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# Safety of multistate series-„*k* out of *n*" systems

**Keywords**
large systems, safety of large systems, asymptotic approach, limit reliability function

**Abstract**

The paper is concerned with mathematical methods in asymptotic approach to systems safety and reliability analysis. In the safety investigation of large-scale systems, the problem of the complexity of their safety functions arises. This problem may be approximately solved by assuming that the number of system components tends to infinity and finding the limit safety function of the system. The article is based on the results of the author's solutions for series–"*m* out of $k_n$" given in [4] – [6].

## 1. System safety – multistate approach

Like in previous paper, following [4] to define the system with degrading components we assume that

$$E_{ij},\ i = 1,2,...,k_n,\ j = 1,2,...,l_n;\ k_n, l_1, l_2,..., l_{k_n} \in N$$

are the system components and all of them have the safety state set {0, 1,…, $z$}, $z \geq 1$, where safety states are ordered, the safety state 0 is the worst and the state $z$ is the best. Moreover $T_{ij}(u)$ is a random variable representing the lifetimes of assets $E_{ij}$ in the safety state subset $\{u, u+1,..., z\}$, $u = 1,2,\dots,z$, while they were in the safety state $z$ at the moment $t = 0$ and $T(u)$ is a random variable representing the lifetime the system in the safety state subset $\{u, u+1,..., z\}$, $u = 1,2,\dots,z$, while it was in the safety state $z$ at the moment $t = 0$.

We also assume that the safety states, the assets and critical system degrade with time $t$, $s_{ij}(t)$ is the asset's $E_{ij}$ safety state and $s(t)$ is the system safety state at the moment $t$, $t \in <0, \infty)$, given that it was in the safety state $z$ at the moment $t = 0$. The above assumptions mean that the safety state of the system with degrading assets may be changed in time only from better to worse.

We denote the safety function of the system by a vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2),..., \boldsymbol{S}(t,z)],$$

with the coordinates defined by safety functions

$$\boldsymbol{S}(t,u) = P(T(u) > t) \text{ for } t \in <0, \infty),\ u = 1, 2,..., z.$$

Obviously $T(u)$ is a random variable representing the lifetime the system in the safety state subset $\{u, u+1,..., z\}$, $u = 1,2,\dots,z$, while it was in the safety state $z$ at the moment $t = 0$.

## 2. Asymptotic approach to systems safety

The asymptotic approach to systems safety is based on investigating limit distributions of a standardized random variable

$$(T(u) - b_n(u)) / a_n(u),$$

where $T(u)$ is the lifetime of the system in the safety state subset $\{u, u+1,..., z\}$ and $a_n(u)$, $b_n(u)$ are some suitably chosen numbers such that $a_n(u) > 0$, $b_n(u) \in (-\infty, \infty)$.

And since

$$P((T(u) - b_n(u)) / a_n(u) > t) =$$
$$= P(T(u) > a_n(u)t + b_n(u)) = \boldsymbol{S}_n(a_n(u)t + b_n(u), u),$$

where $\boldsymbol{S}_n(t,u)$, $u = 1,2,\dots,z$, are coordinates of a safety function of the system, then we assume the following definition.

*Definition 1*

A safety function

$$\mathfrak{I}(t,\cdot)=\left[1,\mathfrak{I}(t,1),\mathfrak{I}(t,2),...,\mathfrak{I}(t,z)\right]$$

is called the limit safety function of the homogeneous regular series–"$m_n$ out of $k_n$" multistate system if there exist normalising constants $a_n(u)$, $b_n(u)$ such that $a_n(u)>0$, $b_n(u)\in(-\infty,\infty)$ for $u=1,2,...,z$ and

$$\lim_{n\to\infty}\boldsymbol{S}^{(m_n)}_{k_n,l_n}\left(a_n(u)t+b_n(u),u\right)=\mathfrak{I}(t,u) \quad \text{for } t\in C_{\mathfrak{I}(u)},$$

where for $u\in\{1,2,...,z\}$, $C_{\mathfrak{I}(u)}$ is the set of continuity points of $\mathfrak{I}(t,u)$ and $\boldsymbol{S}^{(m_n)}_{k_n,l_n}(t,u)$, $u=1,2,...,z$ are the coordinates of the safety function of the multistate homogeneous regular series–"$m_n$ out of $k_n$" system. Hence, for $u=1,2,...,z$ and sufficiently large $n$ we get the following approximate formula:

$$\boldsymbol{S}^{(m_n)}_{k_n,l_n}(t,u)\cong$$
$$\cong\mathfrak{I}\left((t-b_n(u))/a_n(u),u\right),\ t\in(-\infty,\infty). \quad (1)$$

*Definition 2*
A safety function $\mathfrak{I}(t,u)$, $u\in\{1,2,...,z\}$ is called degenerate if there exists $t_0(u)\in(-\infty,\infty)$ such that

$$\mathfrak{I}(t,u)=\begin{cases}1, & t<t_0(u)\\ 0, & t\ge t_0(u).\end{cases}$$

## 3. Safety of series–"$m_n$ out of $k_n$" systems

Suppose that

$$E_{ij},\ i=1,2,...,k_n,\ j=1,2,...,l_n,\ k_n,\ l_n\in N$$

are components of a system having coordinates of ther safety functions

$$S_{ij}(t,u)=P(T_{ij}(u)>t),\ t\in(-\infty,\infty),$$

where $T_{ij}(u)$ are independent random variables representing the lifetimes of $E_{ij}$, having distribution functions

$$F_{ij}(t,\cdot)=[1,F_{ij}(t,1),...,F_{ij}(t,z)]$$

and where

$$F_{ij}(t,u)=P(T_{ij}(u)\le t),\ u=1,2,...,z,\ t\in(-\infty,\infty).$$

*Definition 3*
A system is called regular multistate series–"$m_n$ out of $k_n$" if its lifetime $T(u)$ in the safety state subset $\{u,u+1,...,z\}$ is given by

$$T(u)=T_{(k_n-m_n+1)}(u),\ m_n=1,2,...,k_n,\ u=1,2,...,z,$$

where $T_{(k_n-m_n+1)}(u)$ is the $m_n$-th maximal order statistics in a sample of random variables

$$T_i(u)=\min_{1\le j\le l_n}\{T_{ij}(u)\},\ i=1,2,...,k_n$$

representing the lifetimes of series subsystems of the system in the safety state subset $\{u,u+1,...,z\}$.

*Definition 4*
A multistate regular series–"$m_n$ out of $k_n$" system is called homogeneous if component lifetimes $T_{ij}(u)$ in the safety state subset $\{u,u+1,...,z\}$ have an identical distribution function with the coordinates

$$F(t,u)=P(T_{ij}(u)\le t),\ t\in(-\infty,\infty),$$

$$i=1,2,...,k_n,\ j=1,2,...,l_n,$$

i.e. if its components $E_{ij}$ have the same safety function with the coordinates

$$S(t,u)=1-F(t,u),\ t\in(-\infty,\infty).$$

The safety function of the homogeneous regular series–"$m_n$ out of $k_n$" system is given by

$$\boldsymbol{S}^{(m_n)}_{k_n,l_n}(t,\cdot)=\left[1,\boldsymbol{S}^{(m_n)}_{k_n,l_n}(t,1),...,\boldsymbol{S}^{(m_n)}_{k_n,l_n}(t,z)\right],\ t\in(-\infty,\infty),$$

with the coordinates defined by

$$\boldsymbol{S}^{(m_n)}_{k_n,l_n}(t,u)=1-\sum_{i=0}^{m_n-1}\binom{k_n}{i}[R^{l_n}(t,u)]^i[1-R^{l_n}(t,u)]^{k_n-i} \quad (2)$$

for $t\in(-\infty,\infty)$, $u=1,2,...,z$,
or by formula

$$\overline{\boldsymbol{S}}^{(m_n)}_{k_n,l_n}(t,\cdot)=\left[1,\overline{\boldsymbol{S}}^{(m_n)}_{k_n,l_n}(t,1),...,\overline{\boldsymbol{S}}^{(m_n)}_{k_n,l_n}(t,z)\right],\ t\in(-\infty,\infty),$$

with the coordinates defined by

$$\overline{\boldsymbol{S}}^{(m_n)}_{k_n,l_n}(t,u)=\sum_{i=0}^{k_n-m_n}\binom{k_n}{i}[1-S^{l_n}(t,u)]^i[S^{l_n}(t,u)]^{k_n-i} \quad (3)$$

for $t\in(-\infty,\infty)$, $u=1,2,...,z$,
where $k_n$ is the number of series subsystems of a system and $l_n$ is the number of components in series subsystems.

## 4. A class of limit safety functions of multistate series-"$m_n$ out of $k_n$" systems

Types of limit safety functions of the homogeneous regular series-"$m_n$ out of $k_n$" systems depends on the relation between $m_n$ and the number $k_n$ of series subsystems of a system. We consider following relations:

*Relation 1* $\lim\limits_{n\to\infty} k_n = \infty,\ \lim\limits_{n\to\infty} m_n = m$ = const,

$$\left(\lim_{n\to\infty}\frac{m_n}{k_n}=0\right)$$

*Relation 2* $\lim\limits_{n\to\infty} k_n = \infty,\ \lim\limits_{n\to\infty}\frac{m_n}{k_n}=\lambda,\ 0<\lambda<1,$

*Relation 3* $\lim\limits_{n\to\infty} k_n = \infty,\ \lim\limits_{n\to\infty} k_n - m_n = \bar{m} = \text{const},$

$$\left(\lim_{n\to\infty}\frac{m_n}{k_n}=1\right)$$

*Relation 4* $\lim\limits_{n\to\infty} k_n = k = \text{const},\ \lim\limits_{n\to\infty} m_n = m = \text{const}.$

Types of limit safety functions of the homogeneous regular series–"$m_n$ out of $k_n$" systems depend not only on the *Relations 1-4*, but also on the shape of the system, i.e. on the relations between the number of series subsystems $k_n$ and the number of components in these subsystems $l_n$.

When *Relation 1* holds then we consider the relationships between $k_n$ and $l_n$ on the form

$$k_n = n,\ l_n = c\log^{\rho(n)} n,\ n\in(0,\infty),\ c>0,$$

with the following cases distinguished:

*Case 1.* $k_n = n, |\, l_n - c\log n\,| >> s,\ s>0,\ c>0$,

*Case 2.* $k_n = n,\ l_n - c\log n \approx s,\ s\in(-\infty,\infty),\ c>0.$

The results achieved in [6] for *Relation 1* may be generalized in the form of the following theorem.

*Theorem 1*

If the homogeneous multistate regular series-"$m_n$ out of $k_n$" system is such that

$$\lim_{n\to\infty} k_n = \infty,\ \lim_{n\to\infty} m_n = m = \text{const},\ (\lim_{n\to\infty}\frac{m_n}{k_n}=0)$$

then the only non-degenerate coordinates of limit safety function of the system are:

*Case 1.* $k_n = n, |\, l_n - c\log n\,| >> s,\ s>0,\ c>0$

$$\mathfrak{S}_1^{(m)}(t,u)=1 \text{ for } t<0,$$

$$\mathfrak{S}_1^{(m)}(t,u)=1-\sum_{i=0}^{m-1}\exp(-t^{-\alpha(u)})\frac{t^{-i\alpha(u)}}{i!}$$

for $t\ge 0,\ \alpha(u)>0,$

$$\mathfrak{S}_2^{(m)}(t,u)=1-\sum_{i=0}^{m-1}\exp[-(-t^{\alpha(u)})]\frac{(-t)^{i\alpha(u)}}{i!}$$

for $t<0, \alpha(u)>0,$

$$\mathfrak{S}_2^{(m)}(t,u)=0 \text{ for } t\ge 0,$$

$$\mathfrak{S}_3^{(m)}(t,u)=1-\sum_{i=0}^{m-1}\exp[-\exp(-t)]\frac{\exp[-it]}{i!}$$

for $t\in(-\infty,\infty)$;

*Case 2.* $k_n = n,\ l_n - c\log n \approx s,\ s\in(-\infty,\infty),\ c>0$

$$\mathfrak{S}_4^{(m)}(t,u)=1 \text{ for } t<0,$$

$$\mathfrak{S}_4^{(m)}(t,u)=$$
$$=1-\sum_{i=0}^{m-1}\exp[-\exp[-t^{\alpha(u)}-\tfrac{s}{c}]]\frac{\exp[i(-t^{\alpha}-\frac{s}{c})]}{i!},$$

for $t\ge 0, \alpha(t)>0,$

$$\mathfrak{S}_5^{(m)}(t,u)=1+$$
$$-\sum_{i=0}^{m-1}\exp[-\exp[(-t)^{\alpha(u)}-\tfrac{s}{c}]]\frac{\exp[i((-t)^{\alpha(u)}-\frac{s}{c})]}{i!}$$

for $t<0, \alpha>0,$

$$\mathfrak{S}_5^{(m)}(t,u)=0 \text{ for } t\ge 0,$$

$$\mathfrak{S}_6^{(m)}(t,u)=1-\sum_{i=0}^{m-1}\exp\left[-\exp[\ \beta(u)\,(-t)^{\alpha(u)}-\tfrac{s}{c}]\right]\cdot$$
$$\cdot\frac{\exp\left[i(\beta(u)\,(-t)^{\alpha(u)}-\frac{s}{c})\right]}{i!} \text{ for } t<0,$$

and for $t\ge 0$ $\mathfrak{S}_6^{(m)}(t,u)=$

$$=1-\sum_{i=0}^{m-1}\exp[-\exp(-t^{\alpha(u)}-\tfrac{s}{c})]\frac{\exp[i(-t^{\alpha(u)}-\frac{s}{c})]}{i!},$$

where $\alpha(u)>0, \beta(u)>0,$

$$\mathfrak{S}_7^{(m)}(t,u)=1 \text{ for } t<t_1,\ \mathfrak{S}_7^{(m)}(t,u)=0 \text{ for } t\ge t_2,$$

and for $t_1\le t<t_2$ $\mathfrak{S}_7^{(m)}(t,u)=1+$

$$-\sum_{i=0}^{m-1}\exp\left[-\exp[-\exp[-\tfrac{s}{c}]]\right]\frac{\exp\left[-i\exp[-\frac{s}{c}]\right]}{i!}.$$

When *Relation 2* holds then we consider the relationships between $k_n$ and $l_n$ on the form

$$k_n = n,\ l_n = c\log^{\rho(n)} n,\ n\in(0,\infty),\ c>0,$$

with the following cases distinguished:

*Case1.* $\rho(n) << \dfrac{1}{\sqrt{n}\log\log n},$

*Case 2.* $\rho(n) \approx \dfrac{1}{\sqrt{n}\log\log n},$

*Case 3.* $\rho(n) >> \dfrac{1}{\sqrt{n}\log\log n}$.

This case was more widely discussed in the previous article, where the meaning of <<, >> and ≈ is also explained. Results achieved in [6] for *Relation 2* may be generalized in the form of the following theorem.

*Theorem 2*

If the multistate homogeneous regular series–"$m_n$ out of $k_n$" system is such that

$$\lim_{n\to\infty} k_n = \infty,\ \ \lim_{n\to\infty}\frac{m_n}{k_n} = \lambda,\ \ 0 < \lambda < 1,$$

$$k_n = n,\, l_n = c\log^{\rho(n)} n,\, n \in (0,\infty),\, c > 0,$$

then the the only non-degenerate coordinates of limit safety function of the system are:

*Case 1.* $\rho(n) << \dfrac{1}{\sqrt{n}\log\log n}$

$$\tilde{\mathfrak{I}}_1^{(\lambda)}(t,u) = 1 \text{ for } t < 0,$$

$$\tilde{\mathfrak{I}}_1^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{ct^{\alpha(u)}} \exp[-\frac{x^2}{2}]dx$$

for $t \ge 0, c > 0, \alpha(u) > 0$,

$$\tilde{\mathfrak{I}}_2^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-c|t|^{\alpha(u)}} \exp[-\frac{x^2}{2}]dx$$

for $t < 0, c > 0, \alpha(u) > 0$,

$$\tilde{\mathfrak{I}}_2^{(\lambda)}(t,u) = 0 \text{ for } t \ge 0,$$

$$\tilde{\mathfrak{I}}_3^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-c_1|t|^{\alpha(u)}} \exp[-\frac{x^2}{2}]dx$$

for $t < 0, c_1 > 0, \alpha(u) > 0$,

$$\tilde{\mathfrak{I}}_3^{(\lambda)}(t,u) = \frac{1}{2} - \frac{1}{\sqrt{2\pi}}\int_{0}^{c_2 t^{\alpha(u)}} \exp[-\frac{x^2}{2}]dx$$

for $t \ge 0, c_2 > 0, \alpha > 0$,

$$\tilde{\mathfrak{I}}_4^{(\lambda)}(t,u) = 1 \text{ for } t < -1,$$

$$\tilde{\mathfrak{I}}_4^{(\lambda)}(t,u) = \frac{1}{2} \text{ for } -1 \le t < 1,$$

$$\tilde{\mathfrak{I}}_4^{(\lambda)}(t,u) = 0 \text{ for } t \ge 1;$$

Case 2. $\rho(n) \approx \dfrac{1}{\sqrt{n}\log\log n}$,

$$\overset{+}{\tilde{\mathfrak{I}}}{}_1^{(\lambda)}(t,u) = 1 \text{ for } t < 0,$$

$$\overset{+}{\tilde{\mathfrak{I}}}{}_1^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{ct^{\alpha(u)} - \sqrt{\frac{\lambda}{1-\lambda}}\log\lambda} \exp[-\frac{x^2}{2}]dx$$

for $t \ge 0, c > 0, \alpha(u) > 0$,

$$\overset{+}{\tilde{\mathfrak{I}}}{}_2^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-c|t|^{\alpha(u)} - \sqrt{\frac{\lambda}{1-\lambda}}\log\lambda} \exp[-\frac{x^2}{2}]dx$$

for $t < 0, c > 0, \alpha(u) > 0$,

$$\overset{+}{\tilde{\mathfrak{I}}}{}_2^{(\lambda)}(t,u) = 0 \text{ for } t \ge 0,$$

$$\overset{+}{\tilde{\mathfrak{I}}}{}_3^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-c_1|t|^{\alpha(u)} - \sqrt{\frac{\lambda}{1-\lambda}}\log\lambda} \exp[-\frac{x^2}{2}]dx$$

for $t < 0, c_1 > 0, \alpha > 0$,

$$\overset{+}{\tilde{\mathfrak{I}}}{}_3^{(\lambda)}(t,u) = \frac{1}{2} - \frac{1}{\sqrt{2\pi}}\int_{0}^{c_2 t^{\alpha(u)} - \sqrt{\frac{\lambda}{1-\lambda}}\log\lambda} \exp[-\frac{x^2}{2}]dx$$

for $t \ge 0, c_2 > 0, \alpha(u) > 0$,

$$\overset{+}{\tilde{\mathfrak{I}}}{}_4^{(\lambda)}(t,u) = 1 \text{ for } t < -1,$$

$$\overset{+}{\tilde{\mathfrak{I}}}{}_4^{(\lambda)}(t,u) = \frac{1}{2} - \frac{1}{\sqrt{2\pi}}\int_{0}^{\sqrt{\frac{\lambda}{1-\lambda}}\log\lambda} \exp[-\frac{x^2}{2}]dx$$

for $-1 \le t < 1$,

$$\overset{+}{\tilde{\mathfrak{I}}}{}_4^{(\lambda)}(t,u) = 0 \text{ for } t \ge 1;$$

*Case 3.* $\rho(n) >> \dfrac{1}{\sqrt{n}\log\log n}$

$$\overset{+}{\tilde{\mathfrak{I}}}{}_5^{(\lambda)}(t,u) = 1 \text{ for } t < 0,$$

$$\overset{+}{\tilde{\mathfrak{I}}}{}_5^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\alpha(u)\log t} \exp[-\frac{x^2}{2}]dx$$

for $t > 0, c > 0, \alpha > 0$,

$$\overset{+}{\tilde{\mathfrak{I}}}{}_6^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\alpha(u)\log|t|} \exp[-\frac{x^2}{2}]dx$$

for $t < 0, c > 0, \alpha > 0$,

$$\overset{+}{\tilde{\mathfrak{I}}}{}_6^{(\lambda)}(t,u) = 0 \text{ for } t \ge 0,$$

$$\overset{+}{\tilde{\mathfrak{I}}}{}_7^{(\lambda)}(t,u) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t} \exp[-\frac{x^2}{2}]dx$$

for $t \in (-\infty,\infty)$.

The results achieved in [6] for *Relation 3* may be generalized in the form of the following theorem.

*Theorem 3*
If the multistate homogeneous regular series–“$m_n$ out of $k_n$” system is such that

$$\lim_{n\to\infty} k_n = \infty,\ k_n - m_n = \bar{m}_n,\ \lim_{n\to\infty}\bar{m}_n = \bar{m} = \text{const},$$

then the only non-degenerate coordinates of limit safety function of the system are:

$$\mathfrak{I}_1^{(\bar{m})}(t,u) = \sum_{i=0}^{\bar{m}} \exp[-(-t)^{-\alpha(u)}] \frac{[(-t)^{-\alpha(u)}]^i}{i!}$$
for $t < 0,\ \alpha(u) > 0,$
$$\mathfrak{I}_1^{(\bar{m})}(t,u) = 0 \ \text{ for } t \geq 0,$$

$$\mathfrak{I}_2^{(\bar{m})}(t,u) = 1 \ \text{ for } t < 0,$$
$$\mathfrak{I}_2^{(\bar{m})}(t,u) = \sum_{i=0}^{\bar{m}} \exp[-t^{\alpha(u)}] \frac{(t^{\alpha(u)})^i}{i!}$$
for $t \geq 0,\ \alpha(u) > 0,$

$$\mathfrak{I}_3^{(\bar{m})}(t,u) = \sum_{i=0}^{\bar{m}} \exp[-\exp[t]] \frac{[\exp[t]]^i}{i!}$$
for $t \in (-\infty, \infty).$

The results achieved in [6] for *Relation 4* may be generalized in the form of the following theorem.

*Theorem 4*
If the homogeneous regular series -“$m_n$ out of $k_n$” system is such that

$$\lim_{n\to\infty} k_n = k,\ \lim_{n\to\infty} m_n = m,\ \lim_{n\to\infty} l_n = \infty,$$

then the only non-degenerate coordinates of limit safety function of the system are:

for $\alpha(u) > 0,\ t < 0$ $\mathfrak{I}_8^{(m)}(t,u) = 1 +$
$$-\sum_{i=0}^{m-1} \binom{k}{i} \exp[-i(-t)^{-\alpha(u)}][1 - \exp(-(-t)^{-\alpha(u)})]^{k-i}$$

$$\mathfrak{I}_8^{(m)}(t,u) = 0 \ \text{ for } t \geq 0,$$

$$\mathfrak{I}_9^{(m)}(t,u) = 1 \ \text{ for } t < 0$$

and for $t \geq 0, \alpha(u) > 0$ $\mathfrak{I}_9^{(m)}(t,u) =$
$$= 1 - \sum_{i=0}^{m-1} \binom{k}{i} [\exp(-t^{\alpha(u)})]^i [1 - \exp(-t^{\alpha(u)})]^{k-i},$$

and for $t \in (-\infty, \infty)$ $\mathfrak{I}_{10}^{(m)}(t,u) =$
$$= 1 - \sum_{i=0}^{m-1} \binom{k}{i} [\exp(-\exp(t))]^i [1 - \exp(-\exp(t))]^{k-i}$$

## 5. Conclusion

The article contains complete solutions of the problem of possible limit safety functions for multistate homogeneous series–“$m_n$ out of $k_n$” systems with any safety function of the system components. Described solutions have been obtained under the linear normalization of the system lifetime in the safety state subset $\{u, u+1, ..., z\}$. To apply presented theory in practical safety evaluation of real technical multistate series–“$m_n$ out of $k_n$” system we need to formulate some auxiliary theorems and choose the best case of relation between $m_n$, $k_n$ and the number $l_n$ of components in series subsystems of our system.

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