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A 4-D chaotic hyperjerk system with a hidden attractor, adaptive backstepping control and circuit design

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A novel 4-D chaotic hyperjerk system with four quadratic nonlinearities is presented in this work. It is interesting that the hyperjerk system has no equilibrium. A chaotic attractor is said to be a *hidden attractor* when its basin of attraction has no intersection with small neighborhoods of equilibrium points of the system. Thus, our new non-equilibrium hyperjerk system possesses a hidden attractor. Chaos in the system has been observed in phase portraits and verified by positive Lyapunov exponents. Adaptive backstepping controller is designed for the global chaos control of the non-equilibrium hyperjerk system with a hidden attractor. An electronic circuit for realizing the non-equilibrium hyperjerk system is also introduced, which validates the theoretical chaotic model of the hyperjerk system with a hidden chaotic attractor.

Key words: chaos, chaotic systems, hyperjerk systems, hidden attractors, adaptive control, backstepping control, circuit design

1. Introduction

Various dynamical systems with chaos have been reported in many modelling applications [1–4]. A number of authors have considered applications of chaos in different fields [5–16]. In [5–7], Vaidyanathan discussed chaos control and synchronization of novel 3-D chemical chaotic reactors. In [8–11], Vaidyanathan discussed Cellular Neural Network (CNN) attractors, FitzHugh-Nagumo neurology models, Tokamak systems and Lotka-Volterra population biology models.

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In [12], Vaidyanathan *et al.* discussed backstepping controller design for the anti-synchronization of WINDMI chaotic systems. In [13], Cavusoglu *et al.* proposed a new chaotic system with high dynamic features and designed chaos-based hybrid RSA (CRSA) encryption algorithm design in which RNG and RSA algorithms were used together. In [14], Ravi *et al.* proposed two 3-stage hybrid prediction models to address financial time-series prediction problem. In [15], Khorashadizadeh and Majidi proposed a new method for secure communication based on chaos synchronization using Fourier series expansion for compensation of uncertainties. In [16], Hawchar *et al.* proposed an efficient method for time-variant reliability analysis using principal component analysis and polynomial chaos expansion.

A *jerk equation* is given by an explicit third-order differential equation in Classical Mechanics that describes the dynamics of a single scalar variable x (displacement) and having the general form as

$$\frac{d^3x}{dt^3} = f\left(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}\right). \quad (1)$$

The ODE (1) is called a *jerk equation* because the consecutive derivatives of the displacement (x) are velocity (\dot{x}), acceleration (\ddot{x}), and jerk (\dddot{x}) in Classical Mechanics.

A generalization of the jerk dynamics (1) yields the higher-order differential equation

$$\frac{d^{(n)}x}{dt^n} = f\left(x, \frac{dx}{dt}, \dots, \frac{d^{(n-1)}x}{dt^{n-1}}\right), \quad (n \geq 4). \quad (2)$$

The ODE (2) is called a *hyperjerk equation* since it features time derivatives $\frac{d^{(j)}x}{dt^j}$, ($j = 1, \dots, n-1$) of a jerk function.

By denoting

$$x_1 = x, \quad x_2 = \frac{dx}{dt}, \quad \dots, \quad x_n = \frac{d^{(n-1)}x}{dt^{n-1}}, \quad (3)$$

we can also represent the ODE (2) as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \vdots \\ \dot{x}_{n-1} = x_n, \\ \dot{x}_n = f(x_1, x_2, \dots, x_n). \end{cases} \quad (4)$$

We call the nonlinear system of differential equations (4) as a *hyperjerk system*, when $n \geq 4$.

Several attempts have been made to discover chaotic hyperjerk system [17]. Elementary chaotic hyperjerk flows were found by Munmuangsaen and Srisuchinwong [18]. By applying the concept of memory element, Bao *et al.* presented a simple chaotic memory system with complex dynamics [19]. In [20], Dalkiran and Sprott constructed a hyperjerk system with exponential nonlinear function. Four-dimensional hyperchaotic hyperjerk systems were reported in [21, 22]. All these hyperjerk systems have equilibrium points and the stability types of the unstable equilibrium points of these systems have been discussed as well. We remark that there has been no detailed investigation of hyperjerk systems without equilibrium. In this work, we propose a new non-equilibrium hyperjerk system and discuss its properties, circuit design and applications.

A chaotic attractor is said to be a *hidden attractor* when its basin of attraction has no intersection with small neighborhoods of equilibrium points of the system [23]. Thus, our new non-equilibrium hyperjerk system possesses a hidden attractor. Chaotic systems with hidden attractors have applications in many areas [24–28].

Controlling the state trajectories of chaotic systems have received good attention in the control literature [29–33]. We use adaptive backstepping control [34–38] for the global chaos control of the new non-equilibrium hyperjerk system.

2. A non-equilibrium hyperjerk system

In this work, we introduce a novel hyperjerk equation given by the dynamics

$$\frac{d^4x}{dt^4} = a \frac{d^3x}{dt^3} + bx \frac{dx}{dt} + x \frac{d^3x}{dt^3} + c \left(x^2 + x \frac{d^2x}{dt^2} + 1 \right). \quad (5)$$

We define new state variables as

$$\begin{cases} x_1 = x, \\ x_2 = \dot{x}, \\ x_3 = \ddot{x}. \end{cases} \quad (6)$$

Using (6), our novel hyperjerk equation (5) can be rewritten in system form as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = ax_4 + bx_1x_2 + x_1x_4 + c(x_1^2 + x_1x_3 + 1), \end{cases} \quad (7)$$

where a, b and c are positive parameters.

It is interesting to see that the system (7) is chaotic for the following set of parameters: $a = 2.2$, $b = 5$ and $c = 2$.

For $(a, b, c) = (2.2, 5, 2)$, the Lyapunov exponents of the hyperjerk system (7) are determined by using Wolf's algorithm [43] as $L_1 = 0.1557$, $L_2 = 0$, $L_3 = -1.3455$ and $L_4 = -3.9844$.

The equilibrium points of the hyperjerk system (7) are determined by solving the following system of equations:

$$\begin{cases} x_2 & = 0, \\ x_3 & = 0, \\ x_4 & = 0, \\ ax_4 + bx_1x_2 + x_1x_4 + c(x_1^2 + x_1x_3 + 1) & = 0. \end{cases} \quad (8)$$

From the first three equations in Eq. (8), we obtain

$$x_2 = 0, \quad x_3 = 0, \quad x_4 = 0. \quad (9)$$

Substituting above in the last equation in Eq. (8), we obtain

$$c(x_1^2 + 1) = 0. \quad (10)$$

Because $c > 0$, it is immediate that the system (7) has no equilibrium point. Thus, we deduce that (7) is a chaotic hyperjerk system with a hidden attractor [23].

It is noted that the initial values of the chaotic non-equilibrium hyperjerk system (7) are taken as $X(0) = (0, -5, 0, 0)$ for our numerical simulations.

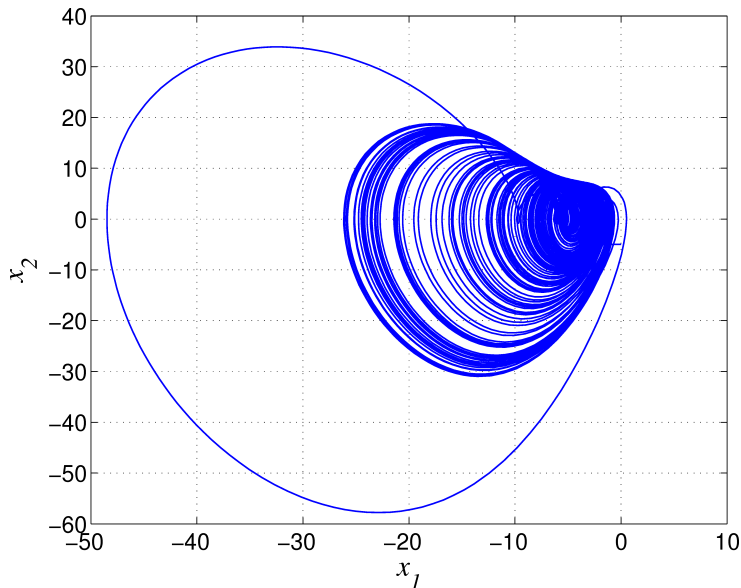


Figure 1: Phase portrait of the chaotic non-equilibrium hyperjerk system on (x_1, x_2) plane for $(a, b, c) = (2.2, 5, 2)$ and $X(0) = (0, -5, 0, 0)$

Figures 1–3 illustrate the two-dimensional projections of the chaotic non-equilibrium hyperjerk system (7) on (x_1, x_2) , (x_1, x_3) , (x_1, x_4) planes respectively.

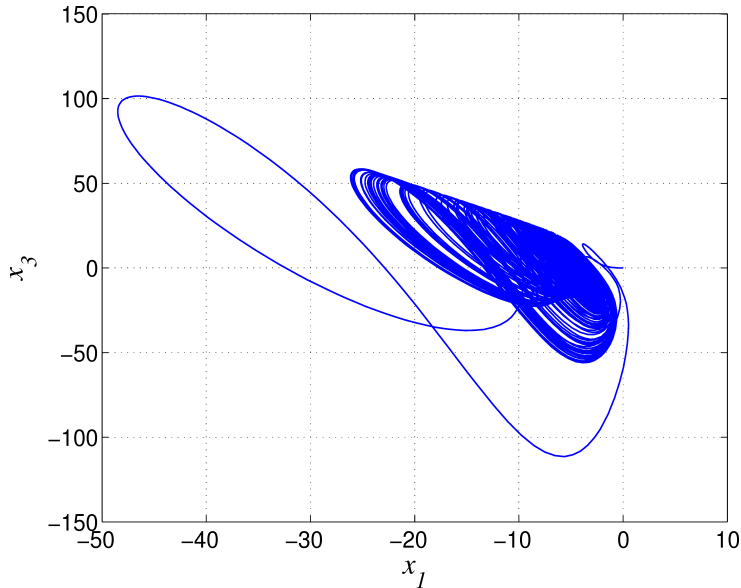


Figure 2: Phase portrait of the chaotic non-equilibrium hyperjerk system on (x_1, x_3) plane for $(a, b, c) = (2.2, 5, 2)$ and $X(0) = (0, -5, 0, 0)$

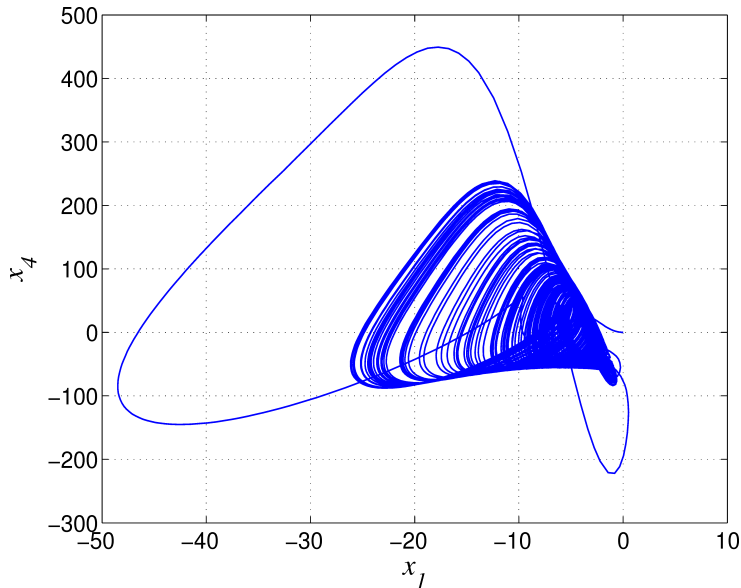


Figure 3: Phase portrait of the chaotic non-equilibrium hyperjerk system on (x_1, x_4) plane for $(a, b, c) = (2.2, 5, 2)$ and $X(0) = (0, -5, 0, 0)$

3. Adaptive control of the new non-equilibrium hyperjerk system via backstepping control

In this section, we design an adaptive feedback control law for globally stabilizing the new non-equilibrium hyperjerk system via backstepping control method.

We take the new non-equilibrium hyperjerk system with a single feedback control given by

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = ax_4 + bx_1x_2 + x_1x_4 + c(x_1^2 + x_1x_3 + 1) + u, \end{cases} \quad (11)$$

where a, b, c are unknown parameters and u is an adaptive feedback control law which uses a time-varying parameter estimate $(A(t), B(t), C(t))$ in lieu of (a, b, c) .

The parameter estimation errors are defined as:

$$e_a(t) = a - A(t), \quad e_b(t) = b - B(t), \quad e_c(t) = c - C(t). \quad (12)$$

It is easy to see that

$$\dot{e}_a = \dot{A}, \quad \dot{e}_b = \dot{B}, \quad \dot{e}_c = \dot{C}. \quad (13)$$

Next, we establish the main adaptive control result of this section.

Theorem 1 *The new non-equilibrium hyperjerk system (11) with unknown parameters (a, b, c) is globally and asymptotically stabilized by the adaptive feedback control law,*

$$\begin{aligned} u(t) = & -5x_1 - 10x_2 - 9x_3 - [A(t) + 4]x_4 - B(t)x_1x_2 - x_1x_4 \\ & - C(t)(x_1^2 + x_1x_3 + 1) - Kz_4, \end{aligned} \quad (14)$$

where K is a positive constant,

$$z_4 = 3x_1 + 5x_2 + 3x_3 + x_4 \quad (15)$$

and the update law for the parameter estimates $(A(t), B(t), C(t))$ is given by

$$\begin{cases} \dot{A}(t) = z_4x_4, \\ \dot{B}(t) = z_4x_1x_2, \\ \dot{C}(t) = z_4(x_1^2 + x_1x_3 + 1). \end{cases} \quad (16)$$

Proof. We prove this result via Lyapunov stability theory [44].

In backstepping control method, we start with a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2, \quad (17)$$

where

$$z_1 = x_1. \quad (18)$$

Differentiating V_1 along (11), we find that

$$\dot{V}_1 = z_1 \dot{z}_1 = x_1 x_2 = -z_1^2 + z_1(x_1 + x_2). \quad (19)$$

We set

$$z_2 = x_1 + x_2. \quad (20)$$

The equation (19) can be simplified as

$$\dot{V}_1 = -z_1^2 + z_1 z_2. \quad (21)$$

Next, we take a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2} (z_1^2 + z_2^2). \quad (22)$$

Differentiating V_2 along (11), we find that

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2x_1 + 2x_2 + x_3). \quad (23)$$

We set

$$z_3 = 2x_1 + 2x_2 + x_3. \quad (24)$$

Using (24), the equation (23) can be simplified as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3. \quad (25)$$

Next, we take a quadratic Lyapunov function

$$V_3(z_1, z_2, z_3) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 = \frac{1}{2} (z_1^2 + z_2^2 + z_3^2). \quad (26)$$

Differentiating V_3 along the dynamics (11), we find that

$$\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 + z_3(3x_1 + 5x_2 + 3x_3 + x_4). \quad (27)$$

We set

$$z_4 = 3x_1 + 5x_2 + 3x_3 + x_4. \quad (28)$$

Using (28), the equation (27) can be simplified as

$$\dot{V}_2 = -z_1^2 - z_2^2 - z_3^2 + z_3 z_4. \quad (29)$$

Finally, we take the quadratic Lyapunov function

$$V(z_1, z_2, z_3, z_4, e_a, e_b, e_c) = V_3(z_1, z_2, z_3) + \frac{1}{2}z_4^2 + \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 + \frac{1}{2}e_c^2. \quad (30)$$

It is clear that V is a positive definite function on \mathbf{R}^7 .

Differentiating V along (11), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4(z_4 + z_3 + \dot{z}_4) - e_a \dot{A} - e_b \dot{B} - e_c \dot{C}. \quad (31)$$

The equation (31) can be expressed in a compact manner as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4 S - e_a \dot{A} - e_b \dot{B} - e_c \dot{C}, \quad (32)$$

where

$$S = z_4 + z_3 + \dot{z}_4 = z_4 + z_3 + 3\dot{x}_1 + 5\dot{x}_2 + 3\dot{x}_3 + \dot{x}_4. \quad (33)$$

It is easy to see that

$$\begin{aligned} S &= 5x_1 + 10x_2 + 9x_3 + (a+4)x_4 + bx_1x_2 + x_1x_4 \\ &+ c(x_1^2 + x_1x_3 + 1) + u. \end{aligned} \quad (34)$$

Substitution of the adaptive control law (14) into (34) yields the result

$$S = [a - A(t)]x_4 + [b - B(t)]x_1x_2 + [c - C(t)](x_1^2 + x_1x_3 + 1) - Kz_4. \quad (35)$$

Using (13), it is easy to simplify (35) as

$$S = e_ax_4 + e_bx_1x_2 + e_c(x_1^2 + x_1x_3 + 1) - Kz_4. \quad (36)$$

Combining (36) and (32), we find that we obtain

$$\begin{aligned} \dot{V} &= -z_1^2 - z_2^2 - z_3^2 - (1+K)z_4^2 + e_a(z_4x_4 - \dot{A}) \\ &+ e_b(z_4x_1x_2 - \dot{B}) + e_c[z_4(x_1^2 + x_1x_3 + 1) - \dot{C}]. \end{aligned} \quad (37)$$

Substituting the update law (16) into (37), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - (1+K)z_4^2, \quad (38)$$

which is a negative semi-definite function on \mathbf{R}^7 .

Using Barbalat's lemma [44], we conclude that $\mathbf{z}(t) \rightarrow \mathbf{0}$ asymptotically as $t \rightarrow \infty$ for all initial conditions $\mathbf{z}(0) \in \mathbf{R}^4$.

Hence, it is immediate that $\mathbf{x}(t) \rightarrow \mathbf{0}$ asymptotically as $t \rightarrow \infty$ for all initial conditions $\mathbf{x}(0) \in \mathbf{R}^4$.

This completes the proof. \square

For numerical simulations, we suppose that the parameter values of the new hyperjerk system (11) are taken as in the chaotic case, *i.e.* $(a, b, c) = (2.2, 5, 2)$. Also, we take $K = 20$.

As initial conditions of the new hyperjerk system (11), we take $x_1(0) = 2.5$, $x_2(0) = 1.2$, $x_3(0) = 3.6$ and $x_4(0) = 1.7$.

Also, as initial conditions of the parameter estimates, we take $A(0) = 4.3$, $B(0) = 10.4$ and $C(0) = 8.7$.

In Figure 4, the asymptotic convergence of the controlled state $\mathbf{x}(t)$ is exhibited, when the controls (14) and (16) are implemented.

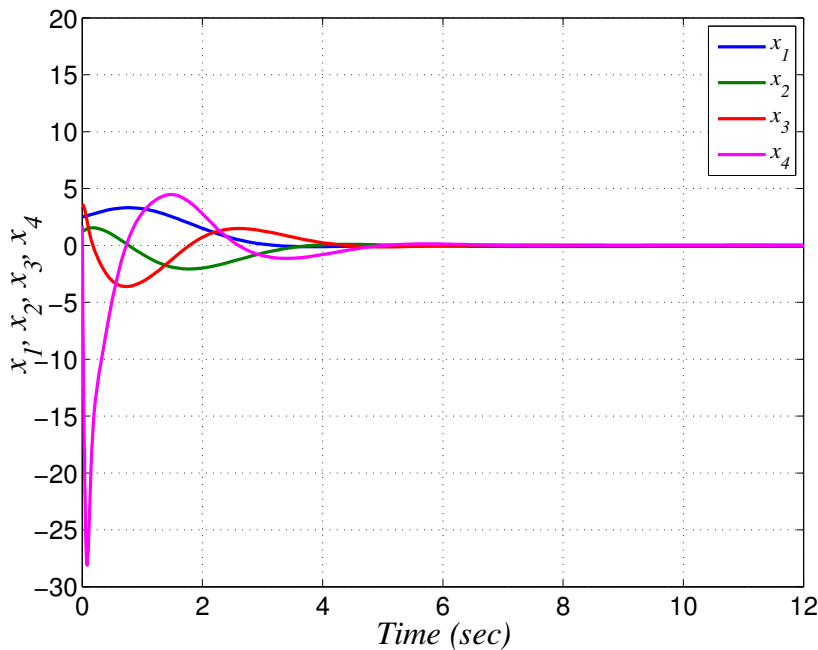


Figure 4: Time-history of the controlled states of the hyperjerk system (11)

4. Circuit implementation of the non-equilibrium hyperjerk system

In this section, we present an electronic circuit realizing new hyperjerk system (7) to verify its feasibility. The designed circuit is shown in Figure 5. Here we have used the general known approach [39–42], in which the state variables

of system (7) correspond to the voltages at operational amplifiers $U_1, U_2, U_3,$ and $U_4,$ respectively.

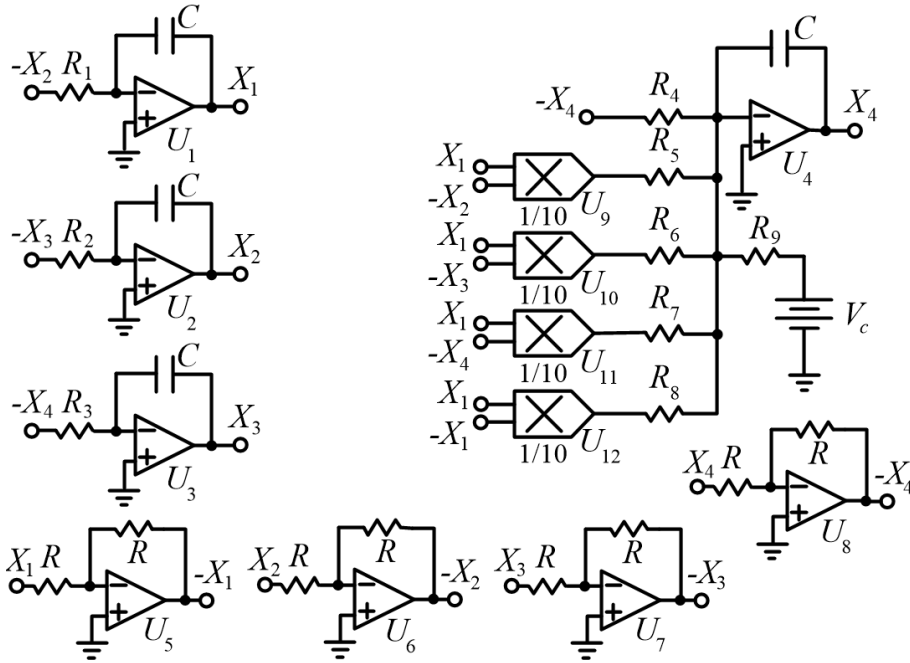


Figure 5: Designed circuit for the non-equilibrium hyperjerk system (7)

By applying Kirchoff’s laws to the designed electronic circuit, we have equations of circuit

$$\left\{ \begin{array}{l} \dot{X}_1 = \frac{1}{R_1 C} X_2, \\ \dot{X}_2 = \frac{1}{R_2 C} X_3, \\ \dot{X}_3 = \frac{1}{R_3 C} X_4, \\ \dot{X}_4 = \frac{1}{R_4 C} X_4 + \frac{1}{10 R_5 C} X_1 X_2 + \frac{1}{10 R_6 C} X_1 X_3 \\ \quad + \frac{1}{10 R_7 C} X_1 X_4 + \frac{1}{10 R_8 C} X_1^2 - \frac{1}{R_9 C} V_c, \end{array} \right. \quad (39)$$

where $X_1, X_2, X_3,$ and X_4 are the voltages at operational amplifiers $U_1, U_2, U_3,$ and $U_4.$ It is simple to see that Eq. (39) matches with Eq. (7).

The circuit is implemented in PSpice with $C = 1$ nF, $R_1 = R_2 = R = 100$ k Ω , $R_3 = 20$ k Ω , $R_4 = 45.45$ k Ω , $R_5 = R_7 = 1$ k Ω , $R_6 = R_8 = 2.5$ k Ω , $R_9 = 2.5$ M Ω , and $V_c = -1 V_{DC}$.

Figures 6, 7, and 8 illustrate phase portraits of the circuit (39) in (X_1, X_2) , (X_1, X_3) and (X_1, X_4) coordinate planes respectively.

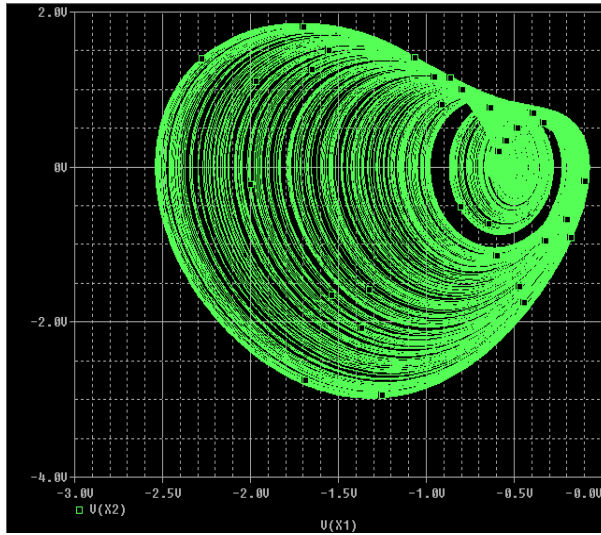


Figure 6: Phase portrait of the designed circuit (39) in (X_1, X_2) -plane

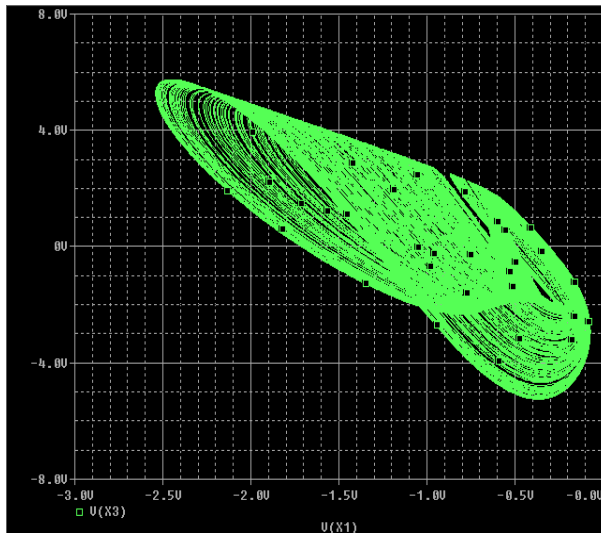


Figure 7: Phase portrait of the designed circuit (39) in (X_1, X_3) -plane

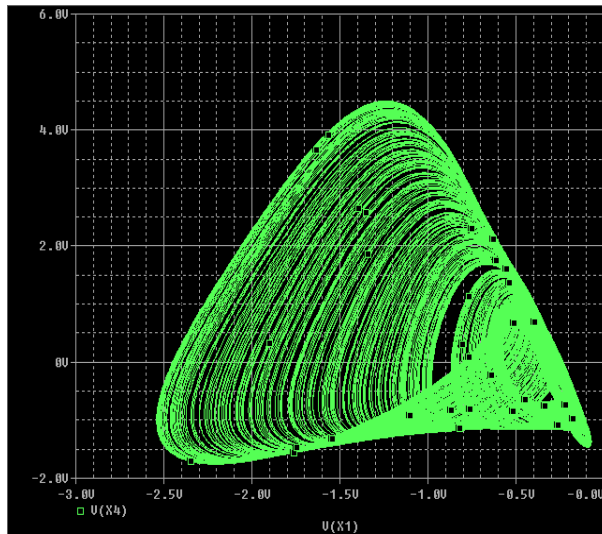


Figure 8: Phase portrait of the designed circuit (39) in (X_1, X_4) -plane

We notice that circuit simulations of the new non-equilibrium hyperjerk system (39) shown in Figures 6–8 show good agreement with the MATLAB simulations of the non-equilibrium hyperjerk system (7) shown in Figures 1–4. This validates the theoretical model of the non-equilibrium hyperjerk system.

5. Conclusion

We presented results for a new 4-D chaotic hyperjerk system with four quadratic nonlinearities. An interesting feature of this new chaotic hyperjerk system is that it has no equilibrium points and hence it possesses a hidden chaotic attractor. We verified chaos in the new hyperjerk system by a positive Lyapunov exponent. Adaptive backstepping controller was designed for the global chaos control of the non-equilibrium hyperjerk system with a hidden attractor and MATLAB simulation was given in detail. An electronic circuit for realizing the non-equilibrium hyperjerk system was implemented in PSpice and circuit simulations were detailed.

References

- [1] A.T. AZAR and S. VAIDYANATHAN: *Advances in Chaos Theory and Intelligent Control*, Springer, Berlin, Germany, 2016.

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- [2] S. VAIDYANATHAN and C. VOLOS: *Advances and Applications in Chaotic Systems*, Springer, Berlin, Germany, 2016.
- [3] S. VAIDYANATHAN and C. VOLOS: *Advances in Memristors, Memristive Devices and Systems*, Springer, Berlin, Germany, 2017.
- [4] A.T. AZAR, S. VAIDYANATHAN and A. OUANNAS: *Fractional Order Control and Synchronization of Chaotic Systems*, Springer, Berlin, Germany, 2017.
- [5] S. VAIDYANATHAN: A novel chemical chaotic reactor system and its adaptive control, *International Journal of ChemTech Research*, textbf8(7), (2015), 146–158.
- [6] S. VAIDYANATHAN: Adaptive synchronization of novel 3-D chemical chaotic reactor systems, *International Journal of ChemTech Research*, **8**(7), (2015), 159-171.
- [7] S. VAIDYANATHAN: Global chaos synchronization of chemical chaotic reactors via novel sliding mode control, *International Journal of ChemTech Research*, **8**(7), (2015), 209–221.
- [8] S. VAIDYANATHAN: Synchronization of 3-cells cellular neural network (CNN) attractors via adaptive control method, *International Journal of PharmTech Research*, **8**(5), (2015), 946–955.
- [9] S. VAIDYANATHAN: Adaptive control of the FitzHugh-Nagumo chaotic neuron model, *International Journal of PharmTech Research*, **8**(6), (2015), 117–127.
- [10] S. VAIDYANATHAN: Synchronization of Tokamak systems with symmetric and magnetically confined plasma via adaptive control, *International Journal of ChemTech Research*, **8**(6), (2015), 818–827.
- [11] S. VAIDYANATHAN: Lotka-Volterra population biology models with negative feedback and their ecological monitoring, *International Journal of PharmTech Research*, **8**(5), (2015), 974–981.
- [12] S. VAIDYANATHAN, C.K. VOLOS, K. RAJAGOPAL, I.M. KYPRIANIDIS and I.N. STOUBOULOS: Adaptive backstepping controller design for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters and its SPICE implementation, *Journal of Engineering Science and Technology Review*, **8** (2), (2015), 74–82.

- [13] U. CAVUSOGLU, A. AKGUL, A. ZENGIN and I. PEHLIVAN: The design and implementation of hybrid RSA algorithm using a novel chaos based RNG, *Chaos, Solitons & Fractals*, **104** (2017), 655–667.
- [14] V. RAVI, D. PRADEEPKUMAR and K. DEB: Financial time series prediction using hybrids of chaos theory, multi-layer perceptron and multi-objective evolutionary algorithms, *Swarm and Evolutionary Computation*, **36** (2017), 136–149.
- [15] S. KHORASHADIZADEH and M-H. MAJIDI: Chaos synchronization using the Fourier series expansion with application to secure communications, *AEU – International Journal of Electronics and Communications*, **82** (2017), 37–44.
- [16] L. HAWCHAR, C.P. EL SOUEIDY and F. SCHOEFS: Principal component analysis and polynomial chaos expansion for time-variant reliability problems, *Reliability Engineering and System Safety*, **167** (2017), 406–416.
- [17] K.E. CHLOUVERAKIS and J.C. SPROTT: Chaotic hyperjerk systems, *Chaos, Solitons and Fractals*, **28** (2006), 739–746.
- [18] B. MUNMUANGSAEN and B. SRISUCHINWONG: Elementary chaotic snap flows, *Chaos, Solitons and Fractals*, **44** (2011), 995–1003.
- [19] B. BAO, X. ZOU, Z. LIU and F. HU: Generalized memory element and chaotic memory system, *International Journal of Bifurcation and Chaos*, **22** (2013), 1350135.
- [20] F.Y. DALKIRAN and J.C. SPROTT: Simple chaotic hyperjerk system, *International Journal of Bifurcation and Chaos*, **26** (2016), 311–318.
- [21] S. VAIDYANATHAN, C. VOLOS, V.T. PHAM and K. MADHAVAN: Analysis, adaptive control and synchronization of a novel 4-D hyperchaotic hyperjerk system and its SPICE implementation, *Archives of Control Sciences*, **25** (2015), 135–158.
- [22] S. VAIDYANATHAN: Analysis, adaptive control and synchronization of a novel 4-D hyperchaotic hyperjerk system via backstepping control method, *Archives of Control Sciences*, **26** (2016), 311–318.
- [23] D. DUDKOWSKI, S. JAFARI, T. KAPITANIAK, N.V. KUZNETSOV, G.A. LEONOV and A. PRASAD: Hidden attractors in dynamical systems, *Physics Reports*, **637** (2016), 1–50.
- [24] M.F. DANCA and N. KUZNETSOV: Hidden chaotic sets in a Hopfield neural system, *Chaos, Solitons & Fractals*, **103** (2017), 144–150.

- [25] J.P. SINGH and B.K. ROY: Coexistence of asymmetric hidden chaotic attractors in a new simple 4-D chaotic system with curve of equilibria, *Optik*, **145** (2017), 209–217.
- [26] S.T. KINGNI, S. JAFARI, V.T. PHAM and P. WOAFO: Constructing and analyzing of a unique three-dimensional chaotic autonomous system exhibiting three families of hidden attractors, *Mathematics and Computers in Simulation*, **132** (2017), 172–182.
- [27] B.C. BAO, H. BAO, N. WANG, M. CHEN and Q. XU: Hidden extreme multistability in memristive hyperchaotic system, *Chaos, Solitons & Fractals*, **94** (2017), 102–111.
- [28] S.T. KINGNI, V.T. PHAM, S. JAFARI and P. WOAFO: A chaotic system with an infinite number of equilibrium points located on a line and on a hyperbola and its fractional-order form, *Chaos, Solitons & Fractals*, **99** (2017), 209–218.
- [29] S. RASAPPAN and S. VAIDYANATHAN: Global chaos synchronization of WINDMI and Coulet chaotic systems by backstepping control, *Far East Journal of Mathematical Sciences*, **67**(2), (2012), 265–287.
- [30] S. VAIDYANATHAN, C.K. VOLOS and V.T. PHAM: Global chaos control of a novel nine-term chaotic system via sliding mode control, *Studies in Computational Intelligence*, **576** (2015), 571–590.
- [31] S. VAIDYANATHAN and S. RASAPPAN: Hybrid synchronization of hyperchaotic Qi and Lü systems by nonlinear control, *Communications in Computer and Information Science*, **131** (2011), 585–593.
- [32] A.I. EGUNJOBI, O.I. OLUSOLA, A.N. NJAH, S. SAHA and S.K. DANA: Experimental evidence of chaos synchronization via cyclic coupling, *Communications in Nonlinear Science and Numerical Simulation*, **56** (2018), 588–595.
- [33] H. PENG, X. WANG, B. SHI, S. ZHANG and B. CHEN: Stabilizing constrained chaotic system using a symplectic psuedospectral method, *Communications in Nonlinear Science and Numerical Simulation*, **56** (2018), 77–92.
- [34] S. VAIDYANATHAN: Analysis, control, and synchronization of a 3-D novel jerk chaotic system with two quadratic nonlinearities, *Kyungpook Mathematical Journal*, **55**(3), (2015), 563–586.

- [35] S. VAIDYANATHAN, C.K. VOLOS, K. RAJAGOPAL, I.M. KYPRIANIDIS and I.N. STOUBOULOS: Adaptive backstepping controller design for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters and its SPICE implementation, *Journal of Engineering Science and Technology Review*, **8**(2), (2015), 74–82.
- [36] S. VAIDYANATHAN: A new 3-D jerk chaotic system with two cubic nonlinearities and its adaptive backstepping control, *Archives of Control Sciences*, **27**(3), (2017), 409–439.
- [37] Q. ZHANG and C. WANG: Robust adaptive backstepping control for a class of constrained non-affine nonlinear systems via self-organizing Hermite-polynomial-based neural network disturbance observer, *Advances in Mechanical Engineering*, **9**(5), (2012), 1–12.
- [38] S. VAIDYANATHAN: A novel 3-D jerk chaotic system with two quadratic nonlinearities and its adaptive backstepping control, *International Journal of Control Theory and Applications*, **9**(1), (2016), 199–219.
- [39] V.T. PHAM, S. JAFARI, C. VOLOS, A. GIAKOUMIS, S. VAIDYANATHAN and T. KAPITANIAK: A chaotic system with equilibria located on the rounded square loop and its circuit implementation, *IEEE Transactions on Circuits and Systems II: Express Briefs*, **63**(9), (2016), 878–882.
- [40] C. VOLOS, J.O. MAAITA, S. VAIDYANATHAN, V.T. PHAM, I. STOUBOULOS and I. KYPRIANIDIS: A chaotic system with equilibria located on the rounded square loop and its circuit implementation, *IEEE Transactions on Circuits and Systems II: Express Briefs*, **64**(3), (2017), 339–343.
- [41] C.K. VOLOS, A. AKGUL, V.T. PHAM and M.S. BAPTISTA: Antimonotonicity, crisis and multiple attractors in a simple memristive circuit, *Journal of Circuits, Systems and Computers*, **27**(2), (2018), Article no. 1850026.
- [42] A. AKGUL, C. LI and I. PEHLIVAN: Amplitude control analysis of a four-wing chaotic attractor, its electronic circuit designs and microcontroller-based random number generator, *Journal of Circuits, Systems and Computers*, **26**(12), (2017), Article no. 1750190.
- [43] A. WOLF, J.B. SWIFT, H.L. SWINNEY and J.A. VASTANO: Determining Lyapunov Exponents From A Time Series, *Physica D*, **16** (1985), 285–317.
- [44] Z. VUKIC, L. KULJACA, D. DONLAGIC and S. TESNJAK: *Nonlinear Control Systems*, Marcel Dekker, New York, USA, 2003.