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# A topological model to assess networks connectivity and reliability: recent development

**Keywords**

network, connectivity, failure, reliability, availability

**Abstract**

The acceleration of the interconnectivity of networks of all sorts brings to the front scene the issue of networks performance measure. Recently, one observed an accelerating course towards quantitative probabilistic models to describe and assess networks' Connectivity, as being the main vector of performance. However, modelling realistic networks is still far from being satisfactorily achieved using quantitative probabilistic models.
On the other hand, little room had been lift to exploring the potential of topological models to develop qualitative and semi- quantitative models in order to assess networks connectivity. In this paper, we are exploring the potential of the topological modelling. The proposed model is based on describing the node-pair connectivity using binary scalars of different orders (tensors). Preliminary results of our explorations sounded very promoting.

## 1. Introduction

This paper is completing the paper presented last year during SSARS-2012 on the same subject [12].
The interconnection between networks of different types is growing as never. This acceleration of interconnectivity of all sorts results in higher requirements on network performances. It brings to the front scene the issue of assessing the performance of networks in design and operation.
Until very recent, "performance" was used to be tackled in terms of "connectivity" which is measured using the "probability" of being connected.
But, what "connected" is?
What is the best measure of this connectivity on both levels: node-pairs and overall the network?
The use of probabilities is a natural choice in order to measure the "connectivity". But, the "connectivity" itself does still need deeper understanding when it is matter of more than two nodes.
Topological modelling has not been explored enough and seems to provide a promising tools to develop qualitative and semi- quantitative models describing the "connectivity".
We do not claim that "connectivity" is the only dimension in the space of "network performance". But, it seems unavoidable starting point.
This paper completes the investigations presented in [12]. In the paper, we focus on the development of some connectivity measures and their applicability rather than on the mathematical nature of these objects.
The proposed model is still in its earliest phase of development. But, it sounds very promising.
The term "performance" would be extended, beyond "connectivity", to "resilience" and to "robustness" without excluding other possible dimensions, such as: the maximum/optimum flux.

## 2. Description

The paper deals with the "connectivity" as classically used in "terminal-pair reliability –TPR" problem. Two major types of network topologies can be mentioned as well: Point-to-Multipoint (PMP) and mesh type networks.
In the paper, the focus is done on the PMP-like modeling. We will not deal with "mesh type" networks. Models treating the Mesh Network are given in [8] [1].
Very often distinction is done between three modes of network *Connectivity* measure*, such as:*

Two-terminal connectivity
It measures the ability of the network to satisfy the communication needs of a specific pair of nodes. *Two-terminal* availability defines the probability that there is at least one available path in the network connecting a specified pair of nodes.
*k*-terminal connectivity
It measures the ability of the network to satisfy the communication needs of a subset *k* of specified nodes. The *k-terminal* availability is defined as the probability that for *k* specified target nodes there is at least one connecting path between each pair of the *k* nodes, in the network.
All-terminal connectivity
It measures the ability of the network to satisfy the communication needs of all nodes in the network. *All-terminal availability* determines the probability that there is at least one path connecting each pair of nodes in the network
Some reliability models for the PMP are given in [2], [11], [17], [20], [21]
The most common manner to represent networks is to use graphs theory notations. A graph $G=(V,E)$ is a well-defined set of vertices (nodes), $V$ , and edges (links), $E$ .
The performance of each node and each link is defined by a failure probability or a failure rate. These failure figures are functions of the used materials, technology, operational conditions and network's topology.
Three links-failure modes are generally identified such as: path loss, shadowing and signal fading, [6]. These are the failure modes recognized by the IEEE 802.16 WG for communication networks. That could be extended to the PMP networks as well.
Exponential models (Poisson's stochastic process) are often proposed [6], to describe failures occurrence.
Often, some authors confuse "Reliability" and "Availability" concepts, e.g. [10].
Some others, [15] recalls: "... Network reliability refers to the reliability of the overall network to provide communication in the event of a failure of a component in the network, and it depends on the sustainability of both hardware and software."
I, myself, would call that aptitude "Network Availability". The concept "Reliability" can't subsist without referencing to "time duration".
The *Connectivity* between a source node and a receptor node could thus be measured by: the availability of at least a path from one node to the other, ($s-t$ model). This is a very necessary measure that we can (/should) determine for each couple of nodes. Still, it is not yet a network OVERALL measure. It is a local one!

We have three problems to overcome:
*1st: the Order of the Connecting path*
A robust modelling (topological or analytical) should be able to integrate the order of the connecting path between any pairs of nodes. The order of the path defines how many edges (/links) exist between the source node and the receptor one.

*2nd: the Multiplicity of the Connecting Paths*
A robust modelling (topological or analytical) should be able to integrate the number of the connecting paths between any pairs of nodes. The multiplicity of the connecting paths determines how many connecting paths exist between a given source node and a given receptor one, whatever are their orders.

*3rd: the Overall Network Connectivity*
A robust modelling (topological or analytical) should be able to develop a measure of the network overall connectivity. The network overall connectivity may be determined for each connectivity order, i.e. in a spectral way.

Identifying clearly the fundamental problems would guide us towards the right directions.
We recall that we are interested in exploring the potential of the topological modelling to *describe connectivity*.

## 3. Overview on the state-of-the-art

Although it is not the principal issue of the paper, this selective overview of the state-of-the-art would serve as a referential background to our work.

We start by the network *Connectivity* metric, $C(t)$, as in [15], such as:

$$C(t) = \frac{N_{con}(t)}{M},$$

where,

$N_{con}(t)$: is the number of connected node-pairs at time $t$, whatever the order of the connecting path of each node-pair.
$M$ : is the total number of node-pairs in the network
For a network containing $N$ nodes, the total number of node-pairs in the network $M$ is equal to:

$$M = \frac{N(N-1)}{2}.$$

Mandiratta, [15], proposed a model to determine the network *connectivity*, based on a given minimal *Connectivity* condition. In that model, $n$ is the minimal acceptable number of connecting nodes in a network containing $N$ nodes. The network *connectivity*, under this minimal connectivity condition, will be measured such that:

$$P(n:N;C) = \sum_{j=n}^{N} \binom{N}{j} p^j q^{N-j}, \quad n > 1$$

where,

$P(n:N;C)$: The probability that the network is available (/connecting) (at least $n$ nodes out of $N$ are available).
$\binom{N}{j}$ : The number of possible combinations. The number of possible sets containing $j$ nodes available out of $N$.
$p$ : The probability that a given node is available (connected)
$q$ : The probability that a given node is unavailable (disconnected)

This constrained unavailability (at least $n$ nodes are connected, $n > 1$) could be one measure of the network connectivity.
Is it really enough?
Would engineers accept having at least $n$ nodes connected out of $N$, $n > 1$, without knowing more about the of the network overall connectivity?

Regarding some critics that the above expression, recognized by Mandiratta [15], does not describe a coherent system, our answer is that the expression itself (the sum) describes a coherent system. But each term in the sum does not.
Indeed the term $p^j q^{N-j}$ is not monotone with $p$ neither with $j$. It has a maximum when:

$$p = \frac{j}{N}$$

But the sum of all these terms is monotone. In *Figure 1*, the availability of a network made of 10 nodes is given as a function of both the node unavailability and the minimum number of available nodes, (according to Mandiratta [15]). The Nodes are supposed identical.

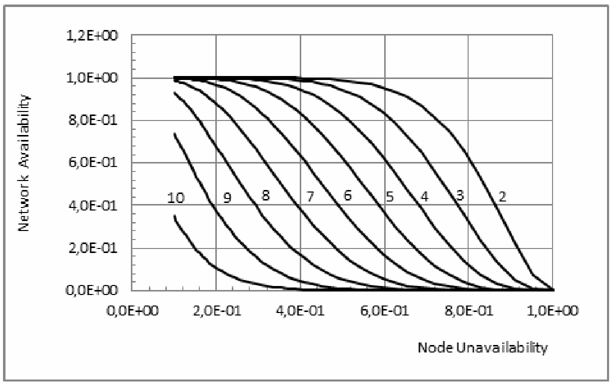


*Figure 1.* Network availability versus node unavailability as a function of the minimum number of available nodes out of 10

It is obvious that the network availability (according to Mandiratta[15]) is monotone and the model describes a coherent system.
Many works have been carried on in order to develop algorithms based on the previous model, [3].
Besides, most of the researchers, working on the determination of the network availability, treat the problem as consecutive k-out-of-N failure (K:N;F) problem, e.g. [14].
The only objective remark I would formulate about the model of Mandiratta is that links unavailability are not explicitly integrated in the model. It looks as if it considers only nodes failures.
Very often in network connectivity, the following assumptions are considered:

- Nodes are completely reliable; only links fail. (failure rates of nodes are by so far smaller than the link failure ones).

- Link failures (nodes as well) are independently random events, [4], [18].
- Sometimes link failures are supposed equally probable. This assumption is often made because no detailed information about link failures is available, whereas information about the average failure is available, [9]

Significant R&D efforts are devoted to the development of numerical algorithms in order to determine the probability that a given network may have a determined level of *connectivity,* [2], [4], [7], [11], [13], [15], [17].

Most of the researchers called this probability "Reliability" of consecutive K-out-of-N system. Recently, this has been evolved to a k-within consecutive-$r \times s$-out-of-$m \times n : F$ system, [14]. This is a generalisation of the problem of consecutive k-ou-of-N failures.

The network availability is generally determined either analytically or numerically. Analytical schemes are limited by the size and the topological complexity of the network. For large and complex network numerical methods such as: Mont-Carlo simulation, neural-network models, or genetic algorithms are developed.

Some interesting applications are given in [16], [19].

Another major task in network design is to optimize network reliability (connectivity, performance, resilience, …) versus cost.

Still, we should first be able to determine and to measure the connectivity.

Three classic meta-heuristic procedures are often recognized for solving large and realistic designs: steepest descent, simulated annealing and genetic algorithms. These procedures are clearly described and compared in [4], with an interesting list of corresponding references.

Three sets of difficulties are generally reported. The 1st is related to failure data availability. The second is relative to the combinatorial aspect of the problem. The 3rd arises from the interdependence between different failure paths (cut-sets).

## 4. The topological model

Networks may be represented using graphs. A graph $G = (V, E)$ is a well-defined set of vertices (nodes), $V$ , and edges (links), $E$ .

A network is a graph containing at least 3 nodes, $N \geq 3$.

We are looking for constructing a connectivity measure dependant on the number of links necessary to link two nodes (order) in a given network.

The proposed connectivity measure $C_{ij}^{n}$ is a (binary) scalar (tensor) of order $n$ describes the connecting state between two independent nodes $i$ and $j$ in a given network, such that: it takes the value 1 if the two nodes are connected, otherwise it takes the value 0. The order $n$ refers to the number of links connecting the two nodes.

The $C_{il}^{1}$ will be called the network identity tensor. It describes the 1st order connecting state between all the nodes, i.e., it determines the couples of nodes that are directly connected. In a certain way, it describes the topology of the network and contains all the information we need to know about the connectivity state of the network.

Some measures will be developed and will show of some interest to assess the connectivity.

<u>*Connectivity binary measure*</u>

A connectivity binary measure, $C_{ij}^{n+1}$, is determined by the following recursive relation:

$$T_{ij}^{n+1} = C_{il}^{1} \,.\, T_{lj}^{n} \text{, and } T_{lj}^{0} = 1 \quad i, l, j \in [1, N]$$

$$C_{ij}^{n+1}(T_{ij}^{n+1}) = 1, \text{ if } \forall T_{ij}^{n+1} > 0 ,$$

$$\text{otherwise, } C_{ij}^{n+1}(T_{ij}^{n+1}) = 0$$

where,

$C_{il}^{1}$ : is the network identity tensor.

$T_{ij}^{n+1}$ : is the total number of cuts of order less than or equal to $n+1$, connecting two nodes $(i, j)$ .

$N$ : is the total number of nodes in the network.

We would constrain only to the treatment of the links by switching $C_{ii}^{n+1}(:)$ as follows:

$$C_{ii}^{n+1}(:) = 0, \quad \forall i \in [1, N]$$

Number of Links

The total number of links $L_N$ in a given network can be determined using the network identity tensor $C^1_{ij}$ as following: $L_N = \sum_{i=1}^{N}\sum_{j=i}^{N} C^1_{ij}$

Connectivity Indicator and Connectivity Ratio

The tensor $T^n_{ij}$ allows determining the number of baths of order $n$ that join the node-pair $(i, j)$.
For each node, one would determine a connectivity indicator, $I^n_i$, and a connectivity ratio, $R^n_i$, of a given order, such that:

$$I^n_i = \left(\sum_{j=1}^{N} C^n_{ij}\right), \text{ and } R^n_i = \frac{I^n_i}{(N-1)}$$

Notice the slight difference with [12] because of the fact that we consider only the failure of links.

Overall Connectivity Indicator and Overall Connectivity Ratio

Based on all nodes' connectivity indicator, one can define a network overall connectivity indicator, $I^n_{overall}$, and connectivity ratio, $R^n_{overall}$, of a given order, such that:

$$I^n_{overall} = \sum_{j=1}^{N} I^n_i, \text{ and } R^n_{overall} = \frac{\sum_{i=1}^{N} R^n_i}{N}$$

At last, one may define the network highest order, $n_\infty$, such that, this is the minimum necessary order of cut-sets (links) so that all nodes may become mutually connected. That is can be described as following:

$$n_\infty = Min.\left(n; C^n_{ij} = 1, \forall i, j \in [1, N]\right)$$

Spectral Connectivity Index

The spectral connectivity index $\theta^n_{ij}$ is a binary scalar whose value determines the minimal connectivity order for the node-pair $(i, j)$. It is defined as following:

$$\theta^{n+1}_{ij} = 1, \text{ if } (C^{n+1}_{ij} - \sum_{l=1}^{n} C^l_{ij}) > 0$$

Otherwise, $\theta^{n+1}_{ij} = 0$.

It can been demonstrated that:

$$\theta^{n \geq n_\infty}_{ij} = 0; \quad \forall i, j \in [1, N]$$

Spectral Distribution

The spectral distribution $\nu^n_i$ determines the number of the minimal order paths connecting the node $(i)$ to the network. It is given by:

$$\nu^n_i = \sum_{j=1}^{N} \theta^n_{ij}$$

Obviously,

$$\sum_{n=1}^{n_\infty} \nu^n_i = N - 1, \quad \forall i \in [1, N]$$

Recalling that:

$$n_\infty = Min.\left(n; C^n_{ij} = 1, \forall i, j \in [1, N]\right)$$

## 5. Study case

In order to demonstrate the interest of the different measures, developed above, a study case is used to illustrate their interest. The studied network is schematically presented in figure 2.
It describes a network composed of 20 nodes and 30 links, exported from [4]. Each node is connected to 3 other nodes. That is one of the most widely used PMP type of connectivity in networks design [2], [11], [20], [21].

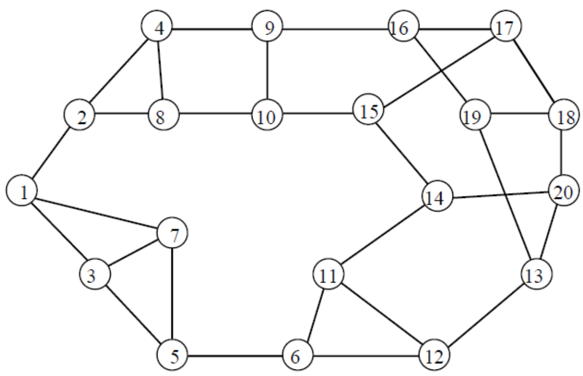


*Figure .2.* Schematic presentation of NET-I [5]

*Network identity tensor*

Examining the presentation given in Fig.2, one may deduce the identity matrix $C^1_{ij}$ expression in the following manner:

$C^1_{ij}$ is equal to 1, for the following $(i, j)$: $(1,2), (1,3), (1,7), (2,1), (2,4), (2,8), (3,1), (3,5),$ $(3,7), (4,2), (4,8), (4,9), (5,3), (5,6), (5,7),$ $(6,5), (6,11), (6,12), (7,1), (7,3), (7,5), (8,2),$ $(8,4), (8,10), (9,4), (9,10), (9,16), (10,8),$ $(10,9), (10,15), (11,6), (11,12), (11,14), (12,6),$ $(12,11), (12,13), (13,12), (13,19), (13,20),$ $(14,11), (14,15), (14,20), (15,10), (15,14),$ $(15,17), (16,9), (16,17), (16,19), (17,15),$ $(17,16), (17,18), (18,17), (18,19), (18,20),$ $(19,13), (19,16), (19,13), (19,18), (20,13),$ $(20,14), (20,18)$

Otherwise $C^1_{ij}$ is equal to 0.

*Number of Links*

To recall that the total number of links $L_N$ is determined as following:

$$L_N = \sum_{i=1}^{N}\sum_{j=i}^{N} C^1_{ij} \, .$$

Using the data given above, we may determine that:

$$L_N = 30.$$

*Connectivity Indicator and Connectivity Ratio*

One recalls that or each node, one would determine a connectivity indicator, $I^n_i$, and a connectivity ratio, $R^n_i$, of a given order, such that:

$$I^n_i = \left(\sum_{j=1}^{N} C^n_{ij}\right), \quad \text{and } R^n_i = \frac{I^n_i}{(N-1)} \, .$$

The connectivity indicator, $I^n_i$, and a connectivity ratio, $R^n_i$, have already been treated and determined previously in [12].

*Overall Connectivity Indicator and Overall Connectivity Ratio*

As mention in the preceding section, both the network overall connectivity indicator, $I^n_{overall}$, and connectivity ratio, $R^n_{overall}$, of a given order have already been fully determined in [12].

*The Network highest order*

The highest order, $n_\infty$, of the network is found to be equal to: $n_\infty = 6$

*Spectral Connectivity Index*

Recalling that the spectral connectivity index $\theta^n_{ij}$ has previously been defined as following:

$$\theta^{n+1}_{ij} = 1, \quad \text{if } (C^{n+1}_{ij} - \sum_{l=1}^{n} C^l_{ij}) > 0$$

Otherwise, $\theta^{n+1}_{ij} = 0$.

The spectral connectivity indices $\theta^n_{ij}$ are given in *Tables 1* to *6*.

*Spectral Distribution*

One recalls that the spectral distribution $\nu^n_i$ is given by:

$$\nu^n_i = \sum_{j=1}^{N} \theta^n_{ij}$$

The values of the spectral distribution $\nu^n_i$ are given in table (7). We can equally verify that:

$$\sum_{n=1}^{6} \nu^n_i = 20 - 1 = 19 \quad \forall i \in [1, N]$$

## 6. Transitions

We would like, in this section, to distinguish between the two types of transitions that this topological model allows us to distinguish: the critical transitions and the degradation transition.

*Critical Transition*

If all the $\nu^n_i$ paths, at a given order $n$, fail as a result of the failure of only one link somewhere in

the network, the node ($i$) may still be connected but with paths of higher order than $n$. This transition in the connectivity of the node ($i$) to a higher order because of the failure of only one link is called a "critical transition". Critical transitions produce failures.
We call that a node connectivity failure of order $n$.
Critical transitions are immediately determined thanks to the binary values of the spectral connectivity index $\theta_{ij}^{n}$.

<u>*Degradation Transition*</u>

If at maximum, $(\nu_i^n - 1)$ paths, at a given order $n$, fail as a result of the failure of only one link somewhere in the network, the node ($i$) may still be connected by at least one bath of order $n$.
We call that a node connectivity degradation of order $n$.

## 7. Loss of connectivity probability

We will focus here on the case of the failure of only one link. We will not be interested in the exact failure probability distribution of the links.
It could be exponential or be another distribution function. In all cases, we suppose that the failure probability distribution function of the link is well defined and is identical for all links.
The number of the minimal order paths connecting a node ($i$) to the network is $\nu_i^n$. Losing all these connecting paths is a critical transition.
Subsequently, The node ($i$) losses connectivity if all these minimal paths of all orders are lost.
The probability that node ($i$) loses its connectivity with the network, $Q_i(t)$, is determined by:

$$Q_i(t) = \sum_{n=1}^{n_\infty} \nu_i^n \cdot Q_i^n(t)$$

where,

$$Q_i^n(t) = (q(t) \cdot \sum_{l=1}^{n} (1 - q(t))^{l-1}) \cdot (1 - q(t))^{(L_N - n)}$$

and
$\nu_i^n$ : is the number of the minimal order paths.
$Q_i^n(t)$ : the probability that only one link fails in a path of order $n$.
$q(t)$ : The probability that a given link is unavailable

## 8. Conclusion

The paper presents a topological model of describing networks connectivity and extends the model to describe transitions.
Compared to the paper [12], the major achievements in the paper are the development of the Spectral Connectivity Index and the Spectral Distribution.
Describing the connectivity in terms of binary tensors allows assessing the connectivity of each node to the network. It allows also distinguishing between the critical transitions and the others (degradation transitions). Once the critical transitions are determined, one may assess the connectivity of a node with its network in probabilistic terms (availability)
More development is underway to describe the transitions and to contribute into the definitions of overall failure probabilities or overall failure rates.

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*Table 1.* The Spectral Connectivity Indices and Spectral Distributions associated to the network NET-I

$n = 1$

| $\theta^1_{ij}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **3** | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **7** | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **8** | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **9** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| **10** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| **11** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| **12** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **13** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| **14** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| **16** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| **17** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| **18** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| **19** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| **20** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\nu_i^1$ | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** | ***3*** |

*Table 2.* the Spectral Connectivity Indices and Spectral Distributions associated to the network NET-I

$n = 2$

| $\theta_{ij}^2$ | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **3** | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| **5** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| **7** | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **8** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| **9** | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **10** | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| **11** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| **12** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| **13** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| **14** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| **15** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| **16** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| **17** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| **18** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| **19** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| **20** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| $\nu_i^2$ | ***3*** | ***4*** | ***2*** | ***3*** | ***3*** | ***4*** | ***2*** | ***3*** | ***5*** | ***5*** | ***4*** | ***4*** | ***5*** | ***6*** | ***6*** | ***5*** | ***5*** | ***4*** | ***4*** | ***5*** |

*Table 3.* the Spectral Connectivity Indices and Spectral Distributions associated to the network NET-I

$n = 3$

| $\theta_{ij}^{3}$ | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| **3** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **5** | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| **7** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **8** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| **9** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| **10** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| **11** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| **12** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| **13** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| **14** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **15** | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| **16** | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| **17** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **18** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **19** | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| **20** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $V_{i}^{3}$ | ***3*** | ***3*** | ***4*** | ***5*** | ***3*** | ***4*** | ***4*** | ***5*** | ***4*** | ***5*** | ***6*** | ***5*** | ***4*** | ***5*** | ***6*** | ***5*** | ***4*** | ***4*** | ***6*** | ***3*** |

*Table 4.* the Spectral Connectivity Indices and Spectral Distributions associated to the network NET-I

$n = 4$

| $\theta_{ij}^{4}$ | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| **2** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| **5** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| **6** | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| **7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| **8** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| **9** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| **10** | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **11** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| **12** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| **13** | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **14** | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **15** | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **16** | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **17** | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **18** | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **19** | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **20** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\nu_i^4$ | *4* | *4* | *4* | *4* | *5* | *5* | *4* | *5* | *5* | *5* | *4* | *4* | *4* | *4* | *2* | *3* | *3* | *3* | *3* | *3* |

*Table 5*. the Spectral Connectivity Indices and Spectral Distributions associated to the network NET-I

$n = 5$

| $\theta_{ij}^5$ | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| **2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| **4** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| **5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| **6** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| **8** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **9** | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **10** | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **11** | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **12** | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **13** | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **14** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **15** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **16** | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **17** | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **18** | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **19** | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **20** | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\nu_i^5$ | *4* | *5* | *4* | *4* | *5* | *3* | *4* | *3* | *2* | *1* | *2* | *3* | *3* | *1* | *2* | *3* | *2* | *2* | *3* | *4* |

*Table 6.* the Spectral Connectivity Indices and Spectral Distributions associated to the network NET-I

$n = 6$

| $\theta_{ij}^{6}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| **2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| **4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| **8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **10** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **12** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **13** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **14** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **15** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **16** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **17** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **18** | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **19** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **20** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\nu_i^6$ | ***2*** | ***0*** | ***2*** | ***0*** | ***0*** | ***0*** | ***2*** | ***0*** | ***0*** | ***0*** | ***0*** | ***0*** | ***0*** | ***0*** | ***0*** | ***0*** | ***2*** | ***3*** | ***0*** | ***1*** |

*Table 7.* Spectral Distributions associated to the network NET-I

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu_i^1$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| $\nu_i^2$ | 3 | 4 | 2 | 3 | 3 | 4 | 2 | 3 | 5 | 5 | 4 | 4 | 5 | 6 | 6 | 5 | 5 | 4 | 4 | 5 |
| $\nu_i^3$ | 3 | 3 | 4 | 5 | 3 | 4 | 4 | 5 | 4 | 5 | 6 | 5 | 4 | 5 | 6 | 5 | 4 | 4 | 6 | 3 |
| $\nu_i^4$ | 4 | 4 | 4 | 4 | 5 | 5 | 4 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 2 | 3 | 3 | 3 | 3 | 3 |
| $\nu_i^5$ | 4 | 5 | 4 | 4 | 5 | 3 | 4 | 3 | 2 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 2 | 2 | 3 | 4 |
| $\nu_i^6$ | 2 | 0 | 2 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 3 | 0 | 1 |
| | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** | ***19*** |