. For a given boundary edge, the chain of boundary edges from both sides of the edge is gathered. The number of edges from each side is limited similarly to the number of layers in the case of surface approximation for a face. Also, crossing of the vertices marked as corners are prohibited.

The next step is to project the gathered set of vertices (from all edges in this chain) onto the surface approximated previously for the face. If those points (after projection) can be approximated with a straight line, no further calculations are performed since the curvature of this contour is already included within the curvature of the surface. Otherwise, the set of points is locally approximated by a polynomial. If successful, the curvature of this contour is calculated and a new metric source is created and inserted into the control space.

The metric tensor for the curvature of contours is formed in a similar manner as in the case of surface curvature. The details of these procedures can be found in [11].

4.2. Creation of control space basing on a set of metric sources

The continuous sizing field stored in the control space is computed from a discrete set of metric sources where each metric source is given as metric tensor defined in 3-D point. In this work, the metric tensor is based on the curvature of approximated surfaces and contours. However, the technique of creating the control space through the introduction of metric sources is more general, and this allows us to incorporate other types of sizing information, including various geometrical properties of the model (like short edges or proximity of entities) or data obtained from other sources (defined by a user or resulting from the adaptation in the simulation of processes).

The preferred type of control space structure is an octree grid. The size of the control space is set based on the bounding box of all vertices from the input polygonal mesh.

In order to transform a discrete irregular set of points into a continuous field defined by the nodes of the control space structure, two approaches can be applied depending on the situation:

1. Local adjustment of the control space using operation of metric intersection. This approach is preferable if the control space is already initialized and some additional metric sources are introduced in order to further adapt the sizing of the mesh. If no predefined sizing field is available, the initial values of nodes of the control space structure are set to be the maximum metric defined by the corresponding parameters of meshing process. Then, each new metric source is sifted through the control space, possibly causing an adaptation of its structure depending on the difference between the metric stored in the source and the metric already set in the nodes of control space. After the adaptation of the CS structure, the nodes of an octree leaf containing the new source are updated

using the procedure of metric intersection. After all metric sources are inserted, an additional operation of smoothing is executed in order to enforce the gradation of metric prescribed by relevant meshing parameters.

2. An alternative method is based on initializing the control space structure directly from the set of discrete metric sources sifted through the octree. The adaptation of the control space structure is based on the disparity between the metric in a list of metric sources for a given leaf. If the difference is too high, the leaf is split, and the list of metric sources is divided accordingly. Finally, after all metric sources are inserted into the control space structure, the procedures of interpolation and extrapolation are applied in order to determine the values of metric in all nodes of the control space.

5. Evaluation of created control space

In order to evaluate the developed technique, several study cases based on algebraic surfaces were prepared and tested. The example test model T_1 (Fig. 3) was created using the following procedure:

1. First, an unstructured mesh was generated for a toroidal surface:

$$\begin{cases} x = (0.05 + 0.02\cos v)\cos u \\ y = (0.07 + 0.01\cos v)\cos u \\ z = 0.03\sin v \end{cases} \tag{5}$$

2. All faces and edges were removed from this mesh, leaving only a set of discrete points.
3. A polygonal mesh (with 48 055 vertices and 37 240 faces) was obtained via surface reconstruction from this cloud of points thanks to PowerCrust [4] software.
4. After the validation procedure combined with elimination of degenerate or too small elements, the resultant polygonal mesh T_1 was created with 23 783 vertices and 24 428 faces.

For the polygonal mesh T_1, a developed procedure of remeshing was executed and evaluated regarding especially the part of creation of the control space structure. The metric sources were gathered from the faces of the polygonal mesh for all three layers of elements used for approximation, using only the first two layers or basing it on the central element only (Sec. 4.1.3) – in further analysis denoted respectively with symbols “012”, “01” and “0”. Additionally, as reference values, the metric sources were also created using the initial algebraic formula for the remeshed toroidal surface – denoted with “F”. Independently from the method of creation of the metric sources, two techniques of control space creation were inspected: (1) local adjustment of control space using metric intersection, and (2) initialization using an average metric (Sec. 4.2) – denoted as “INT” and “AVE” respectively.

For each pair of control space structures being checked, the metric stored therein was retrieved and compared for all nodes of the initial polygonal mesh. In order to

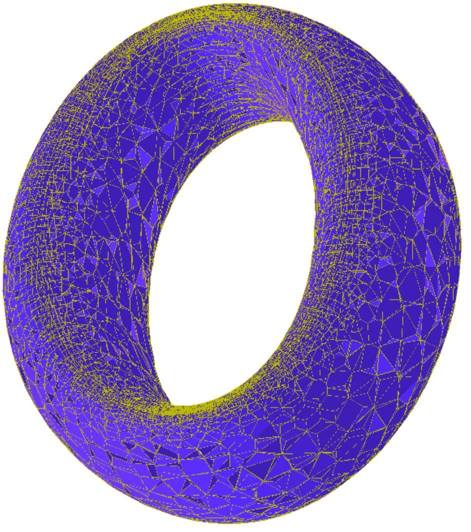


Figure 3. Initial polygonal mesh T_1.

compare non-scalar values of metric tensors $\mathbf{M}_1$ and $\mathbf{M}_2$ obtained from two different control space structures, the following residuals are calculated:

$$\begin{aligned} \mathbf{R_1} &= \mathbf{M}_1^{-1}\mathbf{M}_2 - I \\ \mathbf{R_2} &= \mathbf{M}_2^{-1}\mathbf{M}_1 - I \end{aligned} \tag{6}$$

and the non-conformity coefficient $\delta_{\mathcal{M}}$ is calculated using the Euclidean norm of a matrix [17]

$$\delta_{\mathcal{M}}(\mathbf{M}_1, \mathbf{M}_2) = ||\mathbf{R_1} + \mathbf{R_2}||. \tag{7}$$

The value of $\delta_{\mathcal{M}}$ equal to 0 means perfect match of two metrics. The higher the value, the more different are the metrics.

The results of a series of tests were gathered in Table 1. Each row shows comparison statistics for a pair of control space structures, created using different approaches. $\delta_{\mathcal{M}}^{\min}$ and $\delta_{\mathcal{M}}^{\max}$ present the minimum and maximum values of the metric non-conformity coefficient among all metric comparisons in the vertices of the initial polygonal mesh. The last five columns show the number of comparison results divided into the given intervals.

The results in both INT-0 and AVE-0 (with curvature calculated using an approximation surfaces) correspond closely to the control space structures created using the analytical data (INT-F and AVE-F respectively) which confirms the validity of the presented technique.

For structures denoted with "012" and "01", there is a clear discrepancy between these cases and the reference control space F. This distortion in the metric field also has a visible and negative influence on the quality of the subsequent remeshing

Table 1

Comparison of metric in control space structures created using different approaches.

ACS_1	ACS_2	$\delta_{\mathcal{M}}^{\min}$	$\delta_{\mathcal{M}}^{\max}$	$[0, 0.5)$	$[0.5, 1)$	$[1, 2)$	$[2, 4)$	$[4, \infty)$
INT-F	AVE-F	0.00056	1.1	23273	505	5	0	0
INT-F	INT-0	0.000053	0.3	23783	0	0	0	0
INT-F	INT-01	0.00025	52.2	757	898	2164	3676	16288
INT-F	INT-012	0.00097	61.8	981	1166	2207	3460	15969
AVE-F	AVE-0	0.0016	0.9	23752	31	0	0	0
AVE-F	AVE-01	0.013	20.3	7651	4455	4073	3523	4081
AVE-F	AVE-012	0.0060	35.2	4728	2913	3483	4729	7930

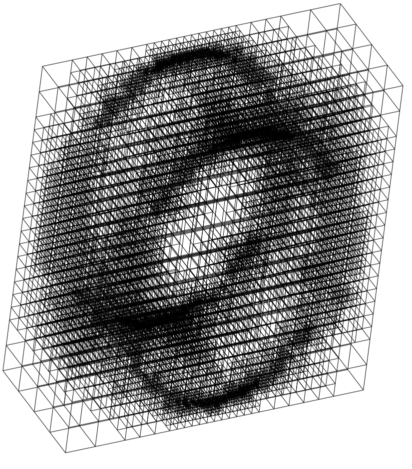
procedure, hindering this process and producing meshes of a lesser quality. The control space structures created with the averaging approach (AVE) seem to be less affected by the inaccuracies in the set of metric sources from which a control space structure is initialized.

The difference between the INT and AVE structures is rather small. The meshes produced using these control spaces have similar structure. The control space structure created using the INT approach has a tendency to produce meshes with slightly smaller elements.

6. Example of influence of the control space on created meshes

Figure 4 presents the result of the remeshing procedure for the tested toroidal model T_1 (Fig. 3).

a)



b)

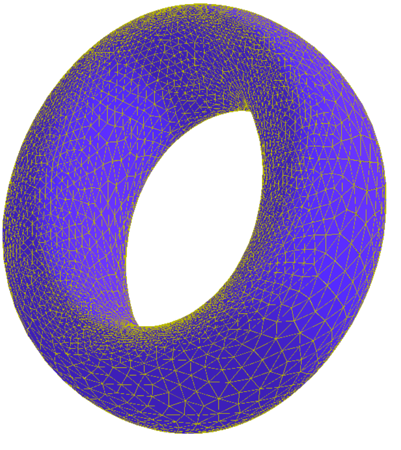


Figure 4. Result of mesh adaptation (NP=5 655, NF=11 310): a) created ACS structure; b) after remeshing.

The process of creating a control space based on a set of metric sources can be additionally adjusted with a number of meshing parameters allowing to easily influence the characteristic of the produced meshes; e.g., maximum anisotropy (controlling the maximum allowable stretching of elements), gradation of the metric (defining how rapidly the metric can be changing throughout the meshing space), or curvature ratio (describing the relation between the curvature radius and the resulting size of elements – the higher the value of this coefficient, the larger the elements). Figure 5 presents the effect of remeshing performed for the value of curvature ratio $\gamma_c = 0.3$ (for Fig. 4 the value of $\gamma_c = 0.15$ was used).

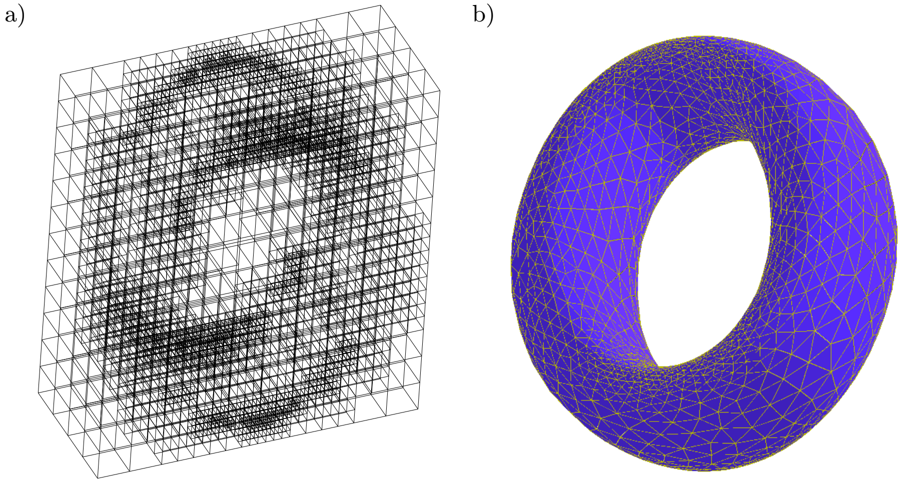


Figure 5. Result of mesh adaptation for altered value of curvature ratio parameter (NP=2 023, NF=4 046): a) created ACS structure; b) after remeshing.

7. Examples

Figures 6 and 7 present the results of the remeshing procedure carried out for a sizing field retrieved from the cubic model with a different density of the initial polygonal mesh (giving comparable structure of the final mesh). The preliminary identification of boundary edges and vertices is based on topological information. The edges (and their vertices) adjacent to more or less than two faces are marked as boundary. Similarly, boundary vertices adjacent to more or less than two boundary edges are marked as corner. After establishing normals for all faces in the input mesh, an additional set of boundaries may be recognized from the sharp edge criterion, where the angles between normals for incident faces are inspected; if the angle is larger than the given threshold value, such an edge is marked as a candidate for boundary edge. The set of candidate sharp edges is then processed in order to promote the creation of continuous sequences of boundary edges by filtering out single candidate edges or completing chains of such edges with additional edges. After recognition of the chains of boundary edges, the set of corner boundary points is updated with the ending vertices of such chains as well as with the vertices where the angle between the adjacent edges is smaller than the given threshold value. After creation of an ACS structure and identification of local surfaces, additional information about the processed model is obtained, which can be utilized to further refine and update the set of boundary edges and vertices.

a) b)

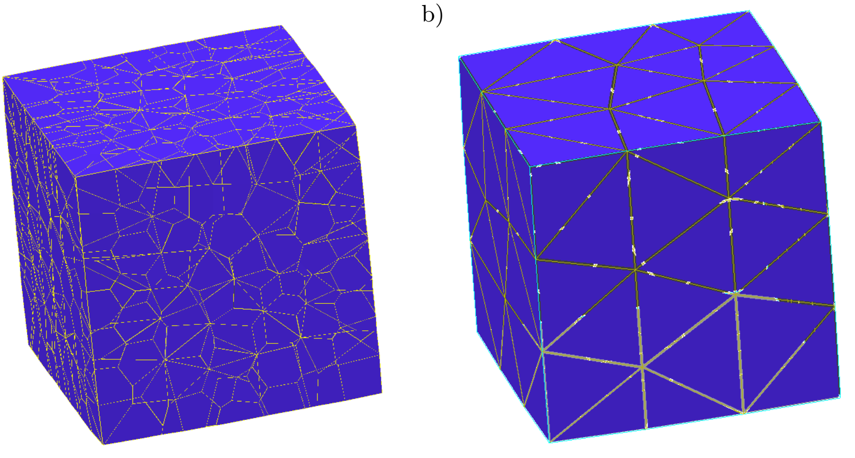


Figure 6. Result of mesh adaptation for a cube: a) initial polygonal mesh (NP=2 592, NF=1 778); b) after remeshing with created sizing space (NP=52, NF=100).

a) b)

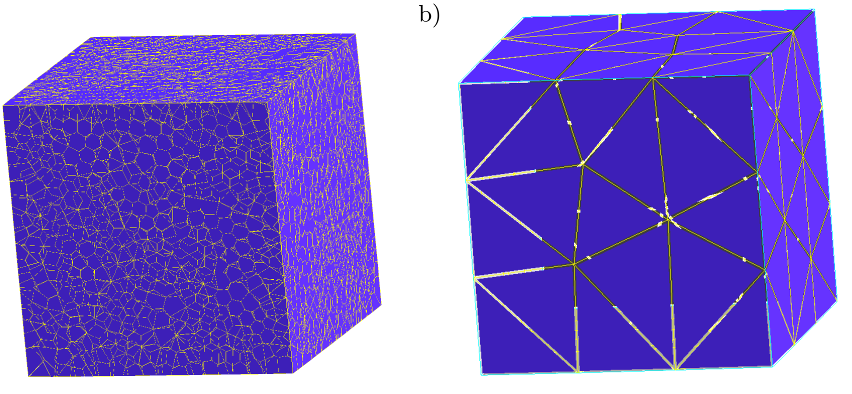


Figure 7. Result of mesh adaptation for a cube: a) initial polygonal mesh (NP=17 571, NF=11 977); b) after remeshing with created sizing space (NP=55, NF=106).

Figure 8 illustrates structure of the mesh obtained after remeshing with the control space created with a higher value of the anisotropy parameter.

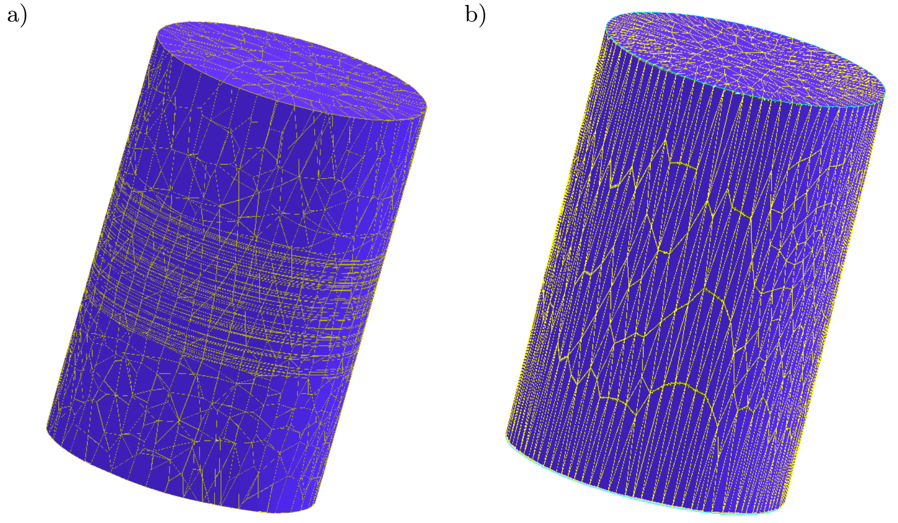


Figure 8. Result of mesh adaptation for a cylinder: a) initial polygonal mesh (NP=8 673, NF=7 222); b) after remeshing with created sizing space (NP=981, NF=1 958).

8. Summary

The article concentrates on the presentation of issues and techniques related to the task of creating a control space structure for remeshing of surface meshes. Proper creation of such structure facilitates the subsequent remeshing procedures (including creation of local reference surfaces and contours, collapsing of edges, geometrical and topological smoothing, splitting of edges, etc.) which will be described in more detail in future articles.

Acknowledgements

The research presented in this paper was partially supported by the AGH grant 11.11.230.015.

References

[1] Alauzet F., Frey P.: *Estimateur d'erreur géométrique et métriques anisotropes pour l'adaptation de maillage. Partie I: aspects théoriques.* Tech. Rep. RR-4759, INRIA Rocquencour, 2003.

[2] Alliez P., Ucelli G., Gotsman C., Attene M.: Recent advances in remeshing of surfaces. In: *Shape Analysis and Structuring, Mathematics and Visualization*, L. Floriani, M. Spagnuolo, eds. Springer, 2008.

[3] Amenta N.: Surface Reconstruction. In: *Point-Based Graphics*, M. Gross, H. Pfister, eds. Morgana Kaufmann Publishers Inc., San Francisco, CA, USA, 2007. ISBN 0123706041, 9780080548821.

[4] Amenta N., Choi S., Kolluri R. K.: The Power Crust, Unions of Balls, and the Medial Axis Transform. *Computational Geometry: Theory and Applications*, **vol. 19**, pp. 127–153, 2000.

[5] Botsch M., Kobbelt L., Pauly M., Alliez P., Lévy B.: *Polygon Mesh Processing.* AK Peters / CRC Press, 2010. ISBN 9781568814261.

[6] Bottasso C.: Anisotropic Mesh Adaptation by Metric-Driven Optimization. *Int. J. Numer. Meth. Engng.*, **vol. 60**, pp. 597–639, 2004.

[7] Dey T. K.: *Curve and Surface Reconstruction: Algorithms with Mathematical Analysis (Cambridge Monographs on Applied and Computational Mathematics).* Cambridge University Press, New York, NY, USA, 2006. ISBN 0521863708.

[8] Frey P.: About surface remeshing. In: *Proc. 9th Int. Meshing Roundtable*, pp. 123–136. Sandia National Laboratories, New Orleans, CA, 2000.

[9] Frey P.: Anisotropic Metrics for Mesh Adaptation. In: *Proc. ECCOMAS 2004.* Jyvaskyla, Finland, 24-28 July 2004.

[10] Głut B., Jurczyk T.: Definition and Interpolation of Discrete Metric for Mesh Generation on 3D Surfaces. *Computer Science, Annual of University of Science and Technology*, **vol. 7**, pp. 89–103, 2005.

[11] Głut B., Jurczyk T.: Preparation of the Sizing Field for Volume Mesh Generation. In: *Proc. 13th Int. Conf. on Civil, Structural and Environmental Engineering Computing.* Chania, Crete, Greece, 2011. Paper 115.

[12] Głut B., Jurczyk T., Kitowski J.: Anisotropic Volume Mesh Generation Controlled by Adaptive Metric Space. In: *Proc. of Int. Conf. NUMIFORM'07*, pp. 233–238. Porto, Portugal, 2007.

[13] Hormann K., Polthier K., Sheffer A.: Mesh Parameterization: Theory and Practice. In: *SIGGRAPH Asia 2008 Course Notes*, 11, pp. v+81. ACM Press, Singapore, 2008.

[14] Jurczyk T., Głut B.: Metric 3D Surface Mesh Generation Using Coordinate Transformation Method. In: *Proc. of Int. Conf. on Computer Methods and Systems CMS'05*, vol. 1, pp. 395–405. Kraków, Poland, 2005.

[15] Jurczyk T., Głut B.: Adaptive Control Space Structure for Anisotropic Mesh Generation. In: *Proc. of ECCOMAS CFD 2006 European Conference on Computational Fluid Dynamics.* Egmond aan Zee, The Netherlands, 2006.

[16] Jurczyk T., Głut B.: The Insertion of Metric Sources for Three-dimensional Mesh Generation. In: *Proc. 13th Int. Conf. on Civil, Structural and Environmental Engineering Computing.* Chania, Crete, Greece, 2011. Paper 116.

[17] Labbé P., Dompierre J., Vallet M. G., Guibault F., Trépanier J. Y.: A universal measure of the conformity of a mesh with respect to an anisotropic metric field. *Int. J. Numer. Meth. Engng*, **vol. 61**, pp. 2675–2695, 2004.

[18] Miranda A. C. O., Martha L. F., Wawrzynek P. A., Ingraffea A. R.: Surface mesh regeneration considering curvatures. *Eng. Comput. (Lond.)*, **vol. 25**(2), pp. 207–219, 2009.

[19] Morigi S.: Feature-sensitive parameterization of polygonal meshes. *Applied Mathematics and Computation*, pp. 1561–1572, 2009.

[20] Rassineux A., Villon P., Savignat J. M., Stab O.: Surface remeshing by local hermite diffuse interpolation. *Int. J. Numer. Meth. Engng*, **vol. 49**(1–2), pp. 31–49, 2000. ISSN 1097-0207.

[21] Surazhsky V., Alliez P., Gotsman C.: Isotropic Remeshing of Surfaces: A Local Parameterization Approach. In: *Proc. 12th Int. Meshing Roundtable*, pp. 215–224. Sandia National Laboratories, Santa Fe, New Mexico, USA, 2003.

[22] Zhu J., Blacker T., Smith R.: Background Overlay Grid Size Functions. In: *Proc. 11th Int. Meshing Roundtable*, pp. 65–74. Sandia National Laboratories, Ithaca, NY, 2002.

Affiliations

Tomasz Jurczyk
AGH University of Science and Technology, Department of Computer Sciences, Krakow, Poland, `jurczyk@agh.edu.pl`

Barbara Głut
AGH University of Science and Technology, Department of Computer Sciences, Krakow, Poland, `glut@agh.edu.pl`

Received: 28.10.2013
Revised: 11.11.2013
Accepted: 11.11.2013