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# Multimodal processes scheduling in mesh-like network environment

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Multimodal processes planning and scheduling play a pivotal role in many different domains including city networks, multimodal transportation systems, computer and telecommunication networks and so on. Multimodal process can be seen as a process partially processed by locally executed cyclic processes. In that context the concept of a Mesh-like Multimodal Transportation Network (MMTN) in which several isomorphic subnetworks interact each other via distinguished subsets of common shared intermodal transport interchange facilities (such as a railway station, bus station or bus/tram stop) as to provide a variety of demand-responsive passenger transportation services is examined. Consider a mesh-like layout of a passengers transport network equipped with different lines including buses, trams, metro, trains etc. where passenger flows are treated as multimodal processes. The goal is to provide a declarative model enabling to state a constraint satisfaction problem aimed at multimodal transportation processes scheduling encompassing passenger flow itineraries. Then, the main objective is to provide conditions guaranteeing solvability of particular transport lines scheduling, i.e. guaranteeing the right match-up of local cyclic acting bus, tram, metro and train schedules to a given passengers flow itineraries.

**Key words:** passengers flow scheduling, multimodal processes, cyclic scheduling, mesh-like structure.

## 1. Introduction

Multimodal processes scheduling are found in different application domains (such as manufacturing, intercity fright transportation supply chains, multimodal passenger transport network combining several unimodal networks (bus, tram, metro, train, etc.) as well as service domains (including passenger/cargo transportation systems, e.g. ferry, ship, airline, AGV, train networks, as well as data and supply media flows, e.g., cloud computing, oil pipeline and overhead power line networks) (Abara 1989; Bielli et al. 2006; Clarke et al. 1996; Friedrich 1999). Multimodal processes executed in Multimodal Transportation Network (MTN), i.e. a set of transport modes which provide connection

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from origin to destination, can be seen as passengers and/or goods flows transferred between different modes to reach their destination (Bocewicz and Banaszak 2013). The throughput of passengers and/or freight depends on geometrical and operational characteristics of MTN.

In that context the solutions of the layout designs exposing the mesh-like structures are frequently observed. A simple example of mesh-like communication network of streets in a city provides a grid of linked nodes defining a more or less ordered pattern (Buhl et al. 2006). The key characteristic of the orthogonal geometry of a proper grid pattern is that any and all streets are equally accessible to traffic and could be chosen at will as alternative routes to a destination. Its inherent advantage is its tendency to yield regular lots in well-packed sequences. This maximizes the use of the land of the block; it does not, however, affect street frequency. Usually the streets in a grid are numbered, lettered, or arranged in alphabetical order. Such a Manhattan-like regular, encompassing repeating design units of transportation structures can be seen in many irrigation and energy/data transmission systems as well as in AGVS' (Hall et al. 2001, Sharma 2012) layouts.

The problems arising in these kinds of networks concern multimodal routing of freight flows and supporting them Multimodal Transportation Processes (MTP) scheduling, are NP-hard (Levner et al. 2010). Since the transportation processes executed along unimodal networks are usually cyclic, hence the multimodal processes supported by them have also periodic character. That means, the periodicity of MTP depends on periodicity of unimodal (local) processes executed in MTN. Of course, the MTP throughput is maximized by minimization of its cycle time. Many models and methods have been considered so far (Levner et al. 2010). Among them, the mathematical programming approach (Abara 1989; Kampmeyer 2006), max-plus algebra (Polak et al. 2004), constraint logic programming (Bocewicz and Banaszak 2013), Petri nets (Song and Lee 1998) frameworks belong to the more frequently used. Most of them are oriented at finding of a minimal cycle or maximal throughput while assuming deadlock-free processes flow. The approaches trying to estimate the cycle time from cyclic processes structure and the synchronization mechanism employed (i.e. mutual exclusion instances) while taking into account deadlock phenomena are quite unique.

In that context our main contribution is to propose a new modeling framework enabling to evaluate the cyclic steady state of a given mesh-like structure (see Fig. 4a) of concurrently interacting cyclic processes (SCCP) encompassing the behavior typical for transportation services supporting passengers traffic in the city (see Fig. 1a)). The following questions are of main interest (Bocewicz and Banaszak 2013): Can the assumed transportation city network, e.g. a metro, functioning meet the passengers' itinerary deadline imposed by scheduled passengers flow processing? Does there exist passengers transport network composed of different bus, tram, metro and train lines enabling to schedule the transportation units as to follow lag-free service of scheduled passengers itinerary processing? So, the main question is: Can the MTP reach their goals subject to constraints assumed on SCCP?

In other words, the paper's objective concerns of MTN infrastructure assessment from the perspective of possible street layout oriented requirements imposed by grid street plan on mesh-like MTP scheduling The rest of the paper is organized as follows: Section 2 introduces a concept of multimodal transportation network encompassing a given streets grid pattern and then provides its representation in terms of systems of concurrently flowing cyclic processes and mesh-like structure models. Section 3 provides the problem formulation. Section 4 discuses the declarative modeling driven approach to multimodal processes scheduling problems. The mesh-like passengers transportation network is considered, and a match-up cyclic processes scheduling principle is proposed. In turn, computational experiments and conclusions are presented in Sections 5 and 6, respectively.

## 2. Multimodal networks

Multimodal Transportation Network (MTN) concerning the organization of city traffic and the network of public transportation can be modeled with focus on the network of city serviced lines and/or routes. Subway or tram lines as well as bus routes form cycles interconnected via common shared interchange stations or closely situated (short walk-distance) transportation mode specific stations. The means of transportation servicing a particular line mode can be seen in turn as transportation processes enabling passengers to move along their destination route.

### 2.1. MMTN modeled in terms of a system of concurrently flowing cyclic processes concept

Multimodal Transportation Network (MTN) concerning the organization of city traffic and the network of public transportation can be modeled with focus on the network of city servicing lines. Subway, tram or bus lines form cycles interconnected via common shared interchange stations or closely situated (short walk-distance) transportation mode specific stations. The means of transportation servicing a particular line mode can be seen in turn as transportation processes enabling passengers to move along their destination route.

Consequently, the MTP network treated as a network of vehicles periodically circulating along cyclic routes (see Fig. 1a) can be modeled in terms of Systems of Concurrently flowing Cyclic Processes (SCCP) shown in Fig. 1b). Vehicles used for passengers transportation follow two directions: North-South (blue line – $mP_1$) and East-West (red line – $mP_2$), while setting routes along which multimodal processes are executed. These routes are composed of fragments of routes of local transportation lines (trams and busses). In the considered case, there are four transportation means: trams (streams $P_1^1$, $P_3^1$) and busses (streams $P_2^1$, $P_4^1$).

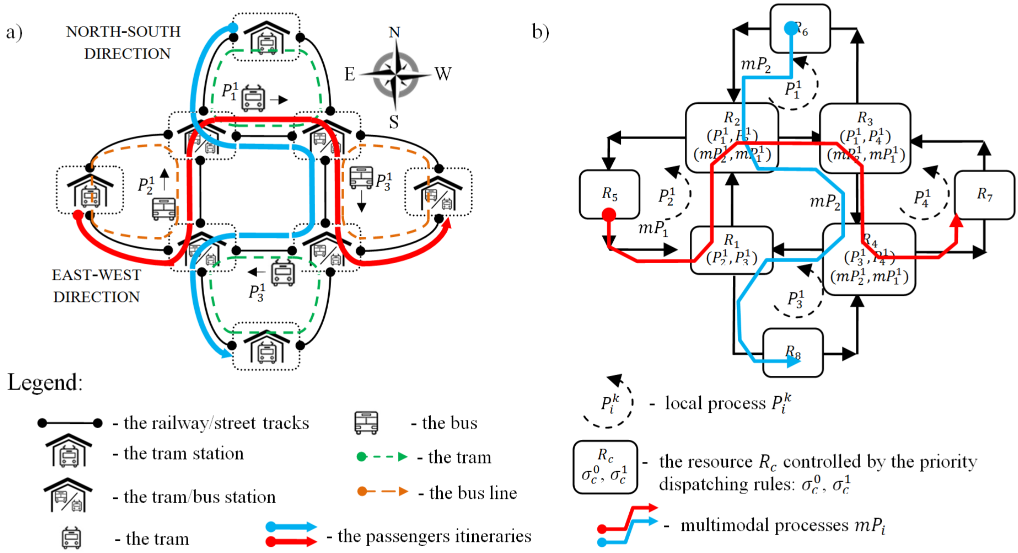

Figure 1: An example of MTP network a), and corresponding SCCP model b).

The considered class of SCCPs consists processes of two categories:

- **local processes** $P_i$, (e.g. $P_1$, $P_2$, $P_3$, $P_4$), whose operations are cyclically repeated along the set of transportation routes (sequences of successively visiting stations – **resources** $R = \{R_1,\dots,R_c,\dots,R_9\}$, $R_c$ – the $c$-th resource). The local cyclic processes $P_i$ contains the set of **streams** $P_i = \{P_i^1,\dots,P_i^k,\dots,P_i^{ls(n)}\}$, $P_i^k$ – the $k$-th stream of the $i$-th local process? $P_i$. The different streams, i.e., vehicles (see Fig. 1a)), following the same route occupy different resources (stations). In the considered case all processes $P_1$, $P_2$, $P_3$, $P_4$, consist of unique streams: $P_1 = \{P_1^1\}$, $P_2 = \{P_2^1\}$, $P_3 = \{P_3^1\}$, $P_4 = \{P_4^1\}$. Transportation routes of corresponding vehicles (see Fig. 1b)) are as follows:

$$p_1^1 = (R_6, R_2, R_3),\ \ p_3^1 = (R_8, R_4, R_1) \ \ – \ \ \text{routes of trams},$$

$$p_2^1 = (R_5, R_1, R_2),\ \ p_4^1 = (R_7, R_3, R_4) \ \ – \ \ \text{routes of busses},$$

where: $R_1$, $R_2$, $R_3$, $R_4$ – resources shared by local processes, $R_5$, $R_6$, $R_7$, $R_8$ – non-shared resources.

The notation below will be used, as well:

$o_{i,j}^k$ – denotes the $j$-th operation executed by the stream $P_i^k$, i.e., operation executed on the $j$-th element (resource) occurring in the route $p_i^k$ (e.g. $o_{1,1}^1$ is the operation executed on the first resource occurring in the route $p_1^1$, i.e., the resource $R_6$),

$t^k_{i,j}$ – denotes the execution time of operation $o^k_{i,j}$,
$x^k_{i,j}(l)$ – the moment of operation $o^k_{i,j}$ beginning in the $l$-th cycle of stream $P^k_i$.

- **multimodal processes** $mP_i$ (e.g. $mP_1$, $mP_2$) representing flows of passengers following they itineraries in MMTN environment. Operations of the multimodal processes are implemented cyclically along routes being compositions of fragments of routes of local processes. In general, each multimodal process consists of a set of streams: $mP_i = \{mP^1_i, \ldots, mP^k_i, \ldots, mP^{lsm(i)}_i\}$, $mP^k_i$ – the $k$-th stream of $mP_i$. In the considered case each multimodal process consist of unique stream: $mP_i = \{mP^1_i\}$, $i = 1,2$, following the route below (see Fig. 1b)):

$$mp^1_1 = ((R_5,R_1,R_2) \frown (R_2,R_3) \frown (R_3,R_4,R_7)) = (R_5,R_1,R_2,R_3,R_4,R_7),$$

$$mp^1_2 = ((R_6,R_2,R_3) \frown (R_3,R_4) \frown (R_4,R_1,R_8)) = (R_6,R_2,R_3,R_4,R_1,R_8).$$

where $(R_5,R_1,R_2)$, $(R_2,R_3)$, $(R_3,R_4,R_7)$ – subsequences of routes $p^1_2$, $p^1_1$, $p^1_4$, defining the transportation sections of $mp^1_1$, $(R_6,R_2,R_3)$, $(R_3,R_4)$, $(R_4,R_1,R_8)$ – subsequences of routes $p^1_1$, $p^1_4$, $p^1_3$, defining the transportation sections of $mp^1_2$. $u \frown v$ – concatenation of sequences $u$ and $v$. If $u = (u_1, \ldots, u_a)$, $v = (v_1, \ldots, v_b)$ and $u_a = v_1$, then $u \frown v = (u_1, \ldots, u_a, \ldots, v_b)$.

The notation below will be also used:

$mo^k_{i,j}$ – denotes the $j$-th operation of the stream $mP^k_i$ from the $i$-th multimodal process,
$mt^k_{i,j}$ – denotes the execution time of the operation $mo^k_{i,j}$,
$mx^k_{i,j}(l)$ – the moment of operation $mo^k_{i,j}$ beginning in the $l$-th cycle of stream $mP^k_i$.

To sum up the local $P_i$ and multimodal $mP_i$ processes can be treated as the set of streams $P^k_i/mP^k_i$ which are executed on the common resources in the same manner. The resources used by streams to execute their operations are determined by routes (sequences) $p^k_i/mp^k_i$.

The local and multimodal processes share common resources, e.g. $R_1$, $R_2$, $R_3$, $R_4$, following the mutual exclusion mode; i.e., guaranteeing that any station (platform, stop) can be occupied at a given moment by only one transportation mode while serving only one flow of passengers stream.

Processes access to shared resources is determined by a set of **priority dispatching rules** $\Theta = \{\Theta^0, \Theta^1\}$, where: $\Theta^i = \{\sigma^i_1, \sigma^i_2, \ldots, \sigma^i_c, \ldots, \sigma^i_m\}$ is the set of priority dispatching rules for local $(i = 0)$ / multimodal $(i = 1)$ processes, and $\sigma^i_c = (s^i_{c,1}, \ldots, s^i_{c,d}, \ldots, s^i_{c,lp(c)})$ – is a sequence (representing one priority dispatching rule) determining an order in which the streams can be executed on the resource $R_c$. For example the priority dispatching rule $\sigma^0_2 = (P^1_2, P^1_3, P^1_1)$ means that an order in which streams of local processes access to $R_2$ follows the sequence: $P^1_2$, $P^1_3$, $P^1_1$, $P^1_2$, $P^1_3$, $P^1_1$, $P^1_2$, … . In the considered case of the system from Fig. 1b), the access to shared resources is determined by rules for:

- local processes: $\sigma_1^0 = (P_2^1, P_3^1)$, $\sigma_2^0 = (P_1^1, P_2^1)$, $\sigma_3^0 = (P_1^1, P_4^1)$, $\sigma_4^0 = (P_3^1, P_4^1)$,
- multimodal processes: $\sigma_2^1 = \sigma_3^1 = \sigma_4^1 = (mP_2^1, mP_1^1)$.

Due to above assumptions, considered processes cannot be preempted; that is, a resource can be released only voluntarily by the process holding it, after that process has completed. Consequently, that means a process waiting for the access to the busy resource cannot release the resource already assigned to him (Bocewicz and Banaszak 2013). Since the processes cannot be preempted; the operation times and sequence of operations performed by the processes do not depend on external disturbances.

Due to above definitions the SCCP behavior is determined by its structure defined by the tuple (Bocewicz and Banaszak 2013):

$$SC = ((R, SL), SM), \tag{1}$$

where:

$R = \{R_1, \ldots, R_c, \ldots, R_m\}$ – the set of resources, $m$ – the number of resources,
$SL = (P, U, O, T, \Theta^0)$ – the structure of local processes where:

$P = \{P_i = \{P_i^1, \ldots, P_i^k, \ldots, P_i^{ls(i)}\} \mid i = 1, \ldots, n\}$ – the set of local processes (streams), $P_i$ – the $i$-th process, $P_i^k$ – the $k$-th stream of the $i$-th local process $P_i$, $n$ – the number of local processes, $ls(i)$ – the number of streams of the $i$-th local process $P_i$,

$U = \{p_i = \{p_i^1, \ldots, p_i^k, \ldots, p_i^{ls(i)}\} \mid i = 1 \ldots, n\}$ – the set of routes of local processes, $p_i^k = (r_1^1, \ldots, r_i^k, \ldots, r_{i,lr(i)}^k)$ – the $k$-th route of the stream $P_i^k, r_i^k \in R$ – resource required for implementing the $j$-th operation of the stream $P_i^k$, $lr(i)$ – is the length of the cyclic process route,

$O = \{O_i = \{O_i^1, \ldots, O_i^k, \ldots, O_i^{ls(i)}\} \mid i = 1, \ldots, n\}$ – the set of sequences of operations in local processes, where: $O_i^k = (o_{i,1}^k, \ldots, o_{i,j}^k, \ldots, o_{i,lr(i)}^k)$ – the sequence of operations in stream $P_i^k$ and $o_{i,j}^k$ – the $j$-th operation of the stream $P_i^k$,

$T = \{T_i = \{T_i^1, \ldots, T_i^k, \ldots, T_i^{ls(i)}\} \mid i = 1, \ldots, n\}$ – the set of operation times in local processes where: $T_i^k = (t_{i,1}^k, \ldots, t_{i,j}^k, \ldots, t_{i,lr(i)}^k)$ – the sequence of operations times in stream $P_i^k$ and $t_{i,j}^k$ – the $j$-th operation time of the stream $P_i^k$,

$\Theta^0 = \{\sigma_1^0, \ldots, \sigma_c^0, \ldots, \sigma_m^0\}$ – the set of priority dispatching rules for local processes, $\sigma_c^0$ – dispatching rule for the resource $R_c$,

$SM = (mP, mU, mO, mT, \Theta^1)$ – structure of multimodal processes, where:

$mP = \{mP_i = \{mP_i^1, \ldots, mP_i^k, \ldots, mP_i^{lsm(i)}\} \mid i = 1, \ldots, w\}$ – the set of multimodal processes (streams), $mP_i$ – the $i$-th process, $mP_i^k$ – the $k$-th stream of the $i$-th multimodal process $mP_i$, $w$ – number of multimodal processes, $lsm(i)$ – number of streams of the $i$-th multimodal process $mP_i$,

$mU = \{mp_i = \{mp_i^1, \ldots, mp_i^k, \ldots, mp_i^{lsm(i)}\} \mid i = 1, \ldots, w\}$ – the set of routes of multimodal processes, $mp_i^k = (mr_1^1, \ldots, mr_i^k, \ldots, mr_{i,lrm(i)}^k)$ – the $k$-th route of the stream $mP_i^k$, $mr_i^k \in R$ – resource required for implementing the $j$-th operation of the stream $mP_i^k$, $lrm(i)$ – is the length of the cyclic process route,

$mO = \{mO_i = \{mO_i^1, \ldots, mO_i^k, \ldots, mO_i^{lsm(i))}\} \mid i = 1, \ldots, w\}$ – the set of sequences of operations in multimodal processes, where: $mO_i^k = (mo_{i,1}^k, \ldots, mo_{i,j}^k, \ldots, mo_{i,lr(i)}^k)$ – the sequence of operations in stream $mP_i^k$ and $mo_{i,j}^k$ – the $j$-th operation of the stream $mPi^k$,

$mT = \{mT_i = \{mT_i^1, \ldots, mT_i^k, \ldots, mT_i^{lsm(i)}\} \mid i = 1, \ldots, w\}$ – the set of operation times in multimodal processes where: $mT_i^k = (mt_{i,1}^k, \ldots, mt_{i,j}^k, \ldots, mt_{i,lr(i)}^k)$ – the sequence of operations times in stream $mP_i^k$ and $mt_{i,j}^k$ – the $j$-th operation time of the stream $mP_i^k$,

$\Theta^1 = \{\sigma_1^1, \ldots, \sigma_c^1, \ldots, \sigma_m^1\}$ – the set of priority dispatching rules for multimodal processes, $\sigma_c^1$ – dispatching rule for the resource $R_c$.

The existing approach to solving the SCCPs scheduling problem is based upon the simulation models, e.g. the Petri nets (Song and Lee 1998), the algebraic models (Kampmeyer 2006) upon the (max,+) algebra or the artificial intelligent methods (Heo et al. 2003). The SCCP driven models, assuming a unique process execution along each cyclic route while allowing to take into account the stream-like flow of local cyclic processes, e.g. buses servicing a given city line, studied in (Bocewicz et al. 2013), mesh-like structures. Therefore, this work can be seen as a continuation of our former investigations conducted in (Bocewicz et al. 2013; Bocewicz and Banaszak 2013). In that context, our paper provides contribution to a time and/or minimal distance path-finding problem (Li 2008; Liu 2010) within the environment of multimodal transportation mesh-like network as well as its possible implementation in the route advisory systems solving the Multi-Criteria, Multi-Modal Shortest Path Problem (Guo 2008).

The behavior imposed by SCCP structure (1) can be described by the cyclic schedule:

$$X' = ((X, \alpha), (mX, m\alpha)) \tag{2}$$

where:

$X = \{x_{1,1}^1, \ldots, x_{i,j}^k, \ldots, x_{n,lr(n)}^{ls(n)}\}$ – the set of moments of operations beginning of local processes operations in the first cycle $(l = 0)$, where: $x_{i,j}^k$ – determines the moment of the operation $o_{i,j}^k$ beginning in the $l$-th cycle: $x_{i,j}^k(l) = x_{i,j}^k + \alpha l$, $\alpha$ – periodicity of local processes executions,

$mX = \{mx_{1,1}^1, \ldots, mx_{i,j}^k, \ldots, mx_{w,lm(w)}^{lsm(w)}\}$ – the set of moments of operations beginning of multimodal processes for $l = 0$, where: $mx_{i,j}^k$ – determines the moment of the operation $mo^k_{i,j}$ beginning in the $l$-th cycle: $mx^k_{i,j}(l) = mx^k_{i,j} + m\alpha l$, $m\alpha$ – periodicity of multimodal processes executions.

An example of the cyclic schedule (2) for SCCP from Fig. 1b) is shown in Fig. 2, where to each subsequent discrete time unit a proper state (snapshot encompassing current resources allocation to processes) is assigned. The transitions between states are determined by variables of the SCCP structure (1) including: dispatching priority rules $\Theta^0$, $\Theta^1$, processes routes $U$, $mU$ and etc. In that context, the behavior of the system characterized by various sequences of subsequently reachable states $S^r$ can be illustrated in a graphical form as the states space $\mathcal{P}$.

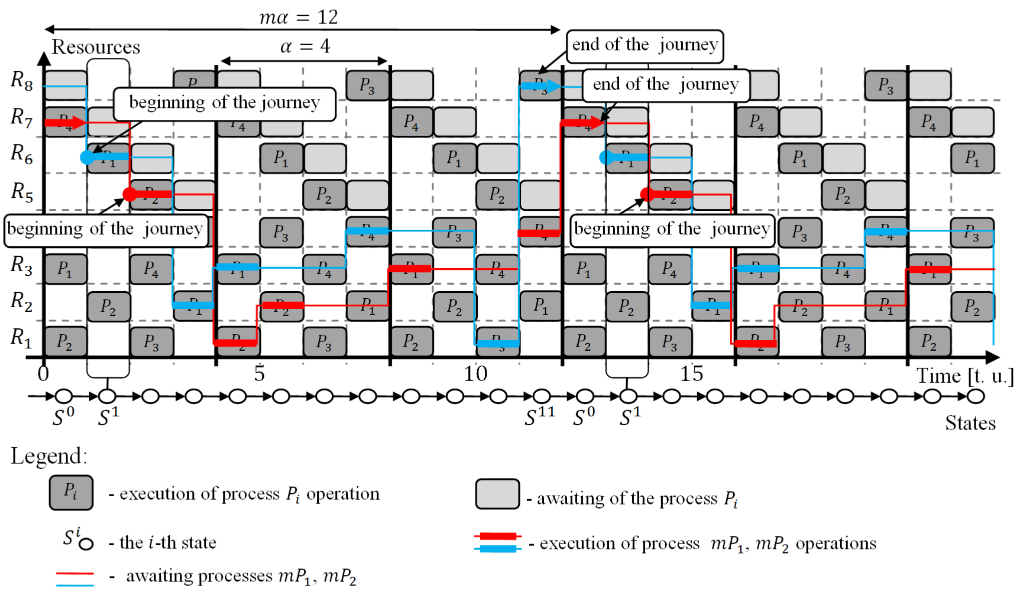


Figure 2: Gantt's chart illustrating cyclic schedule of SCCP from Fig.1 b).

Fig. 3 shows an example illustrating this possibility for the system from Fig. 1b). Assuming the graph-theoretical interpretation of the space $\mathcal{P}$, the diagraph corresponding to it can be represented by the pair $\mathcal{P} = (\mathbb{S}, \mathbb{E})$, where $\mathbb{S}$ means a set of admissible SCCP states (Bocewicz and Banaszak 2012), $\mathbb{E} \subseteq \mathbb{S} \times \mathbb{S}$ means a set of arcs representing transitions between SCCP states (transitions take place according to the function $S^f = \delta(S^e)$ described in (Bocewicz and Banaszak 2012).

Cyclic schedules (see Fig. 2) illustrating cyclic behavior of SCCP can be also recognized by relevant cycles (e.g. digraph $G_1$, see Fig. 3) in the space $\mathcal{P}$. A sequence of states being a part of a cycle is called as a cyclic-steady state. Formally, the **cyclic steady state** is the sequence $D_C = (S^{d_1}, \ldots, S^{d_i}, S^{d_{i+1}}, \ldots, S^{d_{ld}})$ of various admissible states $S^{d_i}$, $S^{d_{i+1}} \in \mathbb{S}$, in which each pair of states satisfies the expression $S^{d_{i+1}} = \delta(S^{d_i})$, $i = 1, \ldots, (ld-1)$ and $S^{d_1} = \delta^{lp}(S^{d_{ld}})$.

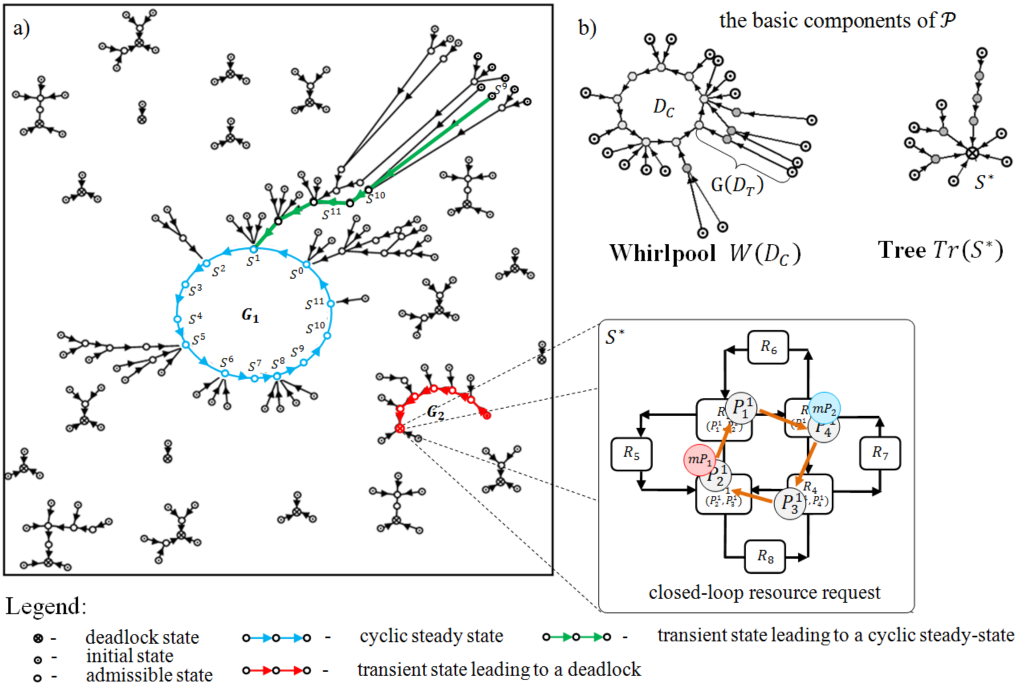


Figure 3: The states space $\mathcal{P}$ imposed by the SCCP structure from Fig 1 a), the basic components of $\mathcal{P}$: Whirlpool $W(DC)$, Tree $Tr(S^*)$ b).

The states of space $\mathcal{P}$ leading to the shared cyclic steady state $D_C$ constitute a connected digraph called **Whirlpool** $W(D_C)$ (Fig. 3)

$$W(D_C) = G(D_C)\dot{\cup}\left(\dot{\cup}_{\forall D_T \in DT(D_C)} G(D_T)\right), \tag{3}$$

where:

$G(D_C)$ – digraph consisting of cyclic steady state $D_C$,

$G(D_T)$ – digraph consisting of sequence of states $D_T$ leading to the cyclic steady state $D_C$,

$D_T \in DT(D_C)$, where: $DT(D_C)$ – the set of all sequences of states leading to $D_C$,

$G_1\dot{\cup}G_2$ – the sum of digraphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2) : G_1\dot{\cup}G_2 = (V_1 \cup V_2, E_1 \cup E_2)$, $\dot{\cup}_{G_i \in G^*} G_i = G_1\dot{\cup}G_2\dot{\cup}\ldots\dot{\cup}G_a$, for $G^* = \{G_1, G_2, \ldots, G_a\}$.

An example of the whirlpool, see Fig. 3, shows clearly that staring processes execution from any state belonging to its set results in a cyclic steady state $D_C$. In the case of a tree, however any state belonging to its set results in a deadlock state $S^*$ (marked with the symbol $\otimes$), which means system interruption caused by a closed-loop resource request occurrence.

An example of a deadlock caused by a closed-loop resource request is illustrated in Fig. 3. In the state $S^*$, the stream $P_1^1$ waits for releasing of the resource $R_3$ by the stream $P_4^1$, while the stream $P_4^1$ waits for releasing the resource $R_4$ by the stream $P_3^1$, while the stream $P_3^1$ waits for releasing the resource $R_1$ by the stream $P_2^1$ and finally while the stream $P_2^1$ waits for releasing of the resource $R_2$ by the stream $P_1^1$. In real life such a situation can be seen when buses (trams) block each other. Therefore, states causing deadlocks constitute the another type of behavior represented by connected digraph called **Tree** (Fig. 3)

$$Tr(S^*) = \dot{\bigcup}_{D_T \in DT(S^*)} G(D_T), \tag{4}$$

where:

$G(D_T)$ – the digraph consisting of sequence of states $D_T$ leading to the deadlock state $S^*$, $D_T \in DT(S^*)$;
$DT(S^*)$ – the set of all sequences of states leading to the deadlock state $S^*$.

Whirlpools and trees are two basic components of the state space $\mathcal{P}$. Whirlpools make it possible to estimate the presence of cyclic steady states (i.e., to determine the collision-free and deadlock-free passengers transportation in MTN environment). In turn the trees enable determining dangerous states that lead to deadlocks (e.g. traffic congestions).

### 2.2. Mesh-like structure

In a special case, SCCP structures may have a mesh-like form. An example of such a structure is shown in Fig. 4a). Structures of this kind consist of repeatable constant fragments of the system (sub-structures $SC_i$). For instance, the structure presented in Fig. 4a) was created as a result of multiple composition of the structure shown in Fig. 4b).

Formally, the mesh-like structure is defined as $SC$ (1) structure, that can be decomposed into the set of isomorphic substructures: $SC^* = \{SC_1, \ldots, SC_i, \ldots, SC_{lc}\}$. In such a case, an assumption is made that:

a) each substructure $SC_i \in SC^*$ of the structure $SC$ is defined by analogy to (1)

$$SC_i = ((Rp_i, SLp_i), SMp_i) \tag{5}$$

where:

$Rp_i$ – the set of resources of sub-structure $SC_i$, $Rp_i \subset R$,
$SLp_i$ – level of local processes of substructure $SC_i$ including local processes $Pp_i \subset P$ and corresponding route sequences: $Up_i \subset U$, of the operation times $Tp_i \subset T$. The set of routes $Up_i$ includes all the resources $Rp_i$. The set of dispatching rules is characterized by $\Theta_i^0 = \{\sigma_{k,i}^0 = (s_{k,1,i}^0, \ldots, s_{k,d,i}^0, \ldots, s_{k,lh(k,0),i}^0) \,|k = 1, \ldots, lk\}$, where $\sigma_{k,i}^0$ – dispatching rule for the resource $R_k \in Rp_i$ in $i$-th substructure, $s_{k,d,i}^0$ – stream of a local process belonging to $Pp_i$, $lh(k,i,0)$ – the length of rule $\sigma_{k,i}^0$.

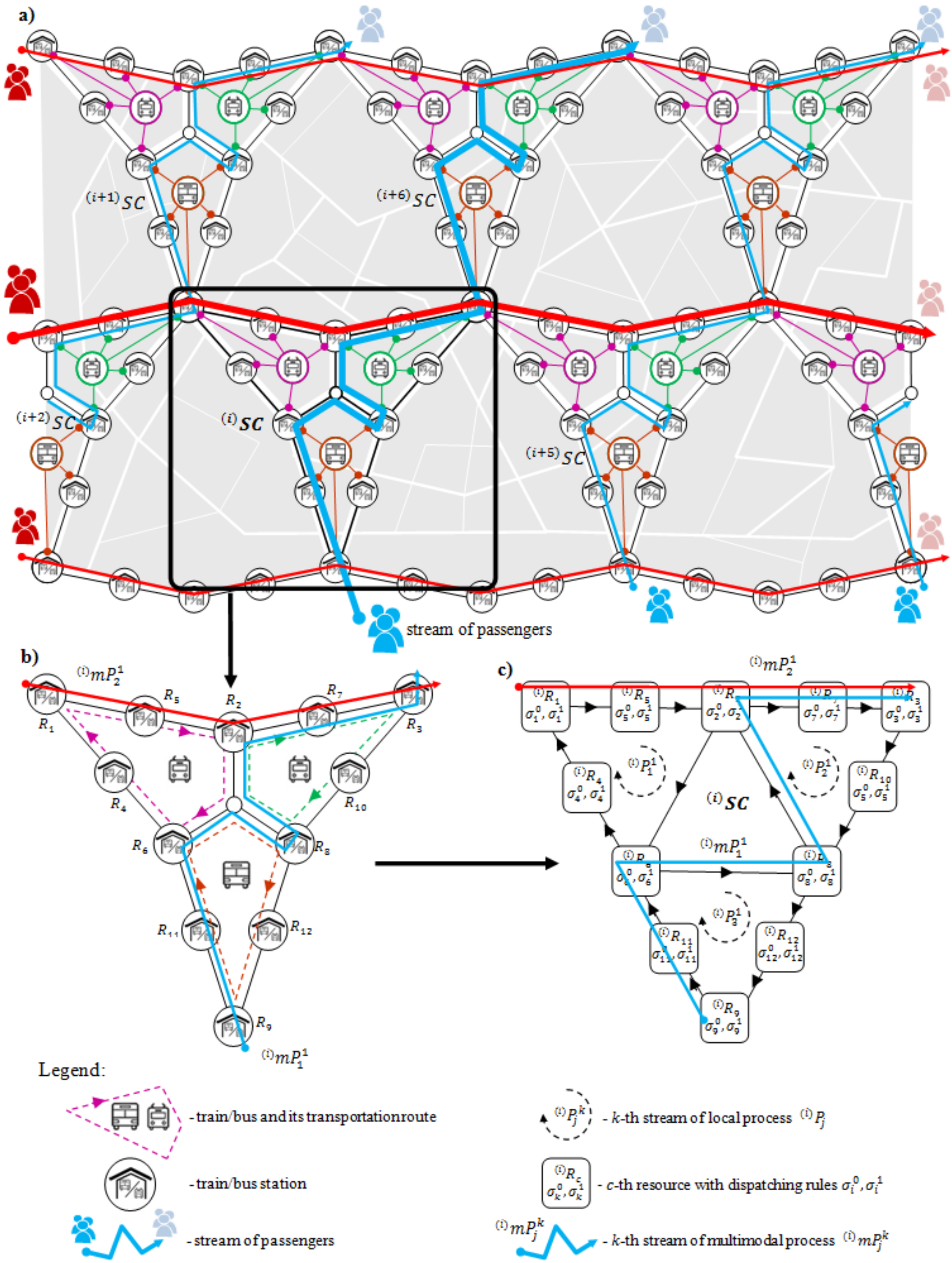


Figure 4: MMTN structure composed of elementary substructures a), elementary isomorphic substructure ${}^{(i)}SC$ b), and its SCCP model c).

$SMp_i$ – level of multimodal processes of substructure $SC_i$, the level includes fragments of $mP_j(a,b)$ of multimodal processes forming the set $mPp_i$, where: $mP_j(a,b)$ – means fragment of the process $mP_j$ related with executing the operation from $a$, $a+1$, ..., $b$. In the substructure $SC_i$ there are only such fragments of multimodal processes which are performed based on local processes $Pp_i$.

b) $SC^* = \{SC_1, \ldots, SC_i, \ldots, SC_{lc}\}$ – is a set of substructures of the structure $SC$ if substructures include all resources $\cup_{i=1}^{lc} Rp_i = R$ of the structure $SC$, and

- $\cup_{i=1}^{lc} Pp_i = P$; $\prod_{i=1}^{lc} Pp_i = \emptyset$ and $\cup_{i=1}^{lc} Up_i = U$; $\prod_{i=1}^{lc} Up_i = \emptyset$ – substructures use all the local processes and only one process occurs in exactly one structure,
- $\prod_{i=1}^{lc} mPp_i = \emptyset$ – each fragment of a multimodal process occurs in exactly one substructure; moreover, within the substructures all fragments of multimodal processes are used.

c) Two sub-structures $SC_a$, $SC_b \in SC^*$ are called isomorphic if:

- each resource $R_a \in Rp_a$ of substructure $SC_a$ is corresponding to exactly one resource $R_b \in Rp_b$ of the structure $SC_b$ : $R_b = f(R_a)$,
- each process $P_a/mP_a$ (local as well as multimodal) of the substructure $SC_a$ is corresponding to exactly one process $P_b/mP_b$ of the structure $SC_b$ : $P_b = f(P_a)$, $mP_b = f(mP_a)$,
- routes $p_b/mp_b$ and $p_a/mp_a$ of the corresponding processes are sequences consisting of corresponding resources,
- each operation $o_{a,j}^h/mo_{a,j}^h$ executed within the substructure $SC_a$ corresponds to exactly one operation $o_{b,j}^h/mo_{b,j}^h$ executed within the substructure $SC_b$ : $o_{a,j}^h = f(o_{b,j}^h)/mo_{a,j}^h = f(mo_{b,j}^h)$; the corresponding operations are executed at the same time: $t_{a,j} = t_{b,j}/mt_{a,j} = mt_{b,j}$,
- dispatching rules $\sigma_a^l/\sigma_b^l$ of the corresponding resources are sequences consisting of elements $s_{a,d}^l/s_{b,d}^l$ indicating the streams of corresponding processes.

The structure shown in Fig. 4a) consists of one type of isomorphic substructures presented in Fig. 4c). The substructures it consists of, denoted as ${}^{(i)}SC$, are corresponding to the structure illustrated in Fig 4. Each of them includes twelve resources $({}^{(i)}R_1 - {}^{(i)}R_{12})$ three local processes $({}^{(i)}P_1, {}^{(i)}P_2, {}^{(i)}P_3)$ and two fragments of multimodal processes (it is assumed that each fragment is related with one stream of multimodal process – ${}^{(i)}mP_1^1, {}^{(i)}mP_2^1$).

## 3. Problem formulation

The considered problem can be formulated in the following way. Given a mesh-like structure $SC$ (1), where values of operation times $T$, $mT$ and dispatching rules $\Theta$ are unknown. An answer to the following question is searched: if there is such a value $T$, $mT$ and $\Theta$ that the cyclic behavior represented by the schedule $X'$ (2) of the relevant $SCCP$ is guaranted?

The mesh-like structure $SC$ (1) can be decomposed into a set of isomorphic substructures $SC^* = \{SC_1, \ldots, SC_i, \ldots, SC_{lc}\}$. Therefore, the selection of parameters $T$, $mT$ and $\Theta$ can be carried out independently for each substructure. If, for every substructure $SC_i$, there is a subset of parameters $T$, $mT$ and $\Theta$ that guarantee its cyclic behavior, then the considered problem should provide an answer to the following question: **is there such a way of composing the substructures $SC^*$, that the cyclic work of the system $SC$ is guaranteed**?

In order to answer this question let us introduce the operator of substructure composition $\oplus$. An assumption is made that the result of two substructures $SC_a$, $SC_b$ composition $SC_a \oplus SC_b$ through mutually shared resources $(Rp_a \cap Rp_b \neq \emptyset)$ results in the structure $SC_a \oplus SC_b = SC_c$ defined as follows

$$SC_c = ((Rp_c, SLp_c), SMp_c) \tag{6}$$

where: $Rp_c = Rp_a \cup Rp_b$ – the set of resources, and

- variables characterizing $SLp_c$ are determined in the following way:

  $Pp_c = Pp_a \cup Pp_b$; $Up_c = Up_a \cup Up_b$; $Tp_c = Tp_a \cup Tp_b$
  
  $\Theta_c^0 = \{\sigma_{k,c}^0 \, | k = 1, \ldots, lk\}$, where:

$$\sigma_{k,c}^l = \begin{cases} \sigma_{k,a}^l & \text{for} \quad R_k \in Rp_a \quad \text{and} \quad R_k \notin Rp_b \\ \sigma_{k,b}^l & \text{for} \quad R_k \in Rp_b \quad \text{and} \quad R_k \notin Rp_a \\ \vartheta(\sigma_{k,a}^l, \sigma_{k,b}^l) & \text{for} \quad R_k \in Rp_a \quad \text{and} \quad R_k \in Rp_b \end{cases} \tag{7}$$

  $\vartheta(\sigma_{k,a}^0, \sigma_{k,b}^0)$ – function determining the dispatching rules for the mutual resource $R_k$ of the composed structures.

- variables characterizing $SMp_c$ are determined in the following way:

  $mPp_c$ – the set including all fragments of multimodal processes of the sets $mPp_a$ and $mPp_b$ except for fragments meeting the condition below.

  If in the set $mPp_a \cup mPp_b$ there are such two fragments: $mP_j(a_{j_1}, a_{j_2})$, $mP_j(b_{j_1}, b_{j_2})$, that $a_{j_2} = b_{j_1}$, then in the set $mPp_c$ these fragments are replaced by the fragment of composed of multimodal process in the form of $mP_j(a_{j_1}, b_{j_2})$. The set $mPp_c$ attained in this way determines the set of routes $mUp_c$, of operations and their execution times $mTp_c$, $\Theta_c^1 = \{\sigma_{k,c}^1 \, | k = 1, \ldots, lk\}$, where $\sigma_{k,c}^1$ is determined analogically as (7).

## 4. Cyclic scheduling of mesh-like SCCP

### 4.1. Determining cyclic steady processes

Fig. 4c) shows arrangement of elementary substructure of the MMTN from Fig. 4a). Considered elementary isomorphic substructures ${}^{(i)}SC$ are coupled by common shared (employed in an mutual exclusion mode) resources. In each substructure ${}^{(i)}SC$ processes are executed in the same manner, i.e. similar operations execute along the similar routes, the same dispatching rules are assigned to the similar resources, etc. Under this assumption the operator of substructures composition $\oplus$ can be seen as a multiple composition of substructures ${}^{(i)}SC$

$$SC = \oplus_{i=1}^{lc}\left({}^{(i)}SC\right) \tag{8}$$

where: $\oplus_{i=1}^{lc}({}^{(i)}SC) = {}^{(1)}SC \oplus \cdots \oplus {}^{(i)}SC \oplus \cdots \oplus {}^{(lc)}SC$ – means composition following (6), (7) i.e. each substructure ${}^{(i)}SC$ is put together with the others by means of integrating the resources belonging to the same set of corresponding resources.

For example, the structure ${}^{(i)}SC$ from Fig. 4c) is put together with the others by the resources ${}^{(i)}R_1$, ${}^{(i)}R_3$, ${}^{(i)}R_9$. The resource ${}^{(i)}R_1$ plays the role of the resource ${}^{(i+2)}R_1$ of the structure ${}^{(i+2)}SC$ and the resource ${}^{(i+1)}R_9$ of the structure ${}^{(i+1)}SC$. In other words, each isomorphic structure such as ${}^{(i)}SC$ shares the following resources with the neighboring structures: ${}^{(i)}R_1$ treated also as ${}^{(i+2)}R_3$ and ${}^{(i+1)}R_9$ (contiguity with ${}^{(i+1)}SC$ and ${}^{(i+2)}SC$), ${}^{(i)}R_3$ treated as ${}^{(i+5)}R_1$ and ${}^{(i+6)}R_9$ and ${}^{(i)}R_9$ treated as ${}^{(i+3)}R_3$ and ${}^{(i+4)}R_1$.

Due to the same manner of process execution, as well as the same manner of substructures composition, the cyclic schedule representing the behavior of the whole SCCP structure can be perceived as a composition of corresponding (isomorphic) schedules:

$$X' = \uplus_{i=1}^{lc}\left({}^{(i)}X'\right) \tag{9}$$

where ${}^{(i)}X'$ – the cyclic schedule of the substructure ${}^{(i)}SC$

$${}^{(i)}X' = \left(({}^{(i)}X, {}^{(i)}\alpha), ({}^{(i)}mX, {}^{(i)}m\alpha)\right) \tag{10}$$

${}^{(i)}X / {}^{(i)}mX$ - - set of the initiation moments of local / multimodal process operations of the substructure ${}^{(i)}SC$,

${}^{(i)}\alpha / {}^{(i)}m\alpha$ – periodicity of local/multimodal processes executions, $\uplus_{i=1}^{lc}({}^{(i)}X') = {}^{(1)}X' \uplus \cdots \uplus {}^{(i)}X' \uplus \cdots \uplus {}^{(lc)}X'$ – composition of schedules ${}^{(i)}X'$, ${}^{(a)}X' \uplus {}^{(b)}X'$ – the operation of integrating the schedule composition ${}^{(a)}X'$, ${}^{(b)}X'$

$$\begin{gathered}{}^{(a)}X' \uplus {}^{(b)}X' = (({}^{(a)}X \cup {}^{(b)}X, lcm({}^{(a)}\alpha, {}^{(b)}\alpha)),\\ \left({}^{(a)}mX \cup {}^{(b)}mX, lcm({}^{(a)}m\alpha, {}^{(b)}m\alpha)\right).\end{gathered} \tag{11}$$

In order to determine the schedule $X'$ it is enough to know the schedule ${}^{(i)}X'$ of the single substructure ${}^{(i)}SC$. However, to make the composition (9) possible, it is necessary to make sure that the operations executed according to ${}^{(i)}X'$, do not lead to deadlocks. And in the mutually shared resources ${}^{(i)}R_1$, ${}^{(i)}R_3$ and ${}^{(i)}R_9$ the streams belonging to various substructures must not collide, i.e. they must be executed alternately.

In order to determine such parameters as dispatching rules ${}^{(i)}\Theta$ and operation times ${}^{(i)}T$, ${}^{(i)}mT$ of the substructure ${}^{(i)}SC$ (Fig. 4c) that guarantee the attainability of the cyclic schedule ${}^{(i)}X'$ within the resultant structure, it is possible to apply the constraint satisfaction problem (Sitek and Wikarek 2008)

$$PS'_{REX_i} = \left(\left(\left\{{}^{(i)}T', {}^{(i)}X', {}^{(i)}\Theta, {}^{(i)}\alpha'\right\}, \{D_T, D_X, D_\Theta, D_\alpha\}\right), \{C_L, C_M, C_D\}\right), \tag{12}$$

where: ${}^{(i)}T'$, ${}^{(i)}X'$, ${}^{(i)}\Theta$, ${}^{(i)}\alpha'$ – decision variables,

${}^{(i)}T' = \left({}^{(i)}T, {}^{(i)}mT\right)$ – the sequence of operation times of substructure ${}^{(i)}SC$,

${}^{(i)}X'$ – the cyclic schedule (10) of substructure ${}^{(i)}SC$,

${}^{(i)}\Theta = \left\{{}^{(i)}\Theta^0, {}^{(i)}\Theta^1\right\}$ – the set of priority dispatching rules for substructure ${}^{(i)}SC$,

${}^{(i)}\alpha' = \left({}^{(i)}\alpha, {}^{(i)}m\alpha\right)$ – the periodicity of local/multimodal processes executions for substructure ${}^{(i)}SC$,

$D_T$, $D_X$, $D_\Theta$, $D_\alpha$ – domains determining admissible value of decision variables: $D_T :\ {}^{(i)}mt^k_{i,j},\ {}^{(i)}t^k_{i,j} \in \mathbb{N}$; $D_X :\ {}^{(i)}mx^k_{i,j},\ {}^{(i)}x^k_{i,j} \in \mathbb{Z}$; $D_\alpha :\ {}^{(i)}m\alpha,\ {}^{(i)}\alpha \in \mathbb{N}$;

$\{C_L, C_M, C_D\}$ – the set of constraints $C_L$ and $C_M$ describing SCCP behavior, $C_L$ – constraints determining cyclic steady state of local processes, i.e. their cyclic schedule, $C_M$ – constraints determining multimodal processes behavior, $C_D$ – constraints that guarantee the smooth implementation of the stream operation executed on mutual resources, (in case of ${}^{(i)}SC$ from Fig. 2a) of the resources ${}^{(i)}R_1$, ${}^{(i)}R_3$ and ${}^{(i)}R_9$).

The solution of the problem (12) is, among other things, the schedule ${}^{(i)})X'$ that meets all the constraints from the given set $\{C_L, C_M, C_D\}$. It means that, if such schedule exists within the substructure ${}^{(i)}SC$, it is possible to smoothly execute the operations of processes occurring in ${}^{(i)}SC$ as well as in neighboring substructures $\left({}^{(i+1)}SC, {}^{(i+2)}SC, \ldots, {}^{(i+6)}SC\right)$.

### 4.2. The conditions for cyclic implementation of processes

The constraints $C_L$, $C_M$ occurring in the problem (12 guarantee deadlock-free and smooth execution of the operations of substructure ${}^{(i)}SC$. They are typical of the relationship between the structure parameters ${}^{(i)}\Theta$, ${}^{(i)}T'$, ${}^{(i)}U$, ${}^{(i)}M$ and its behavior ${}^{(i)}X'$, ${}^{(i)}\alpha'$ (meeting the accepted conditions: mutual exclusion protocol, etc.) and the mutual relationships between local and multimodal processes. In case of the two levels structure model, i.e. including levels $SL$ and $SM$, the constraints $C_L$ and $C_M$ determining ${}^{(i)}x^k_{a,b}/{}^{(i)}mx^k_{a,b}$ were described in (Bocewicz and Banaszak 2013).

The constraints $C_L$, $C_M$ guarantee that in the substructure ${}^{(i)}SC$ from Fig. 4c) the processes will be executed in a cyclic and deadlock-free manner. These constraints, however, cannot ensure the lack of interferences between the operations of neighboring substructure streams $({}^{(i+1)}SC, {}^{(i+2)}SC, \ldots, {}^{(i+6)}SC)$ with the substructure ${}^{(i)}SC$. In order to avoid interferences of this kind, additional constraints $C_D$, are introduced, which describe the relationships between the process operations of the constituted structures. For that purpose the principle of match-up structures coupling is applied.

### 4.3. Principle of match-up structures coupling

The idea of the principle of match-up structures coupling is to attain the cyclic schedule $X'_c$ (that does not lead to any collisions between operations) in the substructure $SC_c$, gained as a result of the composition $SC_a \bigoplus SC_b$. The cyclic schedule is a composition of the schedules $X'_a$, $X'_b$: $X'_c = X'_a \uplus X'_b$ (9) if the following conditions hold:

- the value of the periodicity of schedule $X'_a$ is the total multiple of the periodicity of schedule $X'_b$,
- $m\alpha_a \; MOD \; m\alpha_b = 0$; and $\alpha_a \; MOD \; \alpha_b = 0$,
- the operations of mutual resources $RK = Rp_a \cap Rp_b = \left\{R_{k_1}, \ldots, R_{k_i}, \ldots, R_{k_q}\right\}$ are executed without mutual interferences.

Formally, the constraints that guarantee the lack of interferences while executing the process operations on mutual resources are defined in the following way.

**Constraints for local process operations**. In order to guarantee the smooth process implementation on the resource $R_{k_i} \in RK$ the extension of the conventional constraints of non-superimposition of time intervals is used (Bach et al. 2010). The two operations $o^h_{i,j}$, $o^s_{q,r}$ do not interfere (on the mutually shared resource $R_{k_i}$) if the operation $o^h_{i,j}$ begins (moment $x^h_{i,j}$) after the release (with the delay $\Delta t$) of the resource by the operation $o^s_{q,r}$ (moment $x^s_{q,r^*}$ of the subsequent operation initiation) and releases the resource (moment $x^h_{i,j^*}$ of the subsequent operation initiation) before the beginning of the next execution of the operation $o^s_{q,r}$ (moment $x^s_{q,r} + \alpha$). The collision-free execution of the local process operations is possible if the constraint below is satisfied:

$$\begin{aligned} &\left[(x^h_{i,j} \geqslant x^s_{q,r^*} + k''\alpha_b + \Delta t) \wedge (x^h_{i,j^*} + k'\alpha_a + \Delta t \leqslant x^s_{q,r} + \alpha_b)\right] \\ &\vee \left[(x^s_{q,r} \geqslant x^h_{i,j^*} + k'\alpha_a + \Delta t) \wedge (x^s_{q,r^*} + k''\alpha_b + \Delta t \leqslant x^h_{i,j} + \alpha_a)\right] \end{aligned} \tag{13}$$

where $j^* = (j+1) \; MOD \; lr(i)$, $r^* = (r+1) \; MOD \; lr(q)$,

$$k' = \begin{cases} 0 & \text{when} \quad j+1 \leqslant lr(i) \\ 1 & \text{when} \quad j+1 > lr(i), \end{cases} \qquad k'' = \begin{cases} 0 & \text{when} \quad r+1 \leqslant lr(q) \\ 1 & \text{when} \quad r+1 > lr(q), \end{cases}$$

$\alpha_a/\alpha_b$ – periodicity of schedule $X_a/X_b$; $lr(i)/lr(q)$ – length of process route $P_i/P_q$;
$x^h_{i,j}/x^s_{q,r}$ – initiation moments of the operation $o^h_{i,j}/o^s_{q,r}$ of the structure $SC_a/SC_b$;
$x^h_{i,j^*}/x^s_{q,r^*}$ – initiation moments of operation executed after $o^h_{i,j}/o^s_{q,r}$.

Satisfying the constraint (13) means that on every mutually shared resource of the composed substructures $SC_a$, $SC_b$ the local processes are executed alternately, i.e. they pass each other.

**Constraints for multimodal processes**. In order to guarantee an interference-free implementation of the multimodal processes (when the condition of mutual exclusion is applied) the applied conditions are similar to those used for local processes. Two operations $mo^h_{i,j}$, $mo^s_{q,r}$ can be executed without any interferences on the mutually shared resource $R_{k_i} \in RK$ if one operation is executed between the subsequent executions of the other. In this context, the collision-free execution of the multimodal process operations is possible if the following constraint is satisfied

$$\begin{gathered}\left[(mx^h_{i,j} \geqslant mx^s_{q,r^*} + k''m\alpha_b + \Delta t) \wedge (mx^h_{i,j^*} + k'm\alpha_a + \Delta t \leqslant mx^s_{q,r} + m\alpha_b)\right] \\ \vee \left[(mx^s_{q,r} \geqslant mx^h_{i,j^*} + k'\alpha_a + \Delta t) \wedge (mx^s_{q,r^*} + k''m\alpha_b + \Delta t \leqslant mx^h_{i,j} + m\alpha_a)\right]\end{gathered} \tag{14}$$

where $j^*$, $r^*$, $k'$ and $k''$ defined as in (13),
$mx^h_{i,j}$, $mx^s_{q,r}$ – initiation moments of the operations $mo^h_{i,j}$, $mo^s_{q,r}$ of substructures $SC_a$, $SC_b$, respectively;
$mx^h_{i,j^*}$, $mx^s_{q,r^*}$ – initiation moments of operations executed after $mo^h_{i,j}$, $mo^s_{q,r}$, respectively.

Satisfying the constraint (14) means that on every mutual resource of the composed substructures $SC_a$, $SC_b$ the multimodal processes are executed alternately, i.e. they pass each other.

The constraints (13) and (14) must be satisfied so that the composition of two substructures $SC_c = SC_a \oplus SC_b$ of the known cyclic behaviors, is also characterized by the cyclic behavior $X'_c$. If these constraints are satisfied, the manner of executing operations on mutual resources $R_k$ determines the form of dispatching rules $\sigma^0_{k,c}$ (7), and, to be more exact, the form of functions $\vartheta(\sigma^0_{k,a},\sigma^0_{k,b})$ and $\vartheta(\sigma^1_{k,a},\sigma^1_{k,b})$. The function $\vartheta(\sigma^l_{k,a},\sigma^l_{k,b})$ is determined based on the values of initiation moments of operations executed on the resource $R_k$

$$\begin{gathered}\vartheta\left(\sigma^l_{k,a},\sigma^l_{k,b}\right) = \left(s^l_{k,1,c},\ldots,s^l_{k,j,c},\ldots,s^l_{k,lh_c,c}\right) \text{ when} \\ x^l_{k,1,c} < \cdots < x^l_{k,j,c} < \cdots < x^l_{k,lh_c,c} \quad l \in 0,1\end{gathered} \tag{15}$$

where: $s^l_{k,j,c}$ – $j$-th element of the rule $\sigma^l_{k,c}$ determining the stream of the process of the $l$-th behavior level initiating its operation on the resource $R_k$ at the moment: $x^l_{k,j,c}$;

$s^l_{k,j,c}$ is one of the elements of the rules $\sigma^l_{k,a}$, $\sigma^l_{k,b}$; $x^0_{k,j,c} \in X_a \cup X_b$; $x^1_{k,j,c} \in mX_a \cup mX_b$. In other words, there are such dispatching rules on mutual $R_k$ as the sequence of operations resulting from the schedules $X'_a$, $X'_b$ satisfying the constraints (13) and (14).

### 4.4. Composition approach to mesh-like SCCP scheduling

Introduced the above concepts of structures coupling and schedules, composition can be employed in a new way aimed at SCCP cyclic scheduling. In order to illustrate its main stages let us consider the SCCP shown in Fig. 5a). Assuming the times of operation executions are given, the cyclic schedule of SCCP is found.

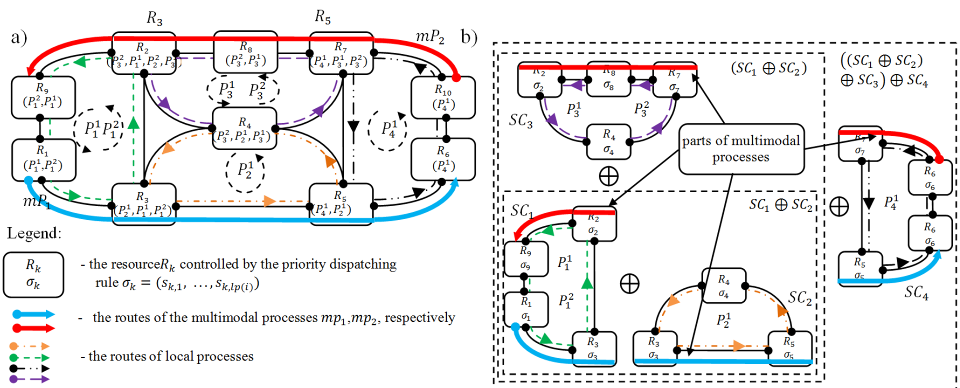


Figure 5: Example of SCCP a) and its decomposition due to the Stage 1 b).

**Stage 1.** Decompose a given SCCP into its elementary constituents, e.g. shown in Fig. 5b).

**Stage 2.** Aiming at the SCCP reconstruction try to join first pair of elementary constituents (Fig. 6a). Consider structures $SC_1$, $SC_2$ corresponding to selected elements and then couple them $SC_1 \oplus SC_2$. The behavior of newly obtained structure $SC_{1-2}$ results in cyclic schedule $X'_{1-2}$ being a composition $X'_1 \cup X'_2$ (11) of schedules obtained as solutions of constraint satisfaction problems: $PS'_{REX_1}$, $PS'_{REX_2}$ (12). In case the set of feasible solutions is empty (that means does not exist any cyclic schedule for considered pair of structures), then STOP; else consider already obtained structure $SC_{1-2}$ and the next remaining elementary constituent, and repeat the Stage 2 (e.g. $SC_{1-2} \oplus SC_3$ – see Fig 6b).

**Stage 3.** STOP in case the all elementary constituents has been coupled due to the input pattern of SCCP.

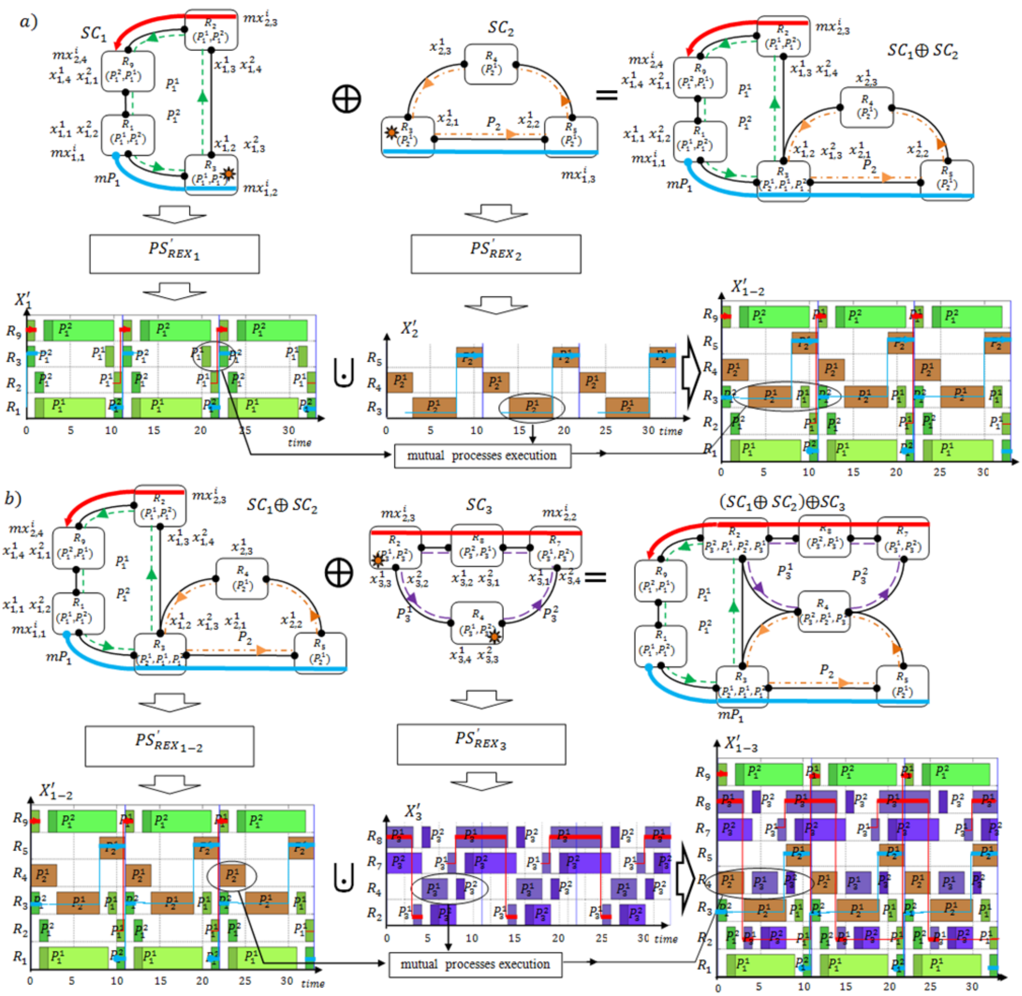


Figure 6: Illustration of first two iterations of the Stage 2 of the SCCP cyclic scheduling method.

Fig. 6 illustrates the second iteration of the Stage 2. The computer implementation of the proposed method is shown on the example considered below.

## 5. Computational experiment

The evaluation of the cyclic behavior (the existence of the schedule $X'$) of the mesh-like structure $SC$ from Fig. 4a) can be obtained as a result of evaluating the parameters of isomorphic structure $^{(i)}SC$ from Fig. 4c). Therefore, the problem $PS'_{REX_i}$ (12) was formulated in which the constraints $C_L$, $C_M$ determining the relationships between the behavior and the structure are formulated according to (Bocewicz and Banaszak 2013). In order to formulate the constraints $C_D$ the principle of match-up structures coupling was applied. In case of constraints $C_D$ it is necessary that they guarantee a collision-free execution of stream operations ${}^{(i)}P_1^1$, ${}^{(i+2)}P_2^1$, ${}^{(i+1)}P_3^1$ (on the resource ${}^{(i)}R_1$), ${}^{(i)}P_2^1$, ${}^{(i+2)}P_1^1$, ${}^{(i+6)}P_3^1$ (on the resource ${}^{(i)}R_3$), ${}^{(i)}P_3^1$, ${}^{(i+3)}P_2^1$, ${}^{(i+4)}P_1^1$ (on the resource ${}^{(i)}R_9$). In order to formulate these constraints, some features of isomorphic elementary substructures ${}^{(i)}SC$ are used. Owing to the fact that streams ${}^{(i)}P_1^1$, ${}^{(i+5)}P_1^1$, ${}^{(i+4)}P_1^1$ (as well as ${}^{(i)}P_2^1$, ${}^{(i+2)}P_2^1$, ${}^{(i+3)}P_2^1$ and ${}^{(i)}P_3^1$, ${}^{(i+1)}P_3^1$, ${}^{(i+6)}P_3^1$) of substructures ${}^{(i)}SC$, ${}^{(i+1)}SC, \ldots, {}^{(i+6)}SC$ are executed in the similar manner, the collision-free streams performance follows from non-simultaneous execution of the operations of streams ${}^{(i)}P_1^1$, ${}^{(i)}P_2^1$, ${}^{(i)}P_3^1$.

The constraints $C_D$ that guarantee this kind of process execution were shown in Fig. 7a) (distinguished by dot dashed lines). The problem $PS'_{REX_i}$, formulated in this manner, was implemented and solved in the constraint programming environment OzMozart (CPU Intel Core 2 Duo 3GHz RAM 4 GB). The first acceptable solution was obtained in less than one second. The result of the problem solution for the substructure from Fig. 7a) are the operation times ${}^{(i)}T$ and their initiation moments ${}^{(i)}X'$, and the dispatching rules ${}^{(i)}\Theta$ shown in the Tab. 1.

To sum up, in the substructure ${}^{(i)}SC$ cyclic behavior is attainable if the operation times have such values and the dispatching rules as those in Tab. 1. The cyclic schedule attainable in this substructure was illustrated in Fig. 7b). It shows that the operations executed on the mutual resources do not superimpose on each other. According to (9) the attained schedule is a component of the schedule $X'$ that characterizes the behavior of the whole structure $SC$.

The schedule $X'$ (9) being a multiple composition of the schedules ${}^{(i)}X'$ is presented in Fig. 8. It is evident that the composition of schedules ${}^{(i)}X'$ of all the substructures of the structure $SC$ does not lead to interferences in the execution of the operation – the schedules ${}^{(i)}X'$ on the resources ${}^{(i)}R_1$, ${}^{(i)}R_3$ and ${}^{(i)}R_9$. On the basis of the obtained schedules it is also possible to determine (according to (15) the dispatching rules for all the resources of the structure $SC$; the rules are presented in Tab. 1.

To sum up, the cyclic behavior in the structure $SC$ is attainable if the operation times and the dispatching rules are such as those in Tab. 1. Referring back to the layout presented in Fig. 4a), the obtained schedule should be treated as an illustration of transportation means (trams/busses) movement (local processes) and the method of executing transportation routes (multimodal processes) in a network consisting of numerous fragments of the same type (Fig. 4b)). It should be emphasized that the periodicity of local processes in the network of this kind amounts to $\alpha = 6$ t.u. (time units), and the times of transporting passengers of a single substructure amount to 10 t.u. (process ${}^{(i)}mP_1^1$) and 9 t.u. (process ${}^{(i)}mP_2^1$).

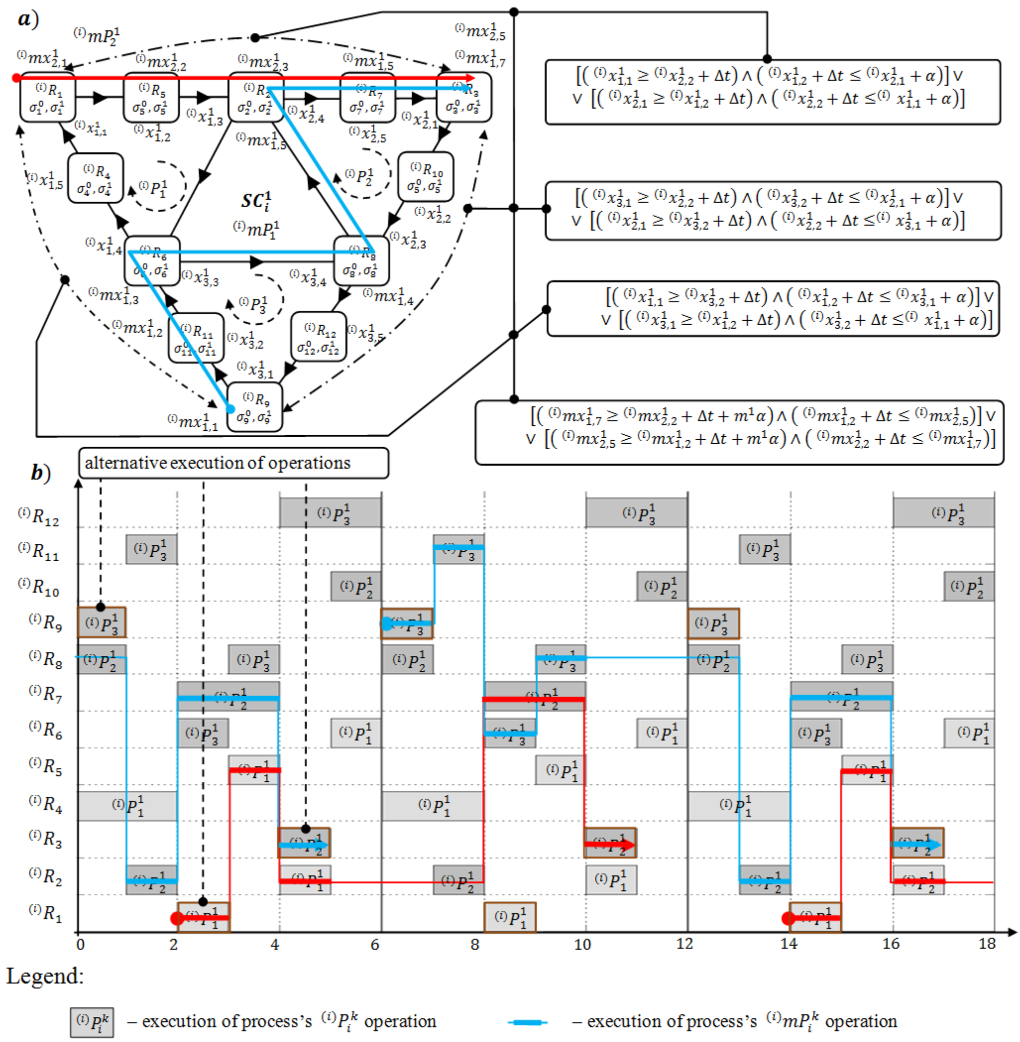


Figure 7: Substructure ${}^{(i)}SC$ with constraints that guarantee the alternate execution of ${}^{(i)}P^1_1$, ${}^{(i)}P^1_2$, ${}^{(i)}P^1_3$ a), cyclic schedule ${}^{(i)}X'$ of the structure ${}^{(i)}SC$ b).

## 6. Conclusions

This paper describes a declarative approach to modeling a multimodal transportation network composed of multiple connecting transport modes, such as bus, tram, light rail, subway and commuter rail, where within each mode, service is provided on separate lines or routes. The considered model of a network of multimodal transportation processes provides a framework to address the needs for transportation networks synchronization while taking into account their capacity and demand requirements. Therefore

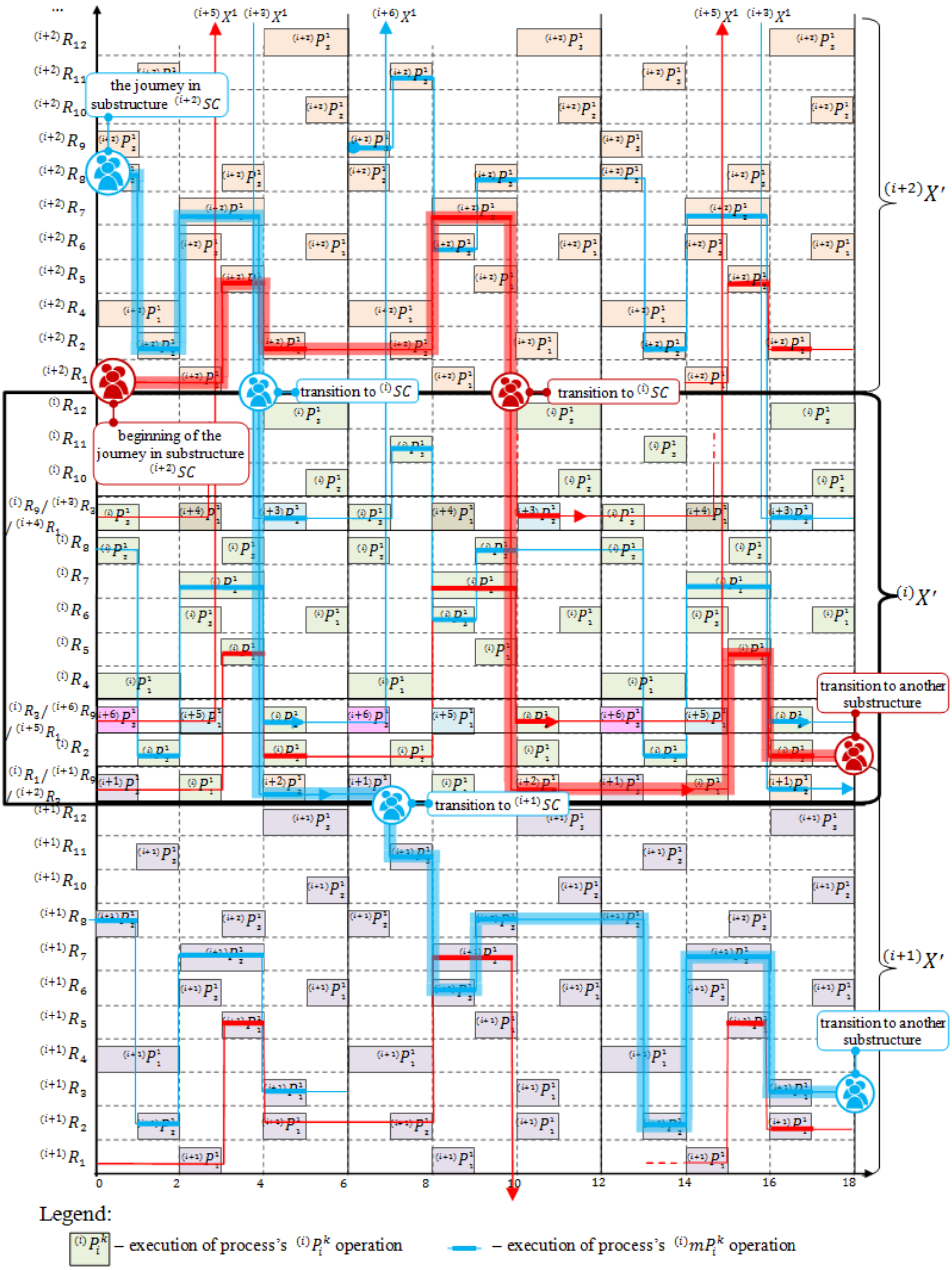


Figure 8: Cyclic schedule for structure *SC* from Fig. 4 .

Table 8: The moments of operations beginning, operation times and the dispatching rules of ${}^{(i)}SC$ from Fig. 7b)

| | j | ${}^{(i)}x^1_{j,1}$ | ${}^{(i)}x^1_{j,2}$ | ${}^{(i)}x^1_{j,3}$ | ${}^{(i)}x^1_{j,4}$ | ${}^{(i)}x^1_{j,5}$ |
|---|---|---|---|---|---|---|
| ${}^{(i)}P^1_1$ | 1 | 2 | 3 | 4 | 5 | 6 |
| ${}^{(i)}P^1_2$ | 2 | 4 | 5 | 6 | 7 | 8 |
| ${}^{(i)}P^1_3$ | 3 | 6 | 7 | 8 | 9 | 10 |

| | j | ${}^{(i)}t^1_{j,1}$ | ${}^{(i)}t^1_{j,2}$ | ${}^{(i)}t^1_{j,3}$ | ${}^{(i)}t^1_{j,4}$ | ${}^{(i)}t^1_{j,5}$ |
|---|---|---|---|---|---|---|
| ${}^{(i)}P^1_1$ | 1 | 1 | 1 | 1 | 1 | 2 |
| ${}^{(i)}P^1_2$ | 2 | 1 | 1 | 1 | 1 | 2 |
| ${}^{(i)}P^1_3$ | 3 | 1 | 1 | 1 | 1 | 2 |

| | j | ${}^{(i)}mx^1_{j,1}$ | ${}^{(i)}mx^1_{j,2}$ | ${}^{(i)}mx^1_{j,3}$ | ${}^{(i)}mx^1_{j,4}$ | ${}^{(i)}mx^1_{j,5}$ | ${}^{(i)}mx^1_{j,6}$ | ${}^{(i)}mx^1_{j,7}$ |
|---|---|---|---|---|---|---|---|---|
| ${}^{(i)}mP^1_1$ | 1 | 6 | 7 | 8 | 9 | 13 | 14 | 16 |
| ${}^{(i)}mP^1_2$ | 2 | 2 | 3 | 4 | 8 | 10 | - | - |

| | j | ${}^{(i)}mt^1_{j,1}$ | ${}^{(i)}mt^1_{j,2}$ | ${}^{(i)}mt^1_{j,3}$ | ${}^{(i)}mt^1_{j,4}$ | ${}^{(i)}mt^1_{j,5}$ | ${}^{(i)}mt^1_{j,6}$ | ${}^{(i)}mt^1_{j,7}$ |
|---|---|---|---|---|---|---|---|---|
| ${}^{(i)}mP^1_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| ${}^{(i)}mP^1_2$ | 2 | 1 | 1 | 1 | 2 | 1 | - | - |

| dispatching rule for local processes | | | |
|---|---|---|---|
| ${}^{(i)}\sigma^0_1$ | $({}^{(i)}P^1_1)$ | ${}^{(i)}\sigma^0_6$ | $({}^{(i)}P^1_1, {}^{(i)}P^1_3)$ |
| ${}^{(i)}\sigma^0_2$ | $({}^{(i)}P^1_1, {}^{(i)}P^1_2)$ | ${}^{(i)}\sigma^0_8$ | $({}^{(i)}P^1_2, {}^{(i)}P^1_3)$ |
| ${}^{(i)}\sigma^0_3$ | $({}^{(i)}P^1_2)$ | ${}^{(i)}\sigma^0_9$ | $({}^{(i)}P^1_3)$ |
| dispatching rule for multimodal processes | | | |
| ${}^{(i)}\sigma^1_1$ | $({}^{(i)}m^1P^1_2)$ | ${}^{(i)}\sigma^1_7$ | $({}^{(i)}m^1P^1_2, {}^{(i)}m^1P^1_1)$ |
| ${}^{(i)}\sigma^1_2$ | $({}^{(i)}m^1P^1_2, {}^{(i)}m^1P^1_1)$ | ${}^{(i)}\sigma^1_9$ | $({}^{(i)}m^1P^1_1)$ |
| ${}^{(i)}\sigma^1_3$ | $({}^{(i)}m^1P^1_2, {}^{(i)}m^1P^1_1)$ | | |

the work focuses on evaluation of the network capability allowing distinguished multimodal processes to continue in order to accomplish trips following an assumed set of multimodal chains connecting transport modes between origins and destinations.

A declarative modeling approach to different transport modes scheduling in mesh-like communication network of streets in a city is considered. Opposite to traditional approach a given network of local cyclic acting transportation services is assumed. In such a regular network encompassing grid pattern, i.e. composed of elementary and structurally isomorphic subnetworks, the passengers pass their origin-destination routes among stations (terminals, platforms) using local lines, i.e. a fleet of transport modes assigned to relevant subnetworks. Since scheduling problem of transportation means can be seen as a blocking job-shop one, i.e. belonging to a class of NP-hard problems, hence the considered case of transportation lines scheduling in mesh-like environments also belongs to NP-hard problems. The solution proposed assumes that schedules of locally acting transportation services will match-up the given, i.e. already planned, schedules of passengers itineraries. The relevant sufficient conditions guaranteeing such a match-up exists were provided. Their implementation, as it was shown enables to consider polynomial complexity cyclic scheduling method aimed at mesh-like multimodal transportation processes.

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