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POWER SYSTEM STATE ESTIMATION IN RECTANGULAR COORDINATE SYSTEM FOR DIFFERENT MODELS OF SYMMETRICAL PHASE SHIFTER.

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Abstract: The paper deals with State Estimation (SE) for a Power System (PS) with a Phase Shifter (PhS). Aim of the paper is to show results of investigations of impact of taking into consideration the specific information about the symmetrical PhS on features of PS SE. During investigations, WLS SE in the rectangular coordinate system is considered. For SE evaluation one utilizes such indices as: the number of iterations in SE, the index of the conditionality of the solved equations (condition number cond(**G**)) and the index of accuracy of SE (rate *A*). Using the IEEE 14-bus test system, investigations are made for representative cases of SE. The results show, that from the viewpoint of condition number cond(**G**), and rate *A* the state estimation with the use of the specific information on PhS is more profitable than SE using only the general model of PhS. Considering number of iterations in SE, the results of investigations are not so unambiguous.

Keywords: model, phase shifter, power system, state estimation.

1. INTRODUCTION

The essential element of dispatcher software is a Power System State Estimation [1] (PSSE) program. Features of PSSE depend on used method and a model of Power System (PS). In the paper, PS with Phase Shifter (PhS) ([2], [3]) is considered. An example of such PS is Polish power system. In the paper, the symmetrical PhS is taken into account. PhSs and particularly the symmetrical one can be differently modeled. The aim of the paper is to show results of investigations of impact of taking into consideration the specific information about the symmetrical PhS on features of PSSE. The paper is next stage of investigations of impact of installation of PhS in PS on PSSE [4], [5]. There are no papers of other authors, in which such investigations are presented.

2. BACKGROUND

2.1. Characteristics of the considered state estimation

1. An objective function is following:

$$J(\mathbf{x}) = \frac{1}{2}[\mathbf{z}-\mathbf{h}(\mathbf{x})]^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{z}-\mathbf{h}(\mathbf{x})], \tag{1}$$

where: $\mathbf{x}$ – a PS state vector,
$\mathbf{z}$ – a vector of measurements,
$\mathbf{h}(\mathbf{x})$ - a vector of functions, representing dependence of measured quantities from $\mathbf{x}$,
$\mathbf{R}$ - a diagonal matrix of measurement covariances.

2. Vector $\mathbf{x}$ is represented in the rectangular coordinate system, i.e.: $\mathbf{x} = [e_1, e_2, \ldots, e_n, f_2, f_3, \ldots, f_n]^{\mathrm{T}}$, where: e_i, f_i are real and imaginary parts of the voltage at bus i ($i \in \{1, 2, \ldots, n\}$), respectively; n is a number of buses.
3. To find a solution of SE, one utilizes the normal-equation set:

$$\mathbf{G}(\mathbf{x}^k)\cdot(\mathbf{x}^{k+1}-\mathbf{x}^k) = -\mathbf{g}(\mathbf{x}^k), \tag{2}$$

where: k - a number of iteration,
$\mathbf{x}^k$ - a solution vector at k-th iteration,

$$\mathbf{G}(\mathbf{x}^k) = \mathbf{H}^T(\mathbf{x}^k)\cdot\mathbf{R}^{-1}\cdot\mathbf{H}(\mathbf{x}^k),\ \ \mathbf{H}(\mathbf{x}) = \frac{\partial\mathbf{h}(\mathbf{x})}{\partial\mathbf{x}},$$

$$\mathbf{g}(\mathbf{x}) = \frac{\partial J(\mathbf{x})}{\partial\mathbf{x}} = -\mathbf{H}^T(\mathbf{x})\mathbf{R}^{-1}[\mathbf{z}-\mathbf{h}(\mathbf{x})].$$

2.2. Indices of state-estimation evaluation [6]

1. The number of iterations in calculation process (n_{it}).
2. The condition number of matrix $\mathbf{G}(\mathbf{x})$ (cond($\mathbf{G}$)) [7], calculated as:

$$\mathrm{cond}(\mathbf{G}) = \lambda_M/\lambda_m, \tag{3}$$

where: λ_m, λ_M - the minimal and maximal eigenvalues of matrix $\mathbf{G}(\mathbf{x})$, respectively.

Condition number cond(**G**) characterizes conditionality of the SE calculation process.

3. Ratio A [8]

$$A = J_e/J_m, \tag{4}$$

where: $J_e = \frac{1}{m}\sum_{i=1}^{m}\left[(\hat{z}_i - z_i^r)/\sigma_i\right]^2$, $J_m = \frac{1}{m}\sum_{i=1}^{m}\left[(z_i - z_i^r)/\sigma_i\right]^2$,

z_i, $\hat{z}_i$, z_i^r - the measured, estimated and real value of i-th measured quantity, respectively,

σ_i - a standard deviation of the measurement of i-th quantity,

m - a number of the measured quantities.

Ratio A characterizes accuracy of the state estimation.

2.3. Characteristics of the considered phase shifter

1. The model of PhS is as it is shown in figure 1 [9]. For that model the following equations are valid:

$$\overline{\mathbf{S}}^*_{ik} = -V_i^2\left(\overline{\mathbf{y}}_{\mathrm{ET}}+\overline{\mathbf{y}}_{\mathrm{BT}}\right) + \overline{\mathbf{V}}_k\overline{\mathbf{V}}_i^*\overline{\mathbf{y}}_{\mathrm{BT}} - \overline{\mathbf{V}}_{\mathrm{BT}}\overline{\mathbf{V}}_i^*\overline{\mathbf{y}}_{\mathrm{BT}} + \overline{\mathbf{V}}_{\mathrm{ET}}\overline{\mathbf{V}}_i^*\overline{\mathbf{y}}_{\mathrm{BT}} \tag{5}$$

$$\overline{\mathbf{S}}^*_{ki} = -V_k^2\overline{\mathbf{y}}_{\mathrm{BT}} + \overline{\mathbf{V}}_i\overline{\mathbf{V}}_k^*\overline{\mathbf{y}}_{\mathrm{BT}} + \overline{\mathbf{V}}_{\mathrm{BT}}\overline{\mathbf{V}}_k^*\overline{\mathbf{y}}_{\mathrm{BT}} \tag{6}$$

$$\overline{\mathbf{S}}^*_{\mathrm{BT}} = V_{\mathrm{BT}}^2\overline{\mathbf{y}}_{\mathrm{BT}} + \overline{\mathbf{V}}_i\overline{\mathbf{V}}_{\mathrm{BT}}^*\overline{\mathbf{y}}_{\mathrm{BT}} - \overline{\mathbf{V}}_k\overline{\mathbf{V}}_{\mathrm{BT}}^*\overline{\mathbf{y}}_{\mathrm{BT}} \tag{7}$$

$$\overline{\mathbf{S}}^*_{\mathrm{ET}} = V_{\mathrm{ET}}^2\overline{\mathbf{y}}_{\mathrm{ET}} - \overline{\mathbf{V}}_i\overline{\mathbf{V}}_{\mathrm{ET}}^*\overline{\mathbf{y}}_{\mathrm{ET}} \tag{8}$$

$$\overline{\mathbf{S}}_{\mathrm{ET}} + \overline{\mathbf{S}}_{\mathrm{BT}} = 0 \tag{9}$$

$$\overline{\mathbf{S}}_i = \overline{\mathbf{S}}_{i-\mathrm{AC}} - \overline{\mathbf{S}}_{ik}\ ,\ \overline{\mathbf{S}}_k = \overline{\mathbf{S}}_{k-\mathrm{AC}} - \overline{\mathbf{S}}_{ki} \tag{10}$$

where: $\overline{\mathbf{S}}_{i-\mathrm{AC}}$, $\overline{\mathbf{S}}_{k-\mathrm{AC}}$ - bus powers at buses i and k, respectively, when there is no PhS in PS.

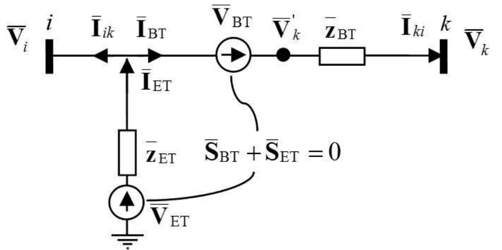

Fig. 1. The model for the considered phase shifter

2. PhS is real, symmetrical one. For that PhS the following relationship has place:

$$t_{ik} = \frac{V_i}{\left|\overline{\mathbf{V}}_i + \overline{\mathbf{V}}_{BT}\right|} = 1. \tag{11}$$

3. INVESTIGATIONS

3.1. Assumptions

1. Calculations are realized with the use of the IEEE 14-bus test system [10].
2. In the test system on the line between bus 5 and bus 4, at bus 5, there is the real symmetrical PhS.
3. One takes into account PSSE in the rectangular coordinate systems. SE_Ph and SE_SY stand for SE with the symmetrical PhS, but in the first case it is SE without taking into consideration the specific information on PhS (i.e. additional equation (9)) and in the second one that information is used.
4. $\overline{\mathbf{z}}_{\mathrm{ET}}$ = 0.01+j 0.1 pu. $\overline{\mathbf{z}}_{\mathrm{BT}}$ = 0.01+j 0.1 pu.
5. Calculations are made for different load variants, defined by sets of values of active and reactive loads and also power injections, which are determined from the formula $W = 0.5W_b + l\cdot W_b$, where W, W_b - the calculated and base values of the considered quantity; $l \in \{0, 0.1, 0.2, \ldots, 1\}$. The variant associated with l is denoted as $V^{0.5+l}$.
6. PhS introduces the phase shift in the range [-20°, 20°] with the step of 0.5°.
7. PSSE is performed for different numbers of Measurement Data (MD), i.e. for: $m_1 = 34$, $m_2 = 53$, $m_3 = 68$ and $m_4 = 104$.
8. Locations of measurement systems are randomly chosen. The number of those locations is the same for the different MD numbers and it is equal to 100.
9. Each item of MD is burden with a small error. Each of such the errors is described by the Gaussian distribution with a zero mean and standard deviation σ, calculated as [11], [12]: $\sigma = 1/3\left[(0.001+0.0025)FS + 0.02\,M\right]$ for active power, $\sigma = 1/3\left[(0.001+0.005)FS + 0.02\,M\right]$ for reactive power, and $\sigma = 1/3\left[(0.0005+0.0025)FS + 0.003\,M\right]$ for voltage magnitude, where FS is a measurement scope, M is a measured value.
10. Features of SE_Ph and SE_SY are investigated for the same: (i) parameters of PhS, (ii) load variants, (iii) MD numbers, (iv) locations of measurement systems, (v) characteristics of small errors burdening MD.

3.2. Results

During investigations, for each of indices: L_{it}, cond(**G**) and A, one calculates relative numbers of SE cases for which values of the mentioned index are in given ranges. i.e. one calculates $L_{r,X,R} = L_{X,R}/L$, where: L is a number of all SE cases, $L_{X,R}$, $L_{r,X,R}$ are respectively, a number of SE cases and a relative number of SE cases for which values of index X are in range R.

Taking into account $L_{r,X,R}$ $X \in \{n_{it}$, cond(**G**), $A\}$ for SE_Ph and SE_SY, relative change of numbers of SE cases in particular ranges of index X as a result of consideration of specific information on PhS is calculated from the formula:

$$\Delta L_{r,X,R} = 100\cdot(L_{r,X,R,SE\_SY} - L_{r,X,R,SE\_Ph})/L_{r,X,R,SE\_Ph} \tag{12}$$

where: L_{r,X,R,SE_Ph}, L_{r,X,R,SE_SY} - the numbers of $L_{r,X,R}$ $X \in \{n_{it}$, cond(**G**), $A\}$ for SE_Ph and SE_SY, respectively.

The earlier- considered relative changes of numbers of SE cases are shown in: (i) figure 2 - $\Delta L_{r,nit,R}$, (ii) figure 3 - $\Delta L_{r,cond(G),R}$, (iii) figure 4 - $\Delta L_{r,A,R}$.

Results of investigations, which are in figure 2, show that from the viewpoint of n_{it}, SE_SY has worse features than SE_Ph. For SE_SY, a number of SE cases with lower numbers of iterations is less than for SE_Ph and a number of SE cases with larger numbers of iterations is larger than for SE_Ph. For $n_{it} > 6$, $\Delta L_{r,nit,R}$ is positive for load variant $V^{0.5}$, and for $n_{it} > 7$, $\Delta L_{r,nit,R}$ is positive for load variant $V^{1.5}$ and increases when n_{it} increases for both load variants.

For m_4, from the viewpoint of n_{it} difference between SE_SY and SE_Ph is much lower than for m_1. Similar situation is when we consider condition number cond(**G**) (Fig. 3). However, in that case, features of SE_SY are better than for SE_Ph. A number of SE cases for SE_SY is larger than for SE_Ph for lower values of cond(**G**) and is lower than for SE_Ph for larger values of cond(**G**).

Analyzing features of PSSE from the viewpoint of ratio A, one can ascertain that more favorable situation is for SE_SY (Fig. 4). One can observe that lower number of SE cases with the largest values of ratio A is for SE_SY than for

SE_Ph. When MD is equal to m_1 and values of ratio A decrease, the values of $\Delta L_{r,A,R}$ increase for $A > 0.5$, and for load variant $V^{0.5}$ as well as for all observed values of A for load variant $V^{1.5}$. When MD is equal to m_4 and values of ratio A decrease, the values of $\Delta L_{r,A,R}$ increase for all values of A and for load variant $V^{0.5}$ as well as for $A > 0.1$, for load variant $V^{1.5}$.

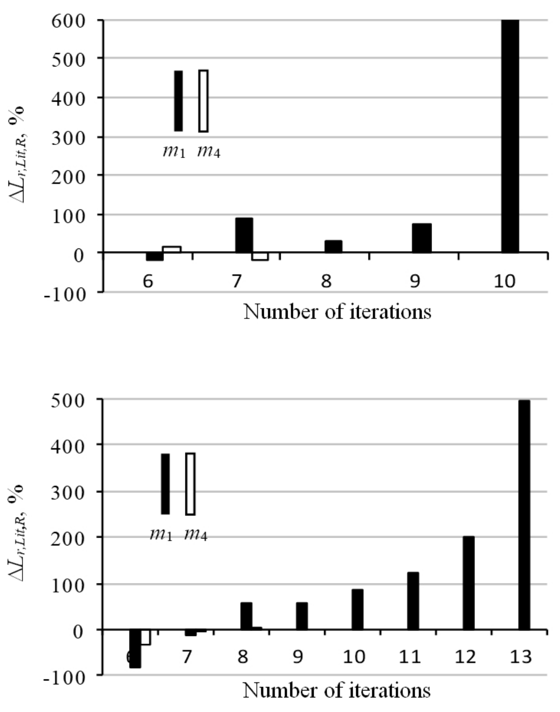

Fig. 2. Changes of numbers of SE cases for the particular numbers of iterations (in percentage) as a result of consideration of the specific information on PhS for load variant: a) $V^{0.5}$, b) $V^{1.5}$.

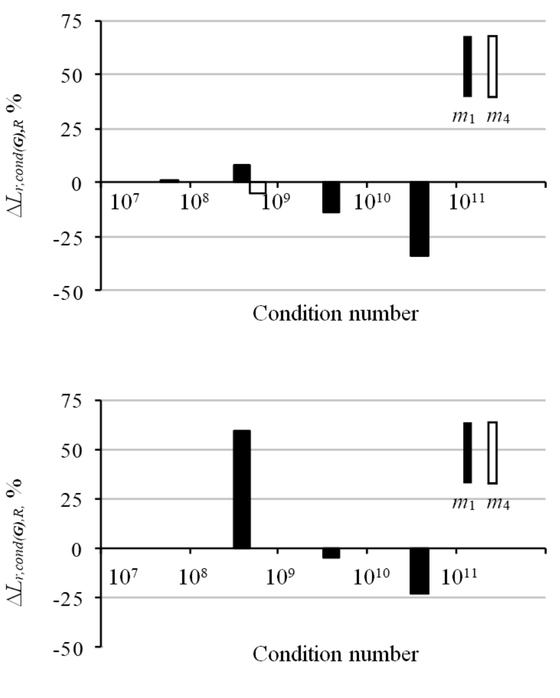

Fig. 3. Changes of numbers of SE cases in the particular ranges of the condition number (in percentage) as a result of consideration of the specific information on PhS for load variant: a) $V^{0.5}$, b) $V^{1.5}$.

In Table 1, there are relative changes of parameters of the indices for SE_SY with respect to suitable parameters of indices for SE_Ph (in percentage). The changes of the parameters shown in Table 1 are determined on the base of the formula: $\Delta p_X = 100 \cdot (p_{X,SE\_SY} - p_{X,SE\_Ph})/p_{X,SE\_Ph}$, where p_{X,SE_SY}, p_{X,SE_Ph} are parameters of index X for SE_SY and SE_Ph, respectively.

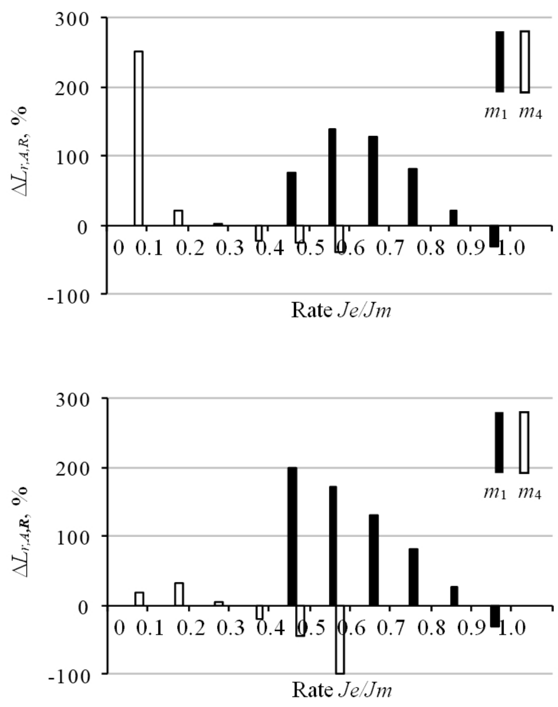

Fig. 4. Changes of numbers of SE cases in the particular ranges of rate Je/Jm (in percentage) as a result of consideration of the specific information on PhS for load variant: a) $V^{0.5}$, b) $V^{1.5}$.

Table 1. Relative changes of parameters of the indices for SE_SY with respect to suitable parameters of indices for SE_Ph (in percentage).

number of MD	Load variant $V^{0.5}$				Load variant $V^{1.5}$			
	m_1	m_2	m_3	m_4	m_1	m_2	m_3	m_4
number of iterations n_{it}								
min	0.0	0.0	0.0	0.0	0.0	0.0	-14.3	0.0
max	40.0	0.0	0.0	0.0	76.9	0.0	0.0	0.0
mean	2.8	-0.6	-0.8	-1.2	4.2	0.1	0.0	0.0
std.dev.	25.7	-1.0	-2.0	-0.9	17.3	2.0	-1.0	1.4
condition number cond(**G**)								
min	-1.8	-1.3	-0.5	0.0	-0.2	-3.4	-0.3	-0.6
max	-12.2	-2.2	-0.5	0.0	-26.8	-3.6	-0.8	-0.5
mean	-9.4	-1.7	-0.6	-0.1	-11.4	-2.8	-1.8	-0.3
std.dev.	-14.8	-2.2	-0.7	-1.8	-13.2	-5.1	-3.3	-0.1
ratio A								
min	-6.1	-8.5	-7.9	-14.7	-15.1	-6.3	-18.1	-0.2
max	-0.1	-2.5	-3.9	-4.9	-0.1	-1.7	0.0	-13.5
mean	-3.7	-3.8	-4.0	-4.2	-3.8	-4.0	-4.2	-4.7
std.dev.	12.3	-1.0	-1.4	-2.4	13.9	0.0	-1.6	-6.6

Analyzing Table 1, one can note, that with the exception of standard deviation of A for m_1 and for all load variants all other changes of parameters of cond(**G**) and ratio A are negative. Other situation is from the viewpoint of number of iterations n_{it}. For MD equal to m_1 and all load variants as well as for MD equal to m_2 or m_4 and load variant $V^{1.5}$ the relative changes of parameters of n_{it} are not less than zero. For other numbers of MD, the relative changes of parameters of n_{it} are not greater than zero. The content of Table 1, confirms previous observations, that the features of SE_SY are better than the features of SE_Ph from the viewpoint of condition number cond(**G**) and ratio *A*. From the viewpoint of number of iterations n_{it} there are many cases when it is the other way round.

4. CONCLUSION

Performing calculations requires appropriate model for object with which these calculations are associated. When there is PhS in PS, PSSE can be performed using a general model of PhS, or in SE calculations also a specific information on this PhS is used. Both the mentioned cases are considered in the paper under assumption that PhS is a symmetrical one. The paper present comparison of features of PSSE for the earlier-presented cases of modeling of PhS. For the aim of the comparative analysis one takes into account such indices characterizing features of PSSE as: number of iterations n_{it}, condition number cond(**G**), and rate *Je*/*Jm* (rate *A*).

Calculations realized for representative cases of PSSE show, that from the viewpoint of condition number cond(**G**), and rate *A* the state estimation with the use of the specific information on PhS is more profitable than the state estimation using only the general model of PhS. Considering number of iterations n_{it} in PSSE, the situation is not so unambiguous. There are cases when PSSE using the specific information on PhS is more profitable and there are cases when PSSE using the general model of PhS is more profitable. There are also such cases when utilization of the specific information on PhS does not change number of iterations n_{it} in a significant way. It should be noted, that on the one hand, utilization of the specific information on PhS enhances quality of SE, what is visible analyzing condition number cond(**G**), and rate *A*. On the other hand, utilization of the additional information on PhS increases the complexity of the PhS model used in SE, what may adversely affect number of iterations n_{it} in PSSE, at least in certain cases.

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ESTYMACJA STANU SYSTEMU ELEKTROENERGETYCZNEGO W PROSTOKĄTNYM UKŁADZIE WSPÓŁRZĘDNYCH DLA RÓŻNYCH MODELI SYMETRYCZNEGO PRZESUWNIKA FAZOWEGO

Praca dotyczy estymacji stanu systemu elektroenergetycznego z przesuwnikiem fazowym. Jest kolejnym etapem badań wpływu obecności przesuwnika fazowego w systemie elektroenergetycznym na właściwości estymacji stanu. Celem pracy jest pokazanie wyników badań wpływu uwzględnienia specyficznej informacji o symetrycznym przesuwniku fazowym na właściwości estymacji stanu. Podczas badań brana jest pod uwagę estymacja stanu najmniejszych ważonych kwadratów w układzie współrzędnych prostokątnych. Dla potrzeb oceny estymacji stanu wykorzystywane są takie wskaźniki, jak: liczba iteracji w procesie obliczeniowym estymacji stanu, wskaźnik uwarunkowanie rozwiązywanych równań (wskaźnik uwarunkowania cond(**G**)) i wskaźnik dokładności estymacji stanu (wskaźnik *A*). W opisywanych badaniach wykorzystywany jest 14-węzłowy system testowy IEEE. Są one przeprowadzane dla reprezentatywnych przypadków estymacji stanu. Otrzymane wyniki pokazują, że z punktu widzenia uwarunkowanie rozwiązywanych równań i dokładności wyników estymacja stanu z uwzględnieniem specyficznej informacji o symetrycznym przesuwniku fazowym jest korzystniejsza aniżeli estymacja stanu uwzględniająca tylko ogólny model przesuwnika fazowego. Biorąc pod uwagę liczbę iteracji w estymacji stanu, wyniki badań nie są tak jednoznaczne.

Słowa kluczowe: estymacja stanu, model, przesuwnik fazowy, system elektroenergetyczny.