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Application of delta function in probabilistic modeling of communication delays in wireless networks – measurement identification and application of the delay model

Abstract

The paper is the second part of the publication [1] which presents the mathematical fundamentals of a new probabilistic model of communication delays in wireless networks. This model is based on a delta function sequence used to describe retransmissions between a transmitter and a receiver [2], [3] which occur when external disturbances influence a wireless transmission. In the paper, there are described a method of identification of the proposed model parameters on the basis of measurements results of delays and verification of the identified values. The model application for describing delays in multi-node wireless networks is presented and illustrated by examples.

Keywords: delta function, wireless networks, probabilistic model of delays, identification of model parameters.

1. Introduction

From measurement and functional point of view, the essential quality parameters of data transmission in the system are connected with delays which arise in a communication channel during sending a message from a transmitter to a receiver [1], [4], [5]. The delays in the channel composed of two elements communicating directly can be characterized in the way shown in Fig. 1.

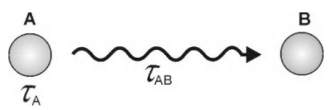

Fig. 1. Sources of delays during transmission from node A to B, τ_A – time necessary to obtain the access to a communication medium, τ_{AB} – time of the message transmission

Let us take that a transmission of a message from the node **A** to **B** is activated at the moment t_0 and this message is received by the node **B** at the moment t_B [1]. The difference between these two moments being the total time of the transmission can be interpreted as the total communication delay of the message transmitted from the node **A** to **B** and may be written as the sum of the partial delays:

$$\tau_{tot} = t_B - t_A + t_A - t_0 = \tau_{AB} + \tau_A \tag{1}$$

where τ_A is the delay associated with the activity inside the node **A** to obtain the access to the communication medium, while τ_{AB} describes the time necessary to transmit a message from **A** to **B**.

The communication delays can be described in a deterministic or probabilistic way. As it has been shown in [1], the deterministic ways are widely used to calculate maximum delays in networks, for example in task scheduling [6] and using the queuing theory [7]. But the probabilistic modeling allows obtaining a better representation of network properties in planning the network structure and efficiency [8], [9], [10], the analysis of energy consumption [6], decreasing delays in networks [11], [12] and the like [13], [14], [15]. Some of the communication errors are caused by external disturbances [16] which are of random character, therefore delays connected with them should be described in the probabilistic way. What is more, delays in measurement systems should be described in a probabilistic way because they can cause specific measurement errors for signals varying in time and other errors arising during the transmission of measured data [1], [17]. All measurement errors should be described in probabilistic categories because they are composed with other errors in the process of uncertainty calculation of measurement results [18].

The method described in the paper [1] represents a probabilistic approach to modeling properties of wireless networks. Its novelty consists in treating the Dirac's delta sequence as a probability density function describing delays which occur when the wireless transmission is disturbed by external factors [2], [19], [16] and it is necessary to retransmit data.

Accordingly with the considerations presented in [1], the access time of the transmitter to the medium τ_A, identified by using the measurement system presented in Section 2, has the normal distribution $g_A(\tau_A)$ which can be written as the convolution of the pattern $g_{Apat}(.)$ and the delta function $\delta(\tau - t_{Aex})$, where t_{Aex} is the expected value of $g_A(\tau_A)$, i.e.: $t_{Aex} = E[g_A(\tau_A)]$. The time τ_{AB}, necessary to send data in the case when retransmissions are needed, can be described by the sequence:

$$g_{AB}(\tau_{AB}) = a_0\delta(\tau_{AB} - b_0) + a_1\delta(\tau_{AB} - b_1) + \ldots + a_k\delta(\tau_{AB} - b_k) \tag{2}$$

where $\delta(.)$ denotes the delta function defined by (3), k is the number of retransmissions (in practice less than 6 [20]). The coefficients $a_0, a_1, \ldots, a_k$ describe the probability of succeeding retransmission occurrence and it is:

$$a_0 + a_1 + \ldots + a_k = 1. \tag{3}$$

The coefficients a_i, $i = 0, 1, \ldots, k$ are non-negative, which means that $0 \leq a_i \leq 1$ and if $i = 0$ then $a_0 = 1$. The coefficients $b_0, b_1, \ldots, b_k$ represent the moments of retransmissions.

Under the assumption that the partial delays in Eq. (1) are uncorrelated random variables described by the probability density functions $g_A(\tau_A)$ and $g_{AB}(\tau_{AB})$, the total delay can be written as the convolution:

$$g_{tot}(\tau_{tot}) = g_A(\tau_A) \otimes g_{AB}(\tau_{AB}). \tag{4}$$

Basing on Eq. (4), one can obtain the probabilistic model of the total delay for the communication chain from Fig. (1) in the case when the messages are retransmitted. The model has the form [1]:

$$g_{tot}(\tau_{tot}) = a_0 g_{Apat}(\tau_{tot} - \tau_0) + \ldots + a_k g_{Apat}(\tau_{tot} - \tau_k) \tag{5}$$

where $\tau_0, \tau_1, \ldots, \tau_k$ denote the delays for which the probability density function (5) takes the local maxima (see Figs. 3 and 4). In the simplest form, the model can be written as:

$$\mathbf{Delay}_{AB}\{g_{Apat}(.), (a_0, \tau_0), (a_1, \tau_1), \ldots, (a_k, \tau_k)\} \tag{6}$$

2. Measurements of delays

The main practical application of the proposed mathematical apparatus is the description of communication delays by the model, parameters of which depend on the properties of factors disturbing the transmission medium. To identify the model, it is necessary to measure delays in selected conditions. The general scheme of the system used for measurements of communication delays in wireless networks is shown in Fig.2.

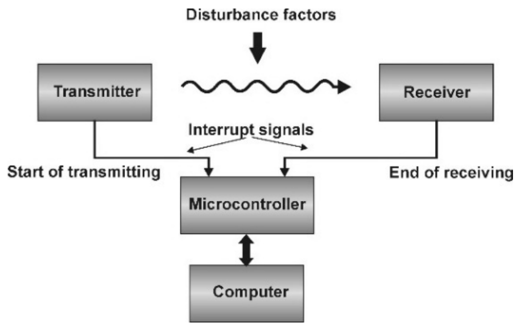

Fig. 2. Scheme of the system for measurements of transmission delays

The investigated communication channel consists of two elements: a transmitter and a receiver working in ZigBee standard [21]. Transmission of data having a constant length is disturbed by different kinds of factors such as walls, electromagnetic or electrostatic fields and other wireless transmissions. The delay, defined as the time between the start of sending the data in the transmitter and the moment when the data are completed in the receiver, is measured by a microcontroller. The measured results are sent to a computer where they are processed to the form of a histogram.

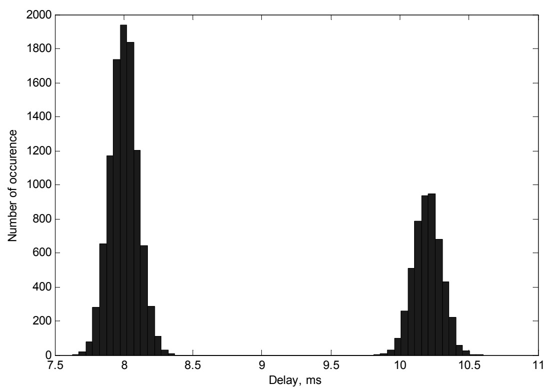

Fig. 3. Exemplary histogram of the total delay with one retransmission obtained from the measurement system

The exemplary histogram shown in Fig. 3 was obtained from the described system in the situation when two nodes communicated witch each other through an obstacle (in that case a concrete wall). The signal transmitted between the ZigBee wireless modules was absorbed by the wall built from reinforced concrete 35 cm thick which caused that a part of communicates had to be retransmitted. The graphical representation of 10 000 delay measurement results in the form of the histogram shown in Fig. 3 consists of 2 modes describing the probability of appearance of the first and the second transmission, respectively.

3. Identification of the delay model

Having measured a set of the delay values obtained for the selected couple of wireless modules in the investigated situation, i.e. when the transmission is disturbed by the chosen factor, one can perform identification of the model parameters according to expression (6). To start with, one has to dispose the pattern $g_{Apat}(.)$ determined for this couple of modules in the way described in [1]. Having it, one can realize the identification procedure in the following steps:

- At first, it is necessary to present both the pattern and the set of delays as two different histograms with the same width of classes.
- Next, the comparison of the first mode of the total delay with the pattern is performed in order to determine the coefficient a_0 and the time displacement τ_0. Both a_0 and τ_0 are obtained in the same process which consists in multiplying the pattern by coefficient a_0 and shifting it along the axis τ_0 as long as the difference between the classes of the first mode and the calibrated pattern comes up to a minimum.
- The procedure described above should be repeated for all the other modes of the histogram.

After realizing the whole procedure, one obtains the coefficients $a_0, a_1, \ldots, a_k$ and $\tau_0, \tau_1, \ldots, \tau_k$ determined for the probability density function $g_{Apat}(.)$, which means that the identification of the total delay model in the sense of expression (6) has been performed. To calculate the inaccuracy of the identified model, it is necessary to determine the total delay histogram on the basis of the obtained parameters using the Monte Carlo method and compare this histogram (let us call it as the template) with the one determined in the measurement way. The mean-squared error of the identification can be calculated as:

$$e_{id} = \frac{1}{N}\sqrt{\sum_{n=1}^{N}\left[y_{meas}(n) - y_{sim}(n)\right]^2} \qquad (7)$$

where $y_{meas}(n)$ is the height of the n-th bar of the total delay histogram obtained in the measurement way, $y_{sim}(n)$ is the height of the n-th bar of the histogram calculated by using the Monte Carlo method, N is the number of all bars.

Example 3. Let us assume that the delay model has been identified in the described way on the basis of measurements performed for ZigBee networks and presented in the form of the histogram from Fig. 3. The obtained parameters of the model can be written in the form (6) as:

$$\mathbf{Delay}_{AB}\left\{N(0,1)\ \mathrm{ms},\ (0.75,\ 8.2\ \mathrm{ms}),(0.25,\ 10.4\ \mathrm{ms})\right\}. \qquad (8)$$

Comparing the histogram with the template composed of the shifted patterns, calculated by using the Monte Carlo method for these parameters, one obtains the error values shown in Fig.4.

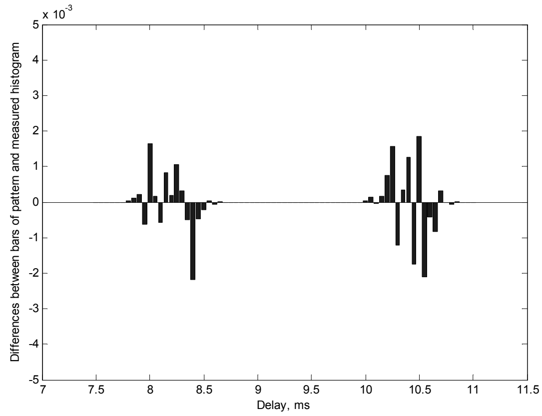

Fig. 4. Differences between the bars of the template and the measured histogram vs. the delay

The mean-squared error calculated on the basis of Eq. (7) $e_{id} = 0.15\mathrm{E}{-3}$. Therefore, one can state that the delay model (6) of the communication channel has been identified in the proposed way with the inaccuracy less than 0.02E-3.

4. Using the model for description of delays in wireless networks

The model described in Section 1 can be used to determine delays in chains of a wireless network composed of more than two nodes, as it is shown in Fig. 5. The communication chain from this figure consists of the nodes **A**,…, **N,** which means that the data are transmitted from **A** to **N** indirectly through nodes: **B**, **C** etc.

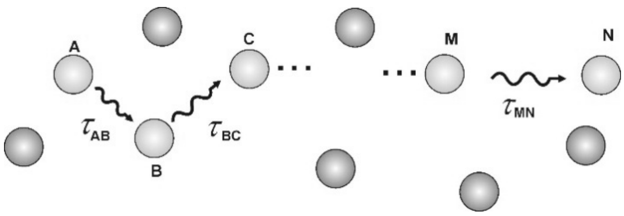

Fig. 5. An exemplary communication chain selected in a wireless network

For the chain selected in Fig. 5, the total delay of the transmission between the nodes A and N can be written as:

$$\tau_{totAN} = \tau_{totAB} + \tau_{totBC} + \ldots + \tau_{totMN} \quad (9)$$

If the partial delays $\tau_{totAB}, \ldots, \tau_{totMN}$ are uncorrelated random variables, the probability density function of the total delay may be determined as the convolution:

$$g_{totAN}(\tau_{totAN}) = g_{totAB}(\tau_{totAB}) \otimes g_{totBC}(\tau_{totBC}) \otimes \ldots \otimes \otimes g_{totMN}(\tau_{totMN}) \quad (10)$$

where $g_{totAB}(\tau_{totAB}), \ldots, g_{totMN}(\tau_{totMN})$ are the probability density functions of the delays arising when the couples of nodes **AB**, …, **MN** communicate directly.

The properties of the proposed model applied to the description of delays in the transmission chain on the basis of Eqs. (9) and (10) are presented on the following example.

Example 1. Let us assume that the communication chain consists of 3 nodes **A**, **B** and **C** as it is shown in Fig. 6. Moreover, all modules used in the chain are of the same construction and the couple **AB** is affected by disturbances, while **BC** is not. The model of the communication delay for the couple **AB** is given by:

$$\mathbf{Delay}_{AB}\{N(0,1)\text{ ms}, (0.75, 13\text{ms}), (0.25, 20\text{ms})\} \quad (11)$$

which means that 25% of messages are retransmitted. The model of transmission for the couple **BC** has the form:

$$\mathbf{Delay}_{BC}\{N(0,1)\text{ ms}, (1, 13\text{ms})\} \quad (12)$$

and describes the communication without any retransmission.

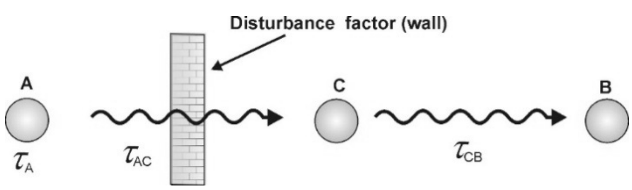

Fig. 6. Scheme of the communication chain consisting of 3 nodes **A**, **B** and **C**

In the considered case, the total delay is the sum:

$$\tau_{totAC} = \tau_{totAB} + \tau_{totBC}, \quad (13)$$

therefore, the probability density function of the total delay can be given by the expression:

$$g_{totAC}(\tau_{totAC}) = g_{totAB}(\tau_{totAB}) \otimes g_{totBC}(\tau_{totBC}). \quad (14)$$

Accordingly with (6), (11) and (12), the model of the delay between the nodes **A** and **B** can be written as the sequence:

$$g_{totAB}(\tau_{totAB}) = a_{0AB}\, g_{Apat}(\tau_{totAB} - \tau_{0AB}) + + a_{1AB}\, g_{Apat}(\tau_{totAB} - \tau_{1AB}) \quad (15)$$

where $g_{Apat}(.)$ is the pattern of the normal distribution $N(0, 1)$ ms, $a_{0AB} = 0.75$, $\tau_{0AB} = 13$ ms, $a_{1AB} = 0.25$, $\tau_{1AB} = 20$ ms. The sequence being the model for the nodes **B** and **C** has the form:

$$g_{totBC}(\tau_{totBC}) = a_{0BC}\, g_{Bpat}(\tau_{totBC} - \tau_{0BC}) \quad (16)$$

where $g_{Bpat}(.)$ is the pattern of the normal distribution $N(0, 1)$ ms, $a_{0BC} = 1$, $\tau_{0BC} = 20$ ms. Making the convolution (14) for the partial models (15) and (16), one obtains the final form of the model of the total delay in the chain from Fig. 6 as:

$$g_{totAC}(\tau_{totAB}) = a_{0AC}\, g_{ABpat}(\tau_{totAC} - \tau_{0AC}) + + a_{1AC}\, g_{ABpat}(\tau_{totAC} - \tau_{1AC}) \quad (17)$$

where the coefficients have the values:

$$a_{0AC} = a_{0AB} \cdot a_{0BC} = 0.75 \cdot 1 = 0.75,$$
$$a_{1AC} = a_{1AB} \cdot a_{0BC} = 0.25 \cdot 1 = 0.25$$
$$\tau_{0AC} = \tau_{0AB} + \tau_{0BC} = 13 + 13 = 26 \text{ ms},$$
$$\tau_{1AC} = \tau_{1AB} + \tau_{0BC} = 20 + 13 = 33 \text{ ms}.$$

Pattern $g_{ABpat}(.)$ is the result of the convolution of the patterns $g_{Apat}(.) = g_{Bpat}(.) = N(0, 1)$ ms, therefore, its standard deviation is: $\sigma_{AB} = \sqrt{\sigma_A^2 + \sigma_B^2} = \sqrt{2}\sigma_A = 1.4 \cdot 1 = 1.4$ ms. In this case, the delay model of the communication chain takes the form:

$$\mathbf{Delay}_{AC}\{N(0, 1.4)\text{ ms}, (0.75, 26\text{ms}), (0.25, 33\text{ms})\}. \quad (18)$$

The transformations presented in Example 1 are relatively sophisticated although the considered communication chain is quite simple. For multi-element chains, one may apply a much easier way of obtaining the total delay parameters basing on the delay histogram determined using the Monte Carlo method [1] as it is shown in Example 2.

Example 2. Let us take that the partial delays of a 3-element communication chain are described as:

$$\mathbf{Delay}_{AB}\{N(0,1)\text{ ms}, (0.7, 13\text{ms}), (0.3, 20\text{ms})\} \quad (19)$$

and

$$\mathbf{Delay}_{BC}\{N(0,1)\text{ ms}, (0.6, 13\text{ms}), (0.4, 20\text{ms})\}. \quad (20)$$

The histogram obtained by using the Monte Carlo method for 10^5 realizations of delays is shown in Fig. 7. The delay model parameters determined on the basis of this histogram have the following values:

$$\mathbf{Delay}_{AB}\{N(0,14)\text{ms}, (0.42, 26\text{ ms}), (0.46, 33\text{ms}), (0.12, 40\text{ms})\}. \quad (21)$$

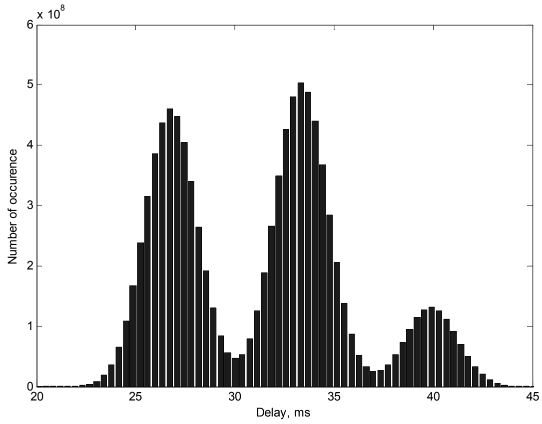

Fig. 7. Delay histogram obtained as the result of Experiment 2

5. Conclusions

The presented identification method of the delay model parameters consist in comparing two histograms: the first one obtained by using a measurement experiment while the second one based on calculations performed by the Monte Carlo Method. The identification inaccuracy is given by the mean-squared error [7].

The identified model of communication channels can be implemented in building models of delays in multi-node wireless networks. A fundamental issue in these networks is that their performance degrades sharply as the number of nodes increases. The performance challenges of multi-node networks have long been recognized and have led to a lot of research on the medium access control (MAC), routing and transport layers of the networking stack [23], [24]. The presented model can be useful in solving these problems.

The important feature of the basic model presented in this paper is the possibility to include the influence of disturbances on the communication delay. The results of investigations of such an influence for different kinds of external disturbances, passive and active, have been presented in [25], [26]. Such mathematical means allow obtaining relatively simple probabilistic descriptions of delays in the networks composed of many nodes. The delay model obtained in this way is useful in the simulative analysis of wireless networks especially in indoor conditions [25], [27].

The similarities between wired and wireless networks can lead to draw the conclusion that the presented mathematical apparatus can also be used to describe the communication delays in wired networks.

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