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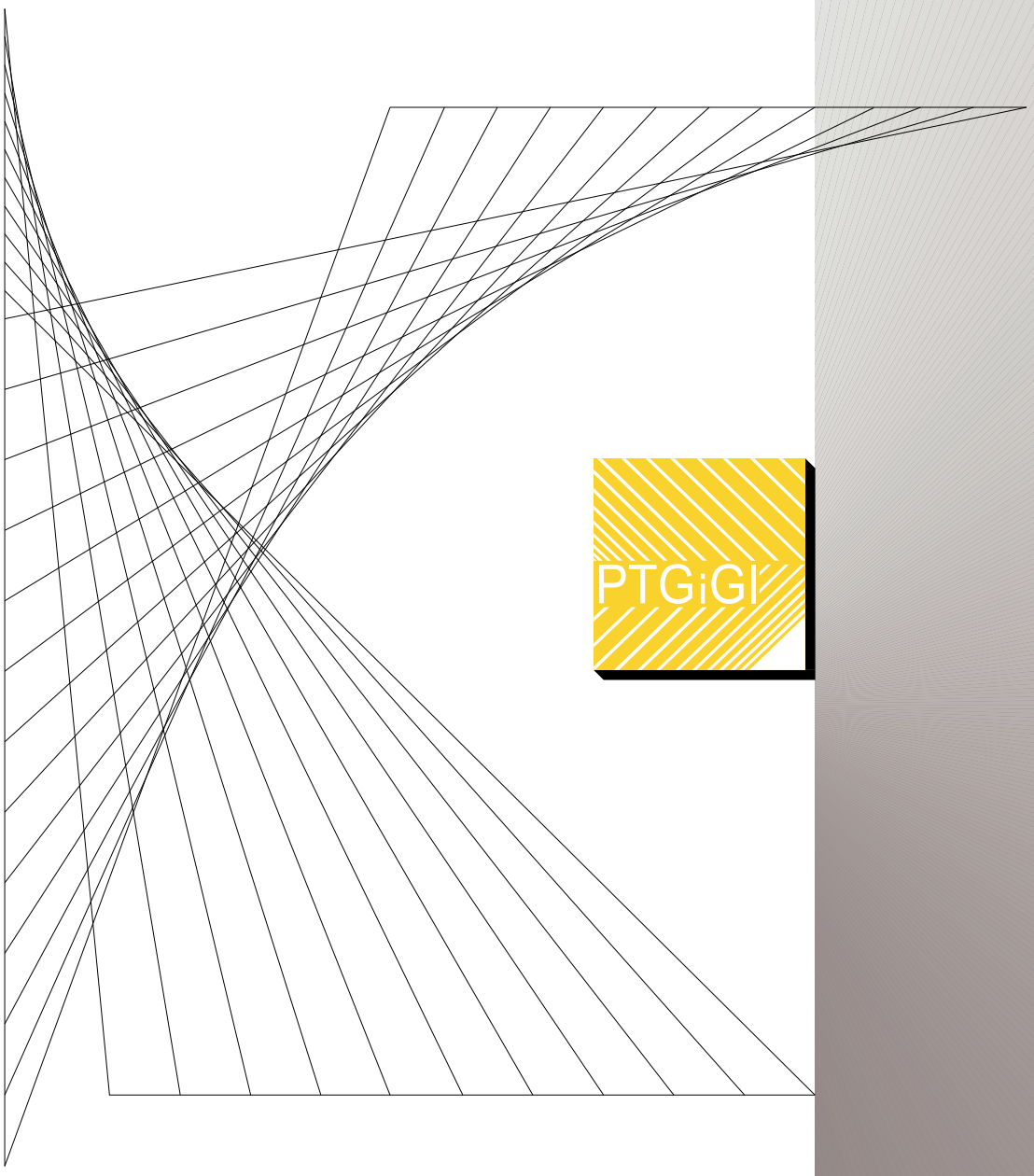

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LEARNING ABOUT THREE-DIMENSIONAL OBJECTS IN A THREE-DIMENSIONAL ENVIRONMENT: IMMERSIVE-ROOM ACTIVITIES FOR PRE-SERVICE MATHEMATICS TEACHERS

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Abstract. Activities in an immersive room were used to teach pre-service math teachers about Platonic solids and were then analysed for their effectiveness. The results show that an immersive room is a promising tool to overcome students' difficulties in comprehending geometry by allowing them a more direct visualization of 3D shapes.

Keywords: Platonic solids, using technology in teaching geometry, immersive room

1 Introduction

The fact that Euclidean geometry is considered both difficult and a less essential discipline in school is well-known and, unfortunately, backed by research [1]. Researchers report a wide spectrum of student difficulties in the comprehension of the nature of geometry and habits of proof and concerning the relationships between various geometric objects [2]. The problem is even more acute when studying three-dimensional objects (prisms, pyramids and other polyhedrons), where typical 2D representations of 3D shapes are often quite difficult to comprehend.

The use of technological tools offers an effective way to overcome part of the difficulties in learning geometry [3], therefore the use of dynamic software in school is becoming more and more widespread. However, its optimal implementation requires in-depth knowledge of the tool on the part of the teacher. Yet, the possibilities that modern technologies offer for visualizing complex ideas advocates their use in teaching and learning [4] and therefore, immersive rooms [5] seem to be a natural extension to the process of actively learning mathematics. This tool is still quite innovative and to date we have found only one example of its use for teaching mathematics [6].

2 Study purpose and method

The purpose of our study was to investigate the effectiveness of an immersive room in teaching Platonic solids to pre-service elementary and secondary mathematics teacher. In total, three activities were designed in which the pre-service teachers were encouraged to explore the structure, nets, and relationships of Platonic solids. The participants (80 in number) were then asked to comment on the effectiveness of the activities regarding their understanding of the topic.

Note that, in Israel, pre-service teachers have generally not met this fascinating and important issue in their didactic courses because the Israeli elementary-school geometry curriculum regards Platonic solids as “additional activities.” This topic is also typically ignored in courses on Euclidean geometry.

3 Activity 1: Why are there just five?

The main aim of this activity is to explore a systematic method of constructing regular convex polyhedra so that students will reach an understanding concerning the number and structure of Platonic solids. Didactically, the activity uses students' prior knowledge about tiling with regular congruent polygons.

The activity begins outside the immersive room. A short review of regular polygons is followed by a verbal description of regular polyhedrons. Typically, students easily present an example of such a shape (a cube in most cases but sometimes a tetrahedron). Students are asked to guess how many regular polyhedra exist. By analogy with the 2D case, they typically suppose that the set of regular polyhedra is infinite.

As additional preparation, students are asked to review the concept of "classic" tiling (tessellations) with regular congruent polygons. By asking leading questions (“How many squares do meet in each junction of tiling?” or “Are five regular triangles enough to fill a junction without a gap?”), they are reminded of (or learn) the basic rule that the combined angles of the adjacent polygons at each vertex must total 360°. A simple analysis of the interplay between the angles of the polygon’s vertex and the number of polygons at each vertex in the tessellations [7] leads students to conclude that there are only three polygons appropriate for regular tiling: squares, regular triangles, and regular hexagons.

At this point, they begin their journey in the specially designed 3D interactive environment. The floor of the immersive room is divided into three parts: the central part is tiled with regular triangles, another part is tiled with squares, and the third is tiled with regular hexagons (see Figure 1).

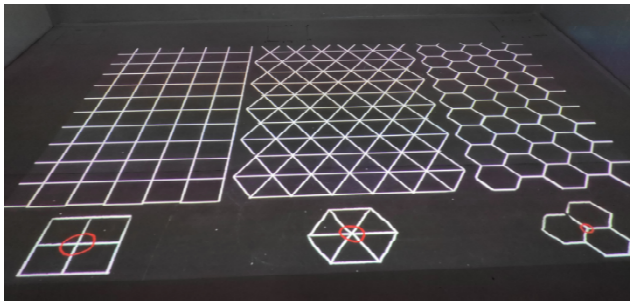

Figure 1: "Tiling" with regular polygons in the immersive room

On the wall above the square tiling, a fragment of the square tessellation is displayed, on the second wall (over the triangular tiling) are three six-triangle tessellation fragments from which one, two, or three triangles have been removed (i.e., what remain are five, four, or three attached triangles, respectively).

Students are asked to describe the situation at the vertex in each case and explain why the pictures have been arranged as they have. Similar questions refer to the diagram of three pentagons meeting at a common vertex situated above the tiling with hexagons.

The discussion at this stage focuses on two interrelated questions:

1. What is a minimum number of regular polygons that can meet at a common vertex to form a polyhedral angle?
2. How many possible polyhedral angles are there in the cases of: regular triangles, squares, and other regular polygons?

In order to "feel" the situation, students touch one of the planar figures on the walls, at which point the figure visually transforms into a polyhedral angle (see example in Figure 2). A further touch completes the figure up to the relevant virtual 3D shape that can be "rotated" to check the regularity of the Platonic Solid. The students note that all the faces are congruent

regular polygons, and each vertex is the meeting point of three (for tetrahedron, cube, and dodecahedron), four (for octahedron), or five (for icosahedron) faces.

After discovering the relevant answer to the first question, students will realize that the angles formed by joining regular *n*-gons where *n* is greater than 5 will be too large to allow the formation of a polyhedral angle. In other words, all regular polyhedra can be constructed only from congruent triangles, squares, or pentagons.

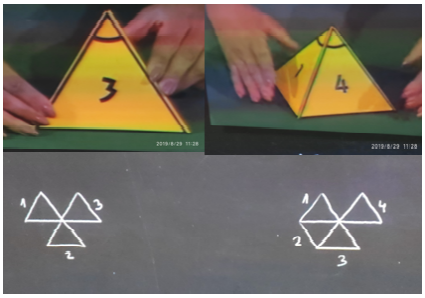

Figure 2: A wall in the immersive room with some polyhedral angles

The conclusion therefore will be that there are exactly five Platonic solids: the cube, with square faces; the dodecahedron, with pentagonal faces; and the tetrahedron, octahedron. and icosahedron, all three with triangular faces.

4 Activity 2: Vertices, edges and faces

This activity is an extension of Activity 1. It uses dynamic pictures to explore issues concerning the net and the main characteristics of 3D shapes, particularly for Platonic solids (see Figure 3).

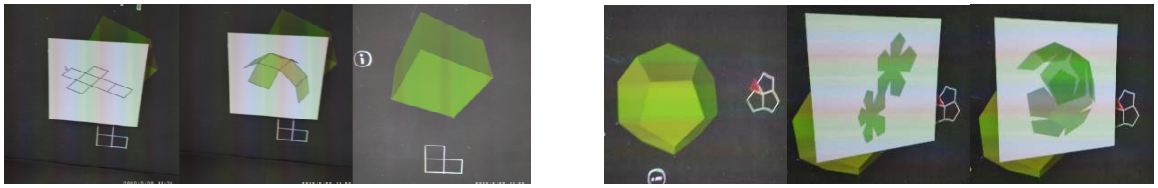

Figure 3. Platonic solids and their nets: the case of a cube and a octahedron

The students are required not just to count the vertices, faces, and edges of each polyhedral, but to verify their findings by determining the interconnection between these numbers. For example, a cube has six faces each with four vertices. But each vertex is counted three times because it is where three faces meet. Thus, in total, there are $\frac{6\times4}{3} = 8$ vertices. This method is much more applicable for those solids that students are less familiar with. For example, an icosahedron has 20 triangular faces. This means 20×3 vertices, each one of which is shared by five faces. Therefore, the shape has a total of $\frac{20\times3}{5} = 12$ vertices. Similarly, the 20×3 edges must be, in fact, divided by two, because each edge is common to two faces, meaning that the shape has 30 edges.

At the completion of this activity in the immersive room, students have filled out a table with the properties of all five Platonic solids. They can verify their results using the touchscreens for the relevant polyhedron. The next stage of this activity takes place outside the immersive room: they are asked to look up the famous Euler formula and to use it to once

more verify their results. Indeed, they will discover that for each Platonic solid, the relationship $V+F=E+2$ (the letters' meanings are clear) is correct.

5 Activity 3: Duality of Platonic solids

We originally designed activities 1 and 2 for pre-service primary school teachers. However, following the intense interest expressed by secondary-school pre-service teachers, we designed a more advanced activity suitable for secondary school to explore the concept of duals.

The table produced after the second activity served as a starting point. Cases of equality between numbers of vertices and faces were noted. For example, a tetrahedron has an equal number of both, and the students constructed a new tetrahedron whose vertices were in the centres of the faces of the original one, thus expressing the self-duality of the tetrahedron. This led them to explore further instances of duals, such as the cube and octahedron: the number of cube faces equals the number of octahedron vertices and vice versa, suggesting that they may be duals of each other. By touching the pictures of these solids on the wall, students can affirm the fact of duality

6 Discussion, conclusions, and further possibilities

All the activities described above were tried with pre-service mathematics teachers. The initial trial did not include any discussion regarding intermediate results, and we found that this was insufficient to build proper comprehension. As a result, we updated the method and conducted a second trial with another group of pre-service teachers. After completing this modified trial, the pre-service teachers (80 in number) evaluated the activity. Almost 100% remarked that their experience in the immersive room was positive. In fact, more than 80% pointed out that the visualization activity in the immersive room provided them with a profound understanding of the issue of Platonic solids and their basic properties, and that their experience in the immersive room profoundly enhanced their understanding of duals.

Certainly, this experience represents a very preliminary pilot project in ascertaining how the immersive room can aid in learning topics in 3D geometry. The success of these initial trials encourages conducting further trials with other populations and on larger scales. Furthermore, this experiment demonstrates that the dynamic visualization of 3D objects in an immersive room definitely helps students to deal with spatial aspects and better understand both the geometric structure of such shapes and the interrelations between them. This implies that an immersive room will aid in teaching other topics connected to spatial reasoning. In fact, the initiative group is presently developing activities that involve the calculation of the volumes of prisms and pyramids and on various basic concepts of non-Euclidian geometry.

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NAUKA O TRÓJWYMIAROWYCH PRZEDMIOTACH W TRÓJWYMIAROWYM ŚRODOWISKU: ZAJĘCIA DLA NAUCZYCIELI MATEMATYKI W PRACOWNI INTERAKTYWNEJ

Zajęcia w pracowni interaktywnej wykorzystano do kształcenia nauczycieli stażystów matematyki na temat brył platońskich, a następnie analizowane ich skuteczności. Wyniki pokazują, że pracownia interaktywna jest obiecującym narzędziem do przezwyciężenia trudności uczniów w zrozumieniu geometrii, umożliwiając im bardziej bezpośrednią wizualizację kształtów 3D.