



DOI: 10.5604/01.3001.0053.9592

Numerical analysis of the thermal behaviour and performance of a brake system with temperature-dependent material properties

M.A. Hassan ^a, M. Ali ^b, O.I. Abdullah ^{c,d,e,*}

^a Mechanical Engineering Department, College of Engineering, University of Kerbala, Iraq

^b Prosthetics and Orthotics Engineering Department, College of Engineering, University of Kerbala, Iraq

^c Energy Engineering Department, College of Engineering, University of Baghdad, Iraq

^d Mechanical Engineering Department, College of Engineering, Gulf University, Sanad 26489, Bahrain

^e Institute of Laser and Systems Technologies, Hamburg University of Technology, Harburger Schloßstraße 28, 21079, Hamburg, Germany

* Corresponding e-mail address: oday.abdullah@tuhh.de

ORCID identifier:  <https://orcid.org/0000-0001-5450-021X> (O.I.A.)

ABSTRACT

Purpose: The brake system is the most significant component of a vehicle because it protects the driver, passengers, other road users, and property on both sides of the road. The basic principle of the disc brake system depends on the friction-based between the brake pads and rotor disc.

Design/methodology/approach: The paper introduced a developed 3D finite element thermal model of the brake system to simulate the heat generated by friction in the vehicle's disc brake.

Findings: The results presented the surface temperature at any instant of the disc brake under various initial velocities when the materials properties of the rotor disc and pad depend on temperature.

Research limitations/implications: The main aim of the present paper is to build a numerical model to simulate the braking process under various initial vehicle velocities and investigate the influence of the material properties when they function on temperature and constant.

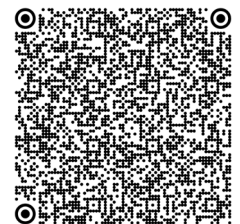
Originality/value: The maximum difference between the two cases (contact and depend on the temperature) was 17 K for the initial velocity of 144,120. Also, it was found out that the percentage differences of the surface temperature increasing with the rise in initial vehicle velocity were 323% and 392.5% when the initial velocity of the vehicle increased from 100 km/h to 144 km/h.

Keywords: Brake system, Thermal analysis, Finite element method, Frictional material, Heat generated

Reference to this paper should be given in the following way:

M.A. Hassan, M. Ali, O.I. Abdullah, Numerical analysis of the thermal behaviour and performance of a brake system with temperature-dependent material properties, Archives of Materials Science and Engineering 122/2 (2023) 58-69. DOI: <https://doi.org/10.5604/01.3001.0053.9592>

METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING



1. Introduction

The brake system plays a crucial role in a vehicle as it ensures the safety of the driver, passengers, and other road users and protects surrounding properties. Based on the principle of friction, the disc brake system efficiently transforms the vehicle's kinetic energy into thermal energy, allowing for rapid deceleration or halting. The effectiveness of a frictional brake system is proportional to the magnitude of the shear force that opposes the disc's rotation. The large heat frictional power rate generated in the brake system leads to a high surface temperature during braking. Consequently, various thermal and mechanical effects types, such as crazing, heat spots, butterfly disc failure, and corrugated rotor disc failure, are predicted to happen in the contact zone [1]. One or more of those effects may lead to premature failure of the brake system. The brake parts absorbed the heat generated according to conduction heat transfer in brake's elements and dissipated to the surrounding by convection and radiation [2]. Therefore, predicting the amounts of heat generated and dissipating heat depends on studying and analysing the thermal transient behaviour of the brake system. Mohammed et al. [3] studied the transient thermal behaviour of mechanical frictional brake systems based on numerical and experimental approaches. Their results illustrated that the optimal friction material is the material that has good thermal properties because it leads to a lower final surface contact temperature. Hasan [4] investigated the effects of the different initial speeds of the vehicle and contact pressure on transient thermal-structural behaviour. The experiments and numerical results showed that the surface temperature was maximum when applied the highest initial speed and pressure. Also, the transient thermal behaviour showed that the surface temperature increased to its peak and then steadily decreased until it reached the lowest point at the end of the braking time. Bena and Sirata [5] analysed the effect of the thickness of the disc on the transient thermal and stress behaviours of the brake disc. The numerical results showed that the maximum surface temperature appeared on the brake contact zone, and the minimum surface temperature appeared on the opposite disc surface. Stojanović et al. [6] discussed the effect of different contact pressures on the magnitude and distribution of the heat generated between the brake disc and pad during the braking process. The result showed that the maximum surface contact temperature was recorded when applying the highest contact pressure. Babu and Solomon [7] investigated the effects of ambient temperature on the transient thermal behaviour of the brake disc system. They found that the maximum surface temperature occurred at each initial period and started slightly decreases along the disc thickness

away from the contact surface. Synák et al. [8] investigated the influence of the repeated braking process on the surface temperature of a brake system component. The result showed that the maximum surface temperature increased dramatically when the number of repeated brakes increased.

Most research studied the thermomechanical transit behaviour of the brake disc, assuming constant materials properties of the disc and pad. The paper presents a comprehensive analysis of the effect of changing the material's properties with surface temperature on the transient thermal behaviour of the disc brake. Furthermore, a comparison was made between the thermal results when assuming constant and variable material properties with temperature during the braking period. The finite element method (COMSOL Multiphysics® 6 software) was used to simulate the braking process and compute the thermal and thermal energy during the braking process.

2. Mathematical modelling

The vehicle moves at initial velocity; when it is necessary to stop the car or slow down. The calliper piston is pressed by brake pad agents, the rotation disc. Thus, the vehicle speed will decrease due to the applied pressure between the brake rotor disc and pad. Therefore, the kinetic energy is converted to heat energy. The total heat generated by the brake system can be expressed as follows [9].

$$Q = 0.5 M (V_1^2 - V_2^2) \quad (1)$$

Q is the total kinetic energy (J), M is the mass of the vehicle, and V1 and V2 are the initial and final velocities of the vehicle (m/s). The governing equations of brake disc and pad temperature distribution under transient thermal state are written in cylindrical coordinates. The finite element method by using COMSOL Multiphysics 6 software solves the temperature field into three dimensions (r,θ,z) using the transit brake rotor disc and pad equation [3];

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha_d} \left(\frac{\partial T}{\partial t} + \omega(t) \frac{\partial T}{\partial \theta} \right) \quad (2)$$

For: $r_d \leq r \leq R_d$, $0 \leq \theta \leq 2\pi$, $0 \leq z \leq \delta_d$, $0 \leq t \leq t_b$.

Where α is thermal-diffusivity, θ is a pad cover angle, and t_b is braking time (second). The brake rotor disc boundary conditions shown in Figure 1 can be written as follows:

$$K_d \left. \frac{\partial T}{\partial t} \right|_{r=R_d} = h [T_\infty - T_{(\theta,z,t)}] \quad (3)$$

$$K_d \left. \frac{\partial T}{\partial r} \right|_{r=r_d} = h [T_{(\theta,z,t)} - T_\infty] \quad (4)$$

For: $0 \leq \theta \leq 2\pi$, $0 \leq z \leq \delta_d$, $0 \leq t \leq t_b$

$$K_d \frac{\partial T}{\partial r} \Big|_{r=\delta_d} = h [T_\infty - T_{(\theta,z,t)}] \quad (5)$$

$$K_d \frac{\partial T}{\partial z} \Big|_{z=0} = h [T_{(\theta,z,t)} - T_\infty] \quad (6)$$

For: $r_d \leq r \leq R_d, 0 \leq \theta \leq 2\pi, 0 \leq t \leq t_b$

While the brake pad boundary conditions can be written as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (7)$$

For: $r_p \leq r \leq R_p, |\theta| < \theta_0, 0 \leq z \leq \delta_p, 0 \leq t \leq t_b$

$$K_p \frac{\partial T}{\partial r} \Big|_{r=R_p} = h [T_\infty - T_{(\theta,z,t)}] \quad (8)$$

$$K_p \frac{\partial T}{\partial r} \Big|_{r=r_p} = h [T_{(\theta,z,t)} - T_\infty] \quad (9)$$

For: $|\theta| < 0.5\theta, 0 \leq z \leq \delta_p, 0 \leq t \leq t_b$

$$K_p \frac{\partial T}{\partial r} \Big|_{\theta=\theta_0} = h [T_\infty - T_{(r,0.5\theta,z,t)}] \quad (10)$$

$$K_p \frac{\partial T}{\partial r} \Big|_{\theta=\theta_0} = h [T_{(r,0.5\theta,z,t)} - T_\infty] \quad (11)$$

For: $r_p \leq r \leq R_p, 0 \leq z \leq \delta_p, 0 \leq t \leq t_b$

$$K_p \frac{\partial T}{\partial z} \Big|_{z=\delta_p} = h [T_\infty - T_{(r,\theta,t)}] \quad (12)$$

For $(r,\theta) \in A_s, 0 \leq t \leq t_b$

where:

$$\omega(t) = V(t) / R_d \quad (13)$$

$$\alpha = K / \rho c \quad (14)$$

R_p and r_p : are the outer and inner stationary pad diameters, ω is the angular velocity of brake rotor disc speed (rad/s). R_d and r_d are the outer and inner rotor disc diameters. The car slows down at a constant deceleration [10] as:

$$V(t) = V_0 \left(1 - \frac{t}{t_b}\right) \quad 0 < t < t_b \quad (15)$$

where: t is the time (s), and t_b is the braking time. The heat generated that the rotor disc and pads absorb can be calculated based on the partition ratio [3,4]:

$$\gamma = \frac{\sqrt{\rho_p c_p K_p}}{\sqrt{\rho_d c_d K_d} + \sqrt{\rho_p c_p K_p}} \quad (16)$$

The expression p and d represents the disc and the pad, respectively. K is the thermal conductivity (W/m.K), C is the specific heat (J/kg.k) at constant pressure, and ρ is the mass density (kg/m³). The heat generated enters into the brake rotor disc and pad according to the partition ratio with an assumption ($T_d = T_p$) are [5,11,12];

$$q_d(r,t)_{z=\delta_p} = (1 - \gamma)\mu\rho_0\omega(t)r \quad (17)$$

For: $r_p \leq r \leq R_p, 0 \leq \theta \leq 2\pi, 0 \leq t \leq t_b$

$$q_p(r,t)_{z=\delta_p} = \gamma\mu\rho_0\omega(t)r \quad (18)$$

For: $r_p \leq r \leq R_p, |\theta| \leq 0.5\theta_0, 0 \leq t \leq t_b$

So, the total heat generated is:

$$q_t = q_d + q_p \quad (19)$$

The amount of heat that enters into the pad is,

$$q_p = \gamma q_t \quad (20)$$

The amount of heat that enters into the rotor disc is,

$$q_d = (1 - \gamma) q_t \quad (21)$$

The amount of heat generated between the brake rotor disc and pad is dissipated to the surrounding by conduction and convection heat transfer is [13],

$$Q_{convection} = hA(T_s - T_a) \quad (22)$$

where:

$$h = 3.691 (v_0/r_{tire})^{0.8} \quad (23)$$

The contact pressure p solves the thermoelastic contact problem (r,t) [14]:

$$contact\ pressure = Q_b / \mu v \quad (24)$$

where: Q_b is the brake frictional power for each brake (watt).

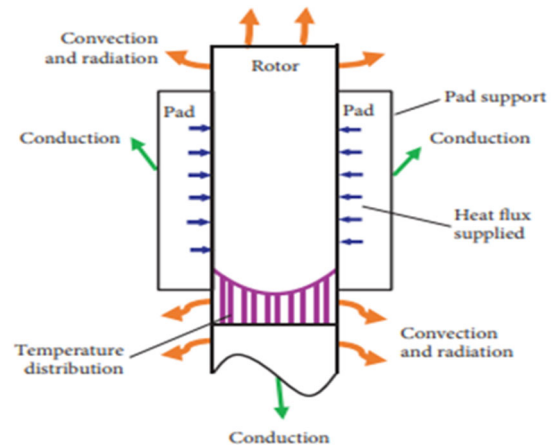


Fig. 1. Brake model boundary condition

3. Finite element simulation

The first step of pre-processing the simulation process is to build the 3-dimensional brake system model using

COMSOL Multiphysics®6 software. The disc brake model has two symmetrical stationary pads with solid rotor discs, as shown in Figure 2. The element mesh type compares tetrahedra, hexahedra, and triangles. For the optimal finite element model, the number of elements was 48710. The final FE model mesh can be seen in Figure 3. The 3-D temperature fields $T_d(r;\theta,z)$ and $T_p(r;\theta,z)$ will be calculated for two assumptions:

1. The thermal properties of the brake disc and pad material are constant and not affected by temperature.
2. The disc and pad thermal material properties are a function of the temperature.

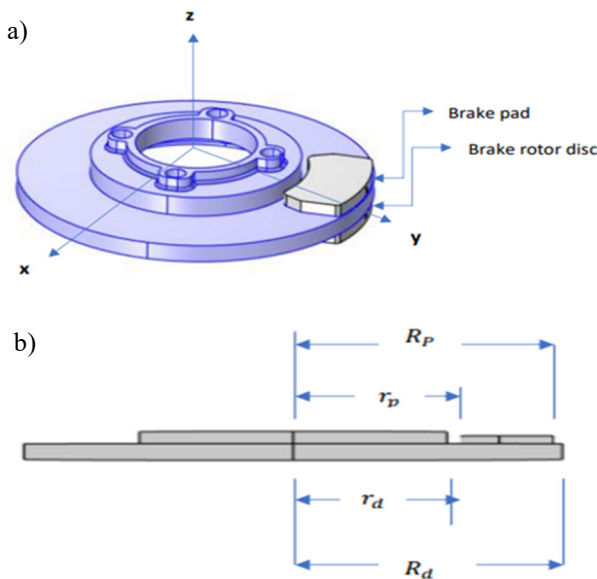


Fig. 2. a) Disc-brake system, b) section of disc-brake

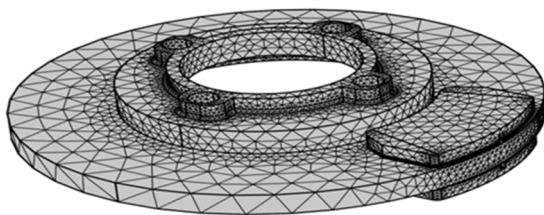


Fig. 3. Finite element model of brake system

The details of the dimensions of the brake model are illustrated in Table 1. The selected initial and final velocities applied during the braking process are (1) 144 to 72 km/h, (2) 120 to 60 km/h, and (3) 100 to 50 km/h. The required

braking time is 4 sec. The driver presses down hard on the brake pedal for 4 seconds to slow down the car of 1200 kg to the final speed. The linear deceleration for the braking interval calculates as the first derivative of vehicle speed. Table 2 shows the brake model operational conditions.

Table 1. Brake system model dimensions

Dimensions, m	Disc	Pad
Outer diameter	0.22	0.21
Inner diameter	0.12	0.14
Thickness	0.013	0.01
Holes number	4	-----
Holes diameter	0.013	-----
Slot length	-----	0.006

Table 2. Model operational condition

Parameters	Value
Initial vehicle speed, km/h	100, 120 and 144
Wheel radius, m	0.276
Vehicle mass, kg	1200
Start braking time, s	0
End braking time, s	4
Initial temperature, K	300

The brake disc material is HT250, and the pad material is resin matrix composite. Tables 3 and 4 illustrate the material properties of the brake disc and pad. The assumptions of Finite Element thermal simulation of the brake system that is considered in the study are:

- the brake model materials are homogeneous and isotropic,
- the heat dissipation is due to conduction and convection heat transfer,
- the visible pad lining area is the contact area,
- neglecting wear of pads and disc,
- the applied pressure is uniform on the contact surface,
- neglecting thermal contact resistance, i.e., perfect contact $T_{disc}=T_{pad}$.

Table 3. The material properties of the brake rotor disc

Temperature, °C	Specific heat, J/Kg.k	Heat conductivity, W/m.k	Thermal expansion, $10^{-6}/k$
0	503	42.4	4.39
20	530	43.1	11.65
100	563	44.2	12.84
200	611	43.6	13.58

Table 4.
The material properties of the brake pad

Temperature, °C	Specific heat, J/Kg.k	Heat conductivity, W/m.k	Thermal expansion, $10^{-6}/k$
20	1200	0.9	10
100	1250	1.1	18
200	1295	1.2	30
300	1320	1.15	32

The main steps achieved by the developed Finite Element Model of the brake system can be summarised as shown in Figure 2. The density and Poisson's ratio of the brake disc are 7280 kg/m^3 and 0.3. The density and Poisson's ratio of the brake pad are 1550 kg/m^3 and 0.25 [15]. Table 5 shows the variation of the friction coefficient with the temperature.

Table 5.
The variation of the friction coefficient with the temperature change

Temperature, °C	The friction coefficient
20	0.37
100	0.38
200	0.41
300	0.39

The developed numerical model was verified by comparing the maximum temperature results with reference [2], where the percentage difference between them was at most 0.4%, as shown in Table 6. The results proved the high

accuracy of the developed model and the high agreement with the results of Ref. [2].

Table 6.
The validation of the developed FE model to find the maximum surface temperature

Maximum temperature of the Brake system at $t=1.2 \text{ s}$	
Ref. [2]	432°C
Result of the present Developed FE model	430.15°C
Difference %	0.42

4. Results and discussions

The main objective of the present analysis is to build a numerical model to simulate the braking process under various initial vehicle velocities and investigate the influence of the material properties when they function on temperature and constant. Also, the distribution of surface temperatures along pads and rotor disc thickness was presented. Figure 4 shows the section along two pads and rotor disc thickness.

In the numerical analysis, it was simulated six different cases. Those cases can be classified into two categories based on the assumption of material properties of the rotor disc and pads: materials properties are dependent and independent of the temperatures. The descriptions of the six cases can be seen in Table 7.

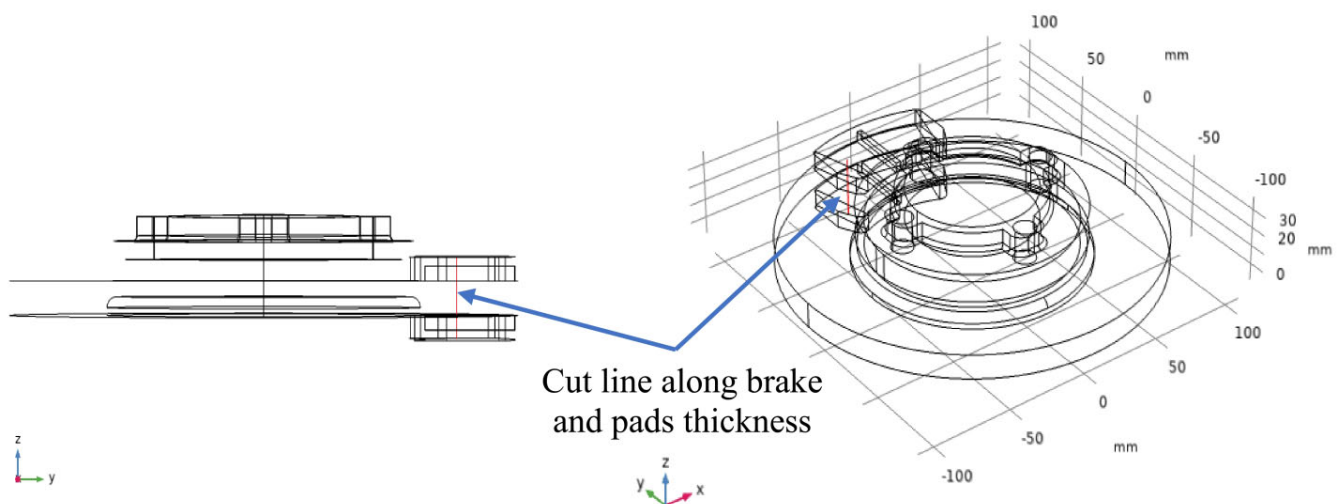


Fig. 4. The UT section of the brake pads and rotor disc thickness

Table 7. The details of the cases used in the numerical analysis

Case No.	Range of Deceleration	Assumption of material properties
1	144 km/h to 72 km/h	Constant with temp.
2	144 km/h to 72 km/h	Depend on temp.
3	120 km/h to 60 km/h	Constant with temp.
4	120 km/h to 60 km/h	Depend on temp.
5	100 km/h to 50 km/h	Constant with temp.
6	100 km/h to 50 km/h	Depend on temp.

Figure 5 shows the variation of the surface temperature of the six cases when the material properties of the brake rotor disc and pad are independent and dependent on temperature. It can be seen that the surface temperature increased until it reached the peak values after the half braking time (2 sec). Then it gradually decreases to the lowest value at the end braking time of 4 s. The maximum surface temperatures occurred when the highest initial velocity was applied, while the minimum surface temperature occurred when the lowest initial velocity was impacted. For the first category, when the materials properties of the brake rotor disc and pad are

independent of the temperature, the maximum surface temperatures for cases (1, 3, and 5) are found to be 615.78 K, 531.88 K, and 468.8 K, respectively. For the second category, when the materials properties of the brake rotor disc and pad are dependent on the temperature, the maximum surface temperatures for cases (2, 4, and 6) are found to be 598.8 K, 521.34 K, and 463.54 K. It can be noticed that the surface temperatures of the first category cases (1, 3, and 5) were higher than surface temperatures of second category cases (2, 4, and 6). Table 6 shows the maximum surface contact temperature of six cases. The maximum difference between the surface temperatures of case 1 and case 2 was approximately 17 K. The maximum difference between the surface temperatures of case 3 and case 4 and case 5 and case 6 are 10.5 K and 5.26 K, respectively. Figures 6-11 illustrate the maximum surface contact temperature of all cases (1-6) when the maximum temperature occurred during the braking process. Figures 12-17 show the variation of temperatures along the brake pads and rotor disc thickness. It can be seen very clearly that the high thermal gradients in the contact zone between two pads and rotor disc and the surface temperatures increased towards the outer disc diameter. At the same time, it decreased away from the sliding surface.

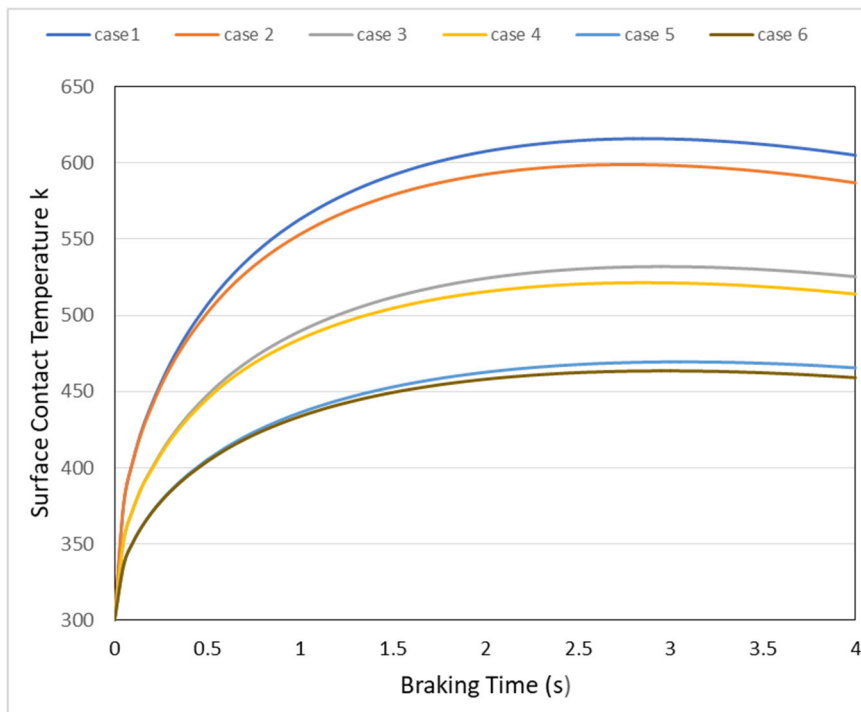


Fig. 5. Surface temperature at applying different initial velocities assuming dependent and independent materials properties on temperature

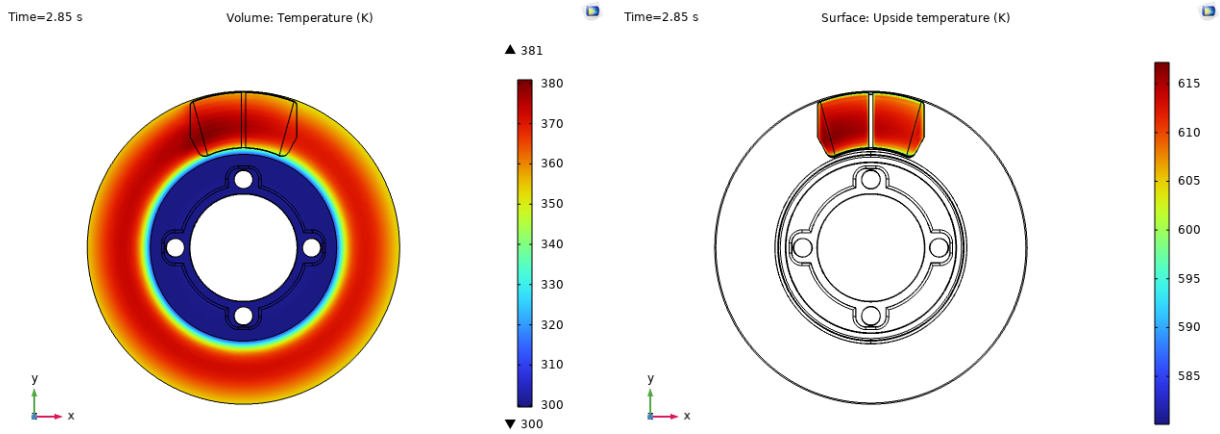


Fig. 6. The distribution of surface temperature at T_{max} for Case 1

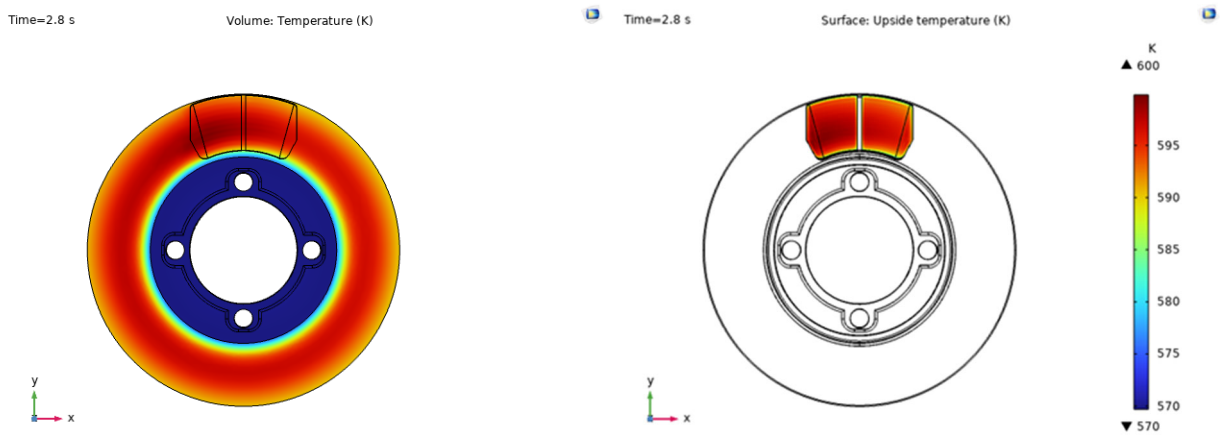


Fig. 7. The distribution of surface temperature at T_{max} for Case 2

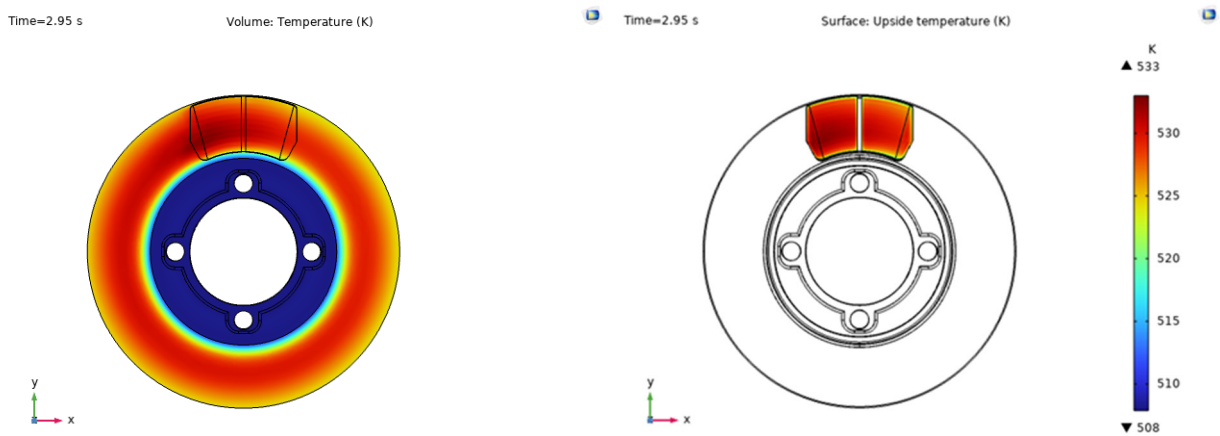


Fig. 8. The distribution of surface temperature at T_{max} for Case 3

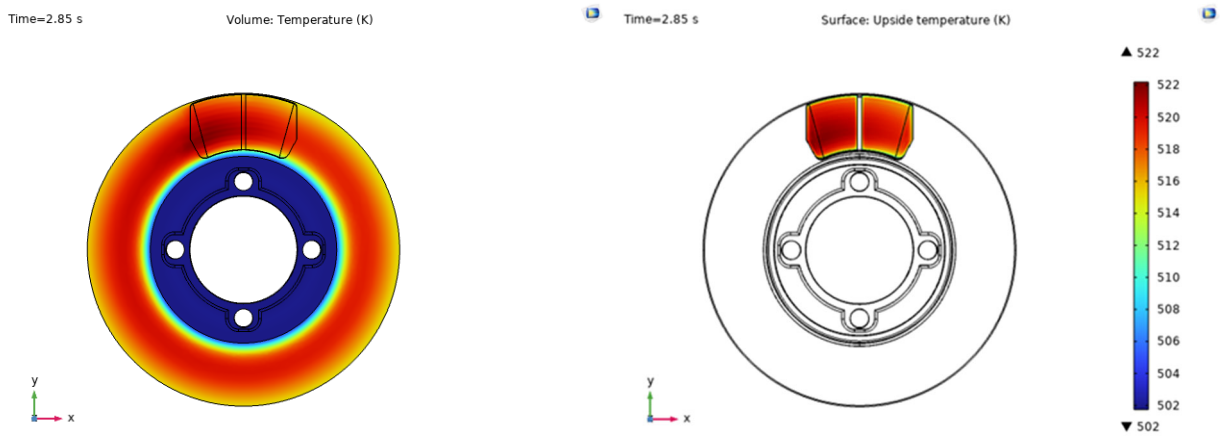


Fig. 9. The distribution of surface temperature at T_{max} for Case 4

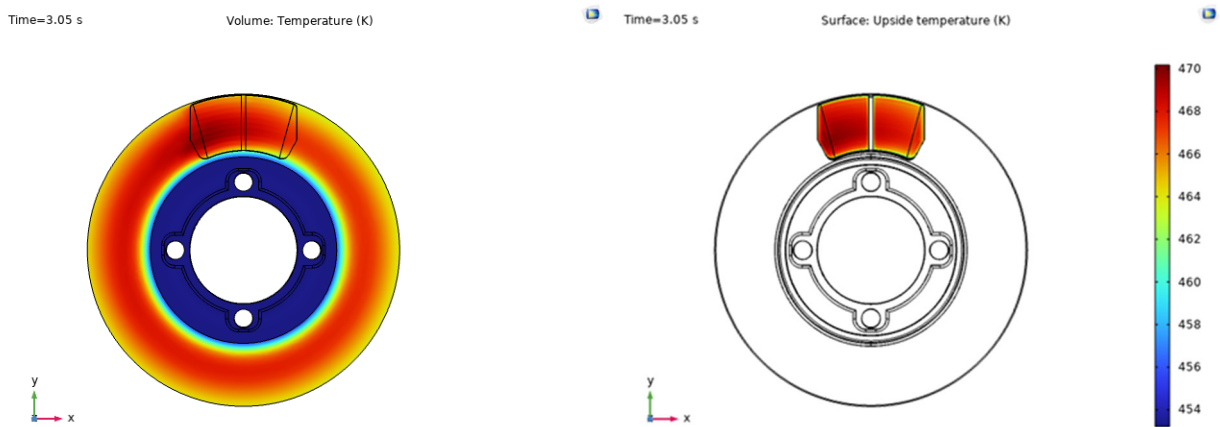


Fig. 10. The distribution of surface temperature at T_{max} for Case 5

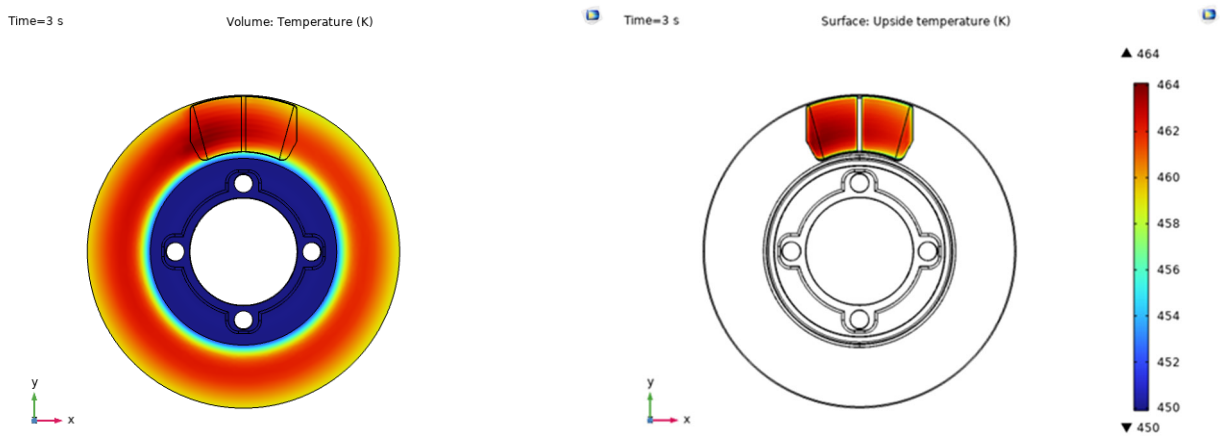


Fig. 11. The distribution of surface temperature at T_{max} for Case 6

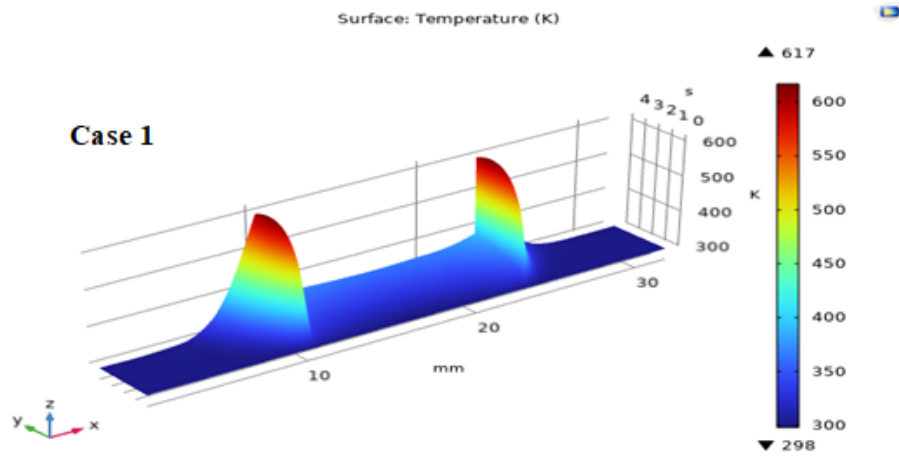


Fig. 12. Temperature variation at T_{max} along brake pads and disc thicknesses (case 1)

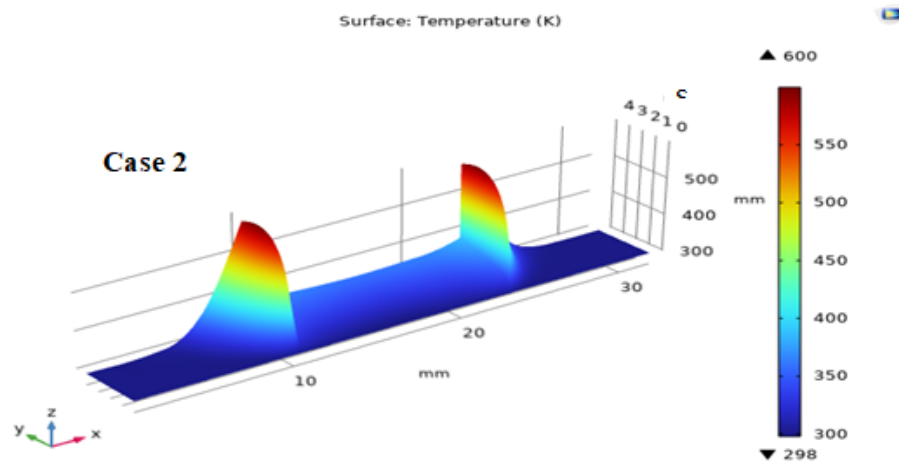


Fig. 13. Temperature variation at T_{max} along brake pads and disc thicknesses (case 2)

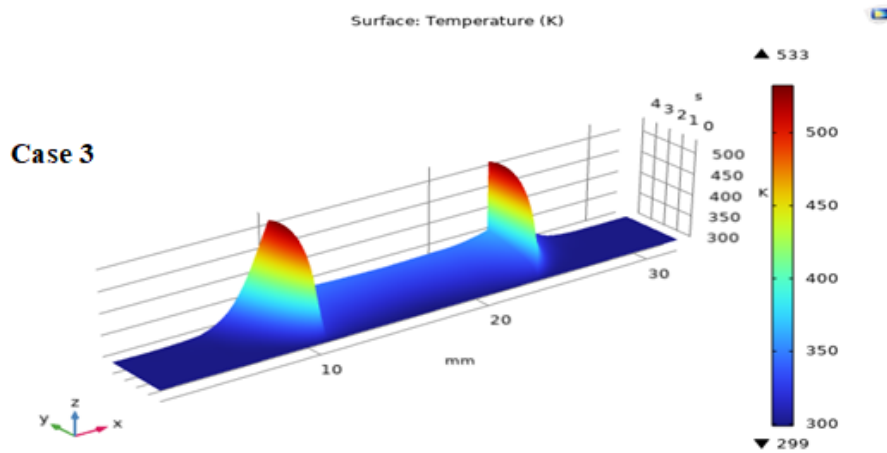


Fig. 14. Temperature variation at T_{max} along brake pads and disc thicknesses (case 3)

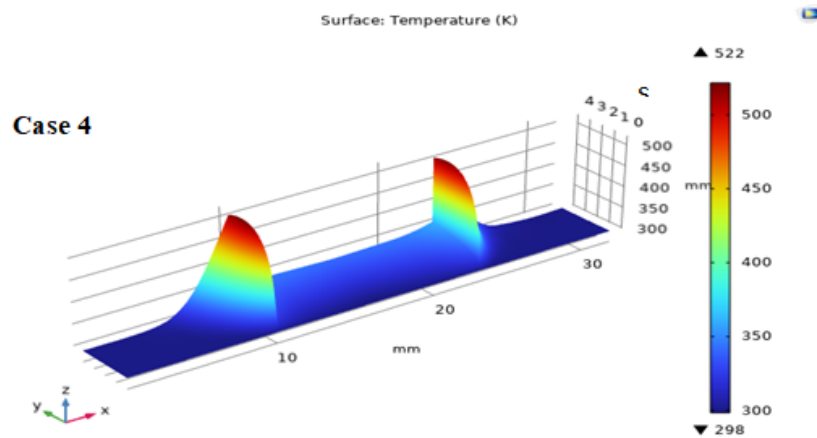


Fig. 15. Temperature variation at T_{max} along brake pads and disc thicknesses (case 4)

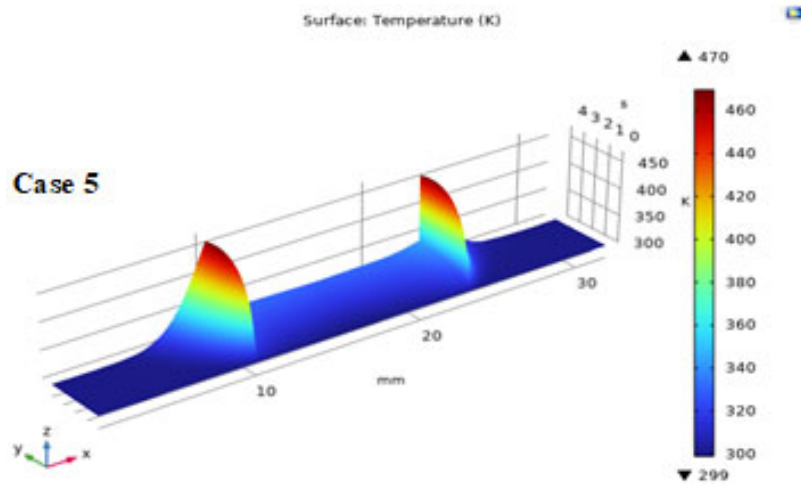


Fig. 16. Temperature variation at T_{max} along brake pads and disc thicknesses (case 5)

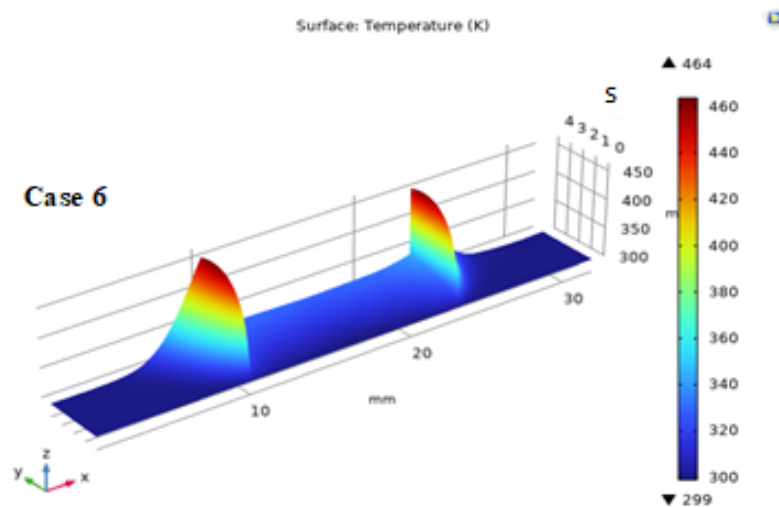


Fig. 17. Temperature variation at T_{max} along brake pads and disc thicknesses (case 6)

5. Conclusions and remarks

In the present research paper, a numerical analysis was developed to investigate the effect of material properties of brake rotor discs and pads that are dependent on temperature on the thermal behaviour and performance of the brake system during the braking process. Based on the transient thermal analysis of the brake system, it can be drawn the following conclusions:

1. The surface temperatures of the first category, when the materials properties of the brake rotor disc and pad are independent of the temperature (cases 1, 3, and 5), are higher than those of the second category (cases 2, 4, and 6). The reason for those results is the proportional relationship between thermal conductivity and temperature. So, when the surface temperature increases, the thermal conductivity increases too.
2. The percentage differences in the maximum surface contact temperature between case 1 and case 5, when increased the initial vehicle velocity from 100 to 144 km/h, was about 323%. While it was about 162% when the changing starting velocity was from 120 to 144 km/h (cases 1 and 3).
3. The surface temperatures along the radial disc direction increased towards the outer rotor disc diameter because the velocity of the point (node) rises according to the increased disc radius on the outer disc diameter.

In future works, the effect of the cracks [16] under the heavy duty working condition of the brake system can be investigated. So, excessive thermal stresses will be generated, and thermal cracks will be initiated and propagated under such conditions. Also, the wear problem of the rotor disc and pads can be studied.

References

- [1] S. Park, K. Lee, S. Kim, J. Kim, Brake-disc holes and slit shape design to improve heat dissipation performance and structural stability, *Applied Sciences* 12/3 (2022) 1171. DOI: <https://doi.org/10.3390/app12031171>
- [2] M. Bashir, A. Qayoum, S. Saleem, Analysis of frictional heating and thermal expansion in a disc brake using COMSOL, *Journal of Physics: Conference Series* 1240/1 (2019) 012094. DOI: <https://doi.org/10.1088/1742-6596/1240/1/012094>
- [3] A.Q. Mohammed, I.Y. Hussain, O.I. Abdullah, Effect of Frictional Material on Thermal Behavior of Brake System, *Tribology in Industry* 44/1 (2022) 64-72. DOI: <https://doi.org/10.24874/ti.1071.03.21.07>
- [4] M.H. Kareem, An Investigation into Thermal Performance of Dry Friction Brake, MSc Thesis, University of Baghdad, Baghdad, Iraq, 2018. DOI: <https://doi.org/10.13140/RG.2.2.30742.60482>
- [5] W. Bena, G. Sirata, Coupled Thermal Stress Analysis of Volvo Truck Disc Brake, *Advances in Materials Science and Engineering* 2022 (2022) 7945264. DOI: <https://doi.org/10.1155/2022/7945264>
- [6] N.R. Stojanović, J.D. Glišović, O.I. Abdullah, I.L. Grujić, S.Ž. Vasiljević, Pressure influence on heating of ventilating disc brakes for passenger cars, *Thermal Science* 24/1A (2020) 203-214. DOI: <https://doi.org/10.2298/TSCI190608314S>
- [7] P. Babu, D.G. Solomon, Simulation of Temperature Distribution in a Brake Pad Ceramic Composite Material, *Journal of The Institution of Engineers (India): Series D* (2023) 1-10. DOI: <https://doi.org/10.1007/s40033-022-00443-w>
- [8] F. Synák, V. Rievaj, M. Kučera, M. Šebök, T. Skrúcaný, Effect of repeated vehicle braking on the warming of selected parts of the vehicle, *Scientific Journal of Silesian University of Technology. Series Transport* 107 (2020) 183-196. DOI: <https://doi.org/10.20858/sjsutst.2020.107.14>
- [9] M. Maniana, A. Azime, F. Errchiqui, A. Tajmouati, Analytical and Numerical Analysis of Thermal Transfer in Disc Brake, *International Journal of Heat and Technology* 40/3 (2022) 693-698. DOI: <https://doi.org/10.18280/ijht.400305>
- [10] S. Sugunarani, V. Santhosh, Design and Thermal Analysis of Brake Rotor with Different Materials, *Journal of Manufacturing Engineering* 15/2 (2020) 044-049. DOI: <https://doi.org/10.37255/jme.v15i2pp44-49>
- [11] H. Dubale, V. Paramasivam, E. Gardie, E.T. Chekol, S.K. Selvaraj, Numerical investigation of thermo-mechanical properties for disc brake using light commercial vehicle, *Materials Today: Proceedings* 46/17 (2021) 7548-7555. DOI: <https://doi.org/10.1016/j.matpr.2021.01.437>
- [12] I.T. Jiregna, H.G. Lemu, Thermal stress analysis of disc brake using analytical and numerical methods, *IOP Conference Series: Materials Science and Engineering* 1201/1 (2021) 012033. DOI: <https://doi.org/10.1088/1757-899X/1201/1/012033>
- [13] O. Hrevtsev, N. Selivanova, P. Popovych, L. Poberezhny, V. Sakhno, O. Shevchuk, L. Poberezhna, I. Murovanyi, A. Hrytsanchuk, O. Romanyshyn, Simulation of thermomechanical processes in disc brakes of wheeled vehicles, *Journal of Achievements in Materials and Manufacturing Engineering* 104/1 (2021) 11-20. DOI: <https://doi.org/10.5604/01.3001.0014.8482>

- [14] J. Thanikachalam, M. Ramkumar, P. Nagaraj, Electromagnetic analysis of magnetorheological brakes, *Journal of Achievements in Materials and Manufacturing Engineering* 76/2 (2016) 61-66. DOI: <https://doi.org/10.5604/17348412.1229481>
- [15] T.J. Mackin, S.C. Noe, K.J. Ball, B.C. Bedell, D.P. Bim-Merle, M.C. Bingaman, D.M. Bomleny, G.J. Chemlir, D.B. Clayton, H.A. Evans, R. Gau, J.L. Hart, J.S. Karney, B.P. Kiple, R.C. Kaluga, P. Kung, A.K. Law, D. Lim, R.C. Merema, B.M. Miller, T.R. Miller, T.J. Nielson, T.M. O'Shea, M.T. Olson, H.A. Padilla, B.W. Penner, C. Penny, R.P. Peterson, V.C. Polidoro, A. Raghu, B.R. Resor, B.J. Robinson, D. Schambach, B.D. Snyder, E. Tom, R.R. Tschantz, B.M. Walker, K.E. Wasielewski, T.R. Webb, S.A. Wise, R.S. Yang, R.S. Zimmerman, Thermal cracking in disc brakes, *Engineering Failure Analysis* 9/1 (2002) 63-76. DOI: [https://doi.org/10.1016/S1350-6307\(00\)00037-6](https://doi.org/10.1016/S1350-6307(00)00037-6)
- [16] S.V. Panin, P.O. Maruschak, P.S. Lyubutin, I.V. Konovalenko, B.B. Ovechkin, Application of meso-and fracture mechanics to material affected by a network of thermal fatigue cracks, *International Journal of Fatigue* 76 (2015) 33-38. DOI: <https://doi.org/10.1016/j.ijfatigue.2014.10.013>



© 2023 by the authors. Licensee International OCSCO World Press, Gliwice, Poland. This paper is an open-access paper distributed under the terms and conditions of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International (CC BY-NC-ND 4.0) license (<https://creativecommons.org/licenses/by-nc-nd/4.0/deed.en>).