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Compressor modeling using Greitzer model validated by pressure oscillations

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Abstract

Nowadays compressors are used in almost every industry. Compressor failure can suspend production of the whole system so the importance of preventing from failures is obvious and essential. One of the most dangerous flow instabilities which is capable of destroying machine in few seconds is surge, which occurs in conditions of low mass flow rate. Greitzer model, apart from its long history, is still most common mathematical model describing surge. It is widely used to predict the surge onset and pressure oscillations during it. However, it is based on parameters that are not directly related to real machine and their choice is not always obvious. Therefore, the calculations may be inaccurate which results in wrong surge prediction. The other approach to Greitzer model is presented in this paper, which in some cases can assure that the process of compressor modeling is more accurate. The applicability of Greitzer surge model for real machines has been analyzed. Method of implementation is based on experimental pressure signal gathered during unstable work of compressor. Presented method is based on experimental compressor characteristic and outlet pressure signal from unstable work of compressor. From that data it is possible to determinate the value of Greitzer model's parameters for selected operational point. Thanks to this method this model could be applied for reliable antisurge protection.

Keywords: Centrifugal compressor; Surge; Greitzer model; Compressor modeling

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Nomenclature

A	–	pressure dimensionless amplitude
A_c	–	compressor pipe area, m^2
A_t	–	throttle pipe area, m^2
A_{in}	–	cross-section area of the passage at the impeller inflow, m^2
A_{out}	–	cross-section area of the passage at the impeller outflow, m^2
B	–	Greitzer model stability parameter
b_1	–	blade height at the leading edge, m
b_2	–	blade height at the trailing edge, m
D_2	–	diameter of the impeller at the trailing edge, m
D_3	–	diameter of the diuser outlet, m
D_{hub}	–	diameter of the hub at the leading edge, m
D_in	–	diameter of the inlet pipe, m
D_{out}	–	diameter of the outlet pipe, m
f_{nom}	–	nominal frequency, Hz
f_{rot}	–	experimental rotational frequency, Hz
G	–	Greitzer model parameter,
L_c	–	compressor pipe length, m
L_c	–	throttle pipe length, m
L_p	–	blade passage length, m
m	–	mass flow, kg/s
$\dot{m}_c$	–	mass flow in compressor pipe, kg/s
$\dot{m}_c$	–	mass flow in throttle pipe, kg/s
$\dot{m}_nom$	–	nominal mass flow, kg/s
p_c	–	pressure of compressor pipe inlet, Pa
p_t	–	pressure of throttle pipe inlet, Pa
p_p	–	plenum pressure, Pa
p_{s-in}	–	signal representing static pressure at the blower inlet, Pa
p_{s-out}	–	signal representing static pressure at the blower outlet, Pa
R_θ	–	volute radius, m
TOA	–	throttle opening area, %
t	–	time, s
t	–	dimensionless time
U_t	–	blade tip speed, m/s
V_p	–	plenum volume, m^3
Z	–	number of blades
ΔA	–	amplitude absolute error
ΔP	–	pressure difference, Pa

Greek symbols

$\Delta\omega$	–	frequency absolute error, Hz
π	–	pressure ratio
π_{nom}	–	nominal pressure ratio
δ	–	blade tip clearance, m
Φ	–	dimensionless mass flow

Φ_c	–	dimensionless mass flow in compressor/compressor performance curve
Φ_t	–	dimensionless mass flow in throttle
ρ_a	–	air density, kg/m^3
ρ	–	density, kg/m^3
Ψ	–	dimensionless pressure rise
Ψ_c	–	dimensionless pressure rise in compressor
Ψ_t	–	dimensionless pressure rise in throttle/ throttle performance curve
ω_h	–	Helmholtz frequency, rad/s

1 Introduction

Centrifugal compressors are turbomachines used in almost all kinds of industry like chemical, petrochemical, aircraft, power, etc. Very often compressor is crucial part of installation and its failure can paralyze whole production and cause severe losses. This is why stable and safe working conditions are so important and require special attention. The existence of flow instabilities decreases safe working range and thus the machine is not working with maximum performance. This also can be counted as loss because machine does not use its full potential. Two most dangerous instabilities are rotating stall and surge and they are often reason of compressor failure. Rotating stall is less destructive phenomenon than surge and it is said to be frequent reason of surge onset, whereas surge can completely destroy machine in few seconds [1–3]. It is very dangerous phenomenon which is still not fully understood in centrifugal compressors and is avoided mainly by provision of large surge margin that strongly limits matching operational range.

1.1 Surge

Surge is flow instability occurring when mass flow is decreased below its critical value. Critical mass flow is different for all compressor rotational speeds and points placed on compressors performance for different speeds creates surge line (Fig. 1).

Surge appears as a pressure and mass flow fluctuations in axial direction. It is one-dimensional instability which affects compressor with pipeline system and it can be visualized as limit cycle oscillations on compressor map [5,6]. Typical deep surge limit cycle and pressure oscillation during surge are presented in Fig. 2 using dimensionless parameters. In this instability annulus-averaged mass flow is unsteady but circumferentially uniform. Sometimes oscillations are so big that reversed mass flow occurs. This is very dangerous because this oscillations can induce significant mechanical and thermal loads. Also, the frequency of oscillations can be close to resonance frequency of machine component. A distinction between

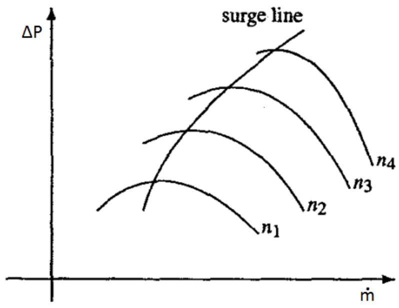

Figure 1: Compressor map with surge line [4].

different kinds of surge depending on fluctuations amplitude is often defined [7]:

1. Mild surge – no reverse flows occurs and the oscillations have frequency slightly less than Helmholtz's frequency [4,5,8,9].
2. Classic surge – no reverse flows occurs, oscillations are bigger than in mild surge but frequency is lower.
3. Modified surge – rotating stall and surge mixed resulting in nonaxisymmetric, unsteady flow [9,10].
4. Deep surge – reverse flow are possible and has biggest fluctuations. Most dangerous of all types.

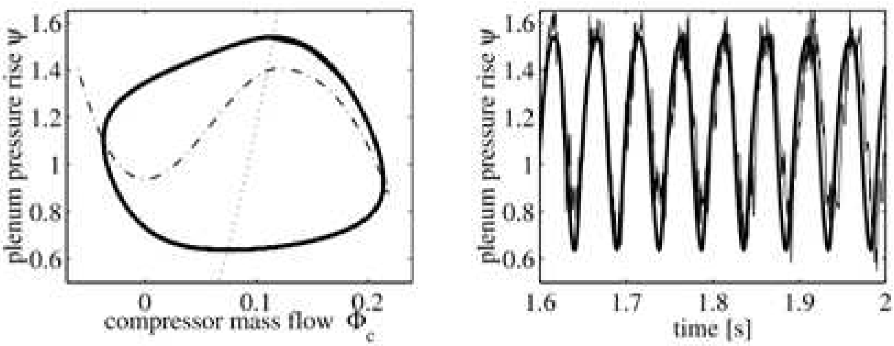

Figure 2: Typical surge limit oscillation on compressor map (left) and pressure rise during surge (right) [11].

It is very important to predict surge onset for purpose of preventing centrifugal compressor from crossing surge line and potential failure. All compressors have defined surge margin which is located at a safe distance before surge line. If we can define surge onset more precisely then this margin can be closer to surge line and as a result machine loses less of its operating range. Surge line can be determined by mathematical models. Since the surge discovery made by Emmons *et al.* [12] in 1955 many mathematical models have been developed. Some of them are presented in Tab. 1 [9].

Table 1: The survey of mathematical models.

Model	Flow type	Compressor type	Flow instability
Badmus [13]	Quasi-1-D Com.	A, C	S
Botros [14]	1-D Com.	A, C	S
Greitzer [15]	1-D Incom.	A	S
Hansen [16]	1-D Incom.	C	S
Fink [17]	1-D Incom.	C	S
Moore-Greitzer [18]	2-D Incom.	A	S, RS
Haynes [19]	2-D Incom.	A	S, RS
Gravdahl-Egeland [6]	2-D Incom.	A	S,RS
Com. – compressible Incom – Incompressible		A – axial C – centrifugal	S – surge RS – rotating stall

1.2 Greitzer model

Most commonly used surge model is Greitzer one [15] developed in 1976 which has been proven by experiment the same year [20]. Despite the fact it has been developed for axial compressors it can be successfully used also in centrifugal compressors what has been proved in [11,16,17,21]. Still many antisurge systems are based on that model and it may be used to define surge margin.

This model is based on compressor scheme which is presented in Fig. 3. It consists of three parts: compressor, plenum, and throttle, and all lower subscripts c, p and t corresponds to this parts, respectively. The compressing part of compressor installation is defined as a pipe with area A_c and length L_c. Compression is accomplished by virtual piston generating pressure rise. Throttle is defined similarly but without piston – it is just pipe which can be closed on its end by decreasing outlet rea. In both mentioned about cases there is significant kinetic energy hence areas and lengths are selected in a way which guarantees dynamics similar to compressor and valve respectively. These pipes are connected by plenum which is big reservoir in which kinetic energy is neglected but has variable pressure p_p. It is defined by its volume V_p. Translating this scheme to real machine, compressor is rotor and diffuser, plenum is everything between diffuser and throttle (so pipline, volute, etc.).

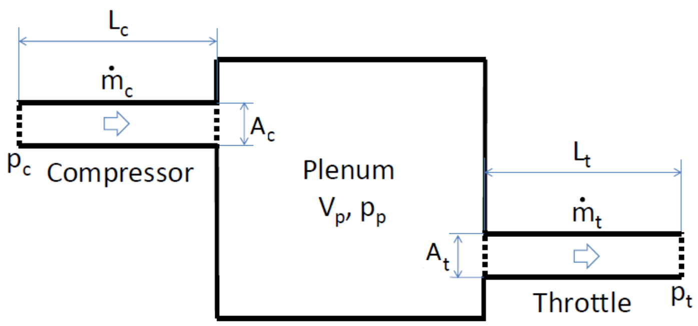

Figure 3: Compressor scheme used in Greitzer model.

Greitzer model also has its assumptions which are pointed below [7]:

1. incompressible flow within pipes, inviscid and one-dimensional, hence compressor mass flow m_c and throttle mass flow m_t are constant along the ducts;
2. isentropic compression in the plenum;
3. constant temperature in the whole compression system;
4. uniform pressure in the whole plenum;
5. valve behavior is quasi-static;
6. gravitational force is neglected.

Original Greitzer model consists of four equations but fourth is used only for axial machines so it will not be considered in this paper. The other three equations can be derived from basic momentum and energy conservation equations. Model operates with dimensionless parameters presented below:
mass flow

$$\Phi = \frac{\dot{m}}{\rho U_{tip} A_c} \ , \tag{1}$$

pressure rise

$$\Psi = \frac{\Delta \mathrm{p}}{\frac{1}{2} {U_{tip}}^2} \ , \tag{2}$$

time

$$\hat{t} = t\omega_h \ , \tag{3}$$

where: A_c - compressor pipe area, Δp – pressure difference; t – time, U_{tip} – blade tip speed, ρ – density, ω_h – Helmholtz frequency, $\dot{m}$ – mass flow.

Greitzer model equations defines:
mass flow in compressor which depends on pressure difference between compressor and plenum

$$\frac{d\Phi_c}{d\hat{t}} = B\left(\Psi_c - \Psi_p\right) \ , \tag{4}$$

throttle mass flow

$$\frac{d\Phi_t}{d\hat{t}} = \frac{B}{G}(\Psi_p - \Psi_t) \ , \tag{5}$$

and pressure rise in plenum depending on difference of mass flows from compressor and throttle

$$\frac{\mathrm{d}\Psi_p}{d\hat{t}} = \frac{1}{B}\left(\Phi_c - \Phi_t\right) \ . \tag{6}$$

Greitzer model has also two parameters: B and G. B (parameter) is called stability parameter because its value determinates if system is stable or not. Its physical interpretation is slope of compressor performance curve. Bigger B corresponds to bigger slope and thus less stable system

$$B{=}\frac{U_{tip}}{2L_c\omega_h} \ , \tag{7}$$

$$G = \frac{A_c L_t}{A_t L_c} \ , \tag{8}$$

where: A_t – throttle pipe area, L_c – compressor pipe length, L_t – throttle pipe length.

It is generally known that due to negligible throttle dynamics (which appears as very low value of parameter G because of very low throttle length L_t) Eq. (5) can be neglected. If parameter G is very low then according to this equation even very low pressure gradient induce big mass flow. This means that throttle follows system changes very. This is commonly used manner done for example in [6,11,22–27]. Therefore, because of lack of dynamic behavior in throttle it is assumed that pressure in plenum, Ψ_p, is equal to throttle pressure, Ψ_t.

1.3 Greitzer model drawbacks

It is clear that the Greitzer model is very simplified comparing to real machine. Some of assumptions are very inaccurate. For example neglecting kinetic energy in plenum is obviously significant misstatement. That is why some models enhance it by coping with mentioned misstatements. They mostly improve pipeline dynamics like in Yoon *et al.* model [24], Goyne and Allarie [26], Helvoirt and de Jager [22] or Helvoirt *et al.* [28].

Another significant drawback is ambiguous way of specifying Greitzer model parameters. It is hard to determinate L_c and A_c in way guarantying constant dynamics. Area of compressor can be set for example as outlet passage area and length is sometime approximated with length of blade or blade passage. According to [28] most commonly used method of determining this parameter is to iterate it and choose the one best suits to machine. But it is clear that the model depends on the whole system including pipeline, throttle etc. The method proposed by Helvoirt in [29] using approximate realizations is accurate but complicated.

1.4 Aim of study

The aim of this study is to present a simple and universal method of matching the Greitzer model simulation with a given compressor based on pressure signals. This solution provides extensive range of antisurge system adjustments that will make it efficient and reliable. This paper is not presenting new experimental results but is using one presented in [30–33].

2 Method

In this paper the results of experimental investigations of single stage centrifugal blower has been used to simulate centrifugal blower by two equation Greitzer model. Pressure signals of surge from centrifugal blower have been collected and analyzed in previous studies, and research results obtained on this machine considering surge onset was published in [30–33].

2.1 Experimental setup

Experimental investigations were performed on single stage centrifugal blower DP1.12 [30–33] which cross section is shown in Fig. 4.
Air entered blower by inlet pipe, A, of diameter $D_{in} = 300$ mm and then was accelerated in Witoszyński nozzle, B, [34] in purpose of aligning velocity profile on rotor, C, inlet. Downstream of rotor air entered parallel, vaneless diffuser, D. After diffuser air flew through circular volute, E, of variable radius R_θ changing from 5 mm to outlet pipe diameter $D_{out} = 150$ mm with radius increasing streamwise. Outlet pipe consisted of two straight elements of lengths 250 mm and 3750 mm and was connected by elbow. At the end of pipe the throttle was placed.

Motor which was driving blower was asynchronous AC motor (400 V/15 kVA). Blower design point was attained at nominal rotation frequency $f_{nom} = 120$ Hz, nominasl mass flow $\dot{m}_{nom} = 0.8$ and nominal pressure ratio $\pi_{nom} = 1.12$ and was

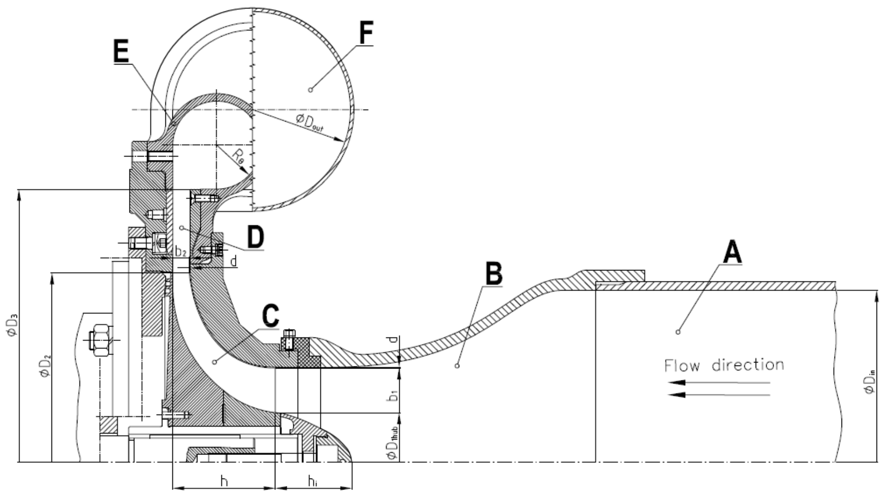

Figure 4: Cross section of centrifugal blower DP1.12.

designed for ambient inlet conditions. In this experiment blower worked at lower conditions of rotational frequency $f_{rot} = 100$ Hz, mass flow $\dot{m} = 0.75$ kg/s and pressure ratio $\pi = 1.08$ due to damage risk. At surge rotational speed yielded the impeller tip speed equal to $U_{tip} = 103$ m/s. All parameters presented in this section are summarized in Tab. 2.

Table 2: Parameters of the centrifugal blower DP1.12.

$D_{in} = 300$ mm	$Z = 23$	$D_3 = 476$ mm	$\pi = 1.08$
$D_{hub} = 86.3$ mm	$\delta = 0.8$ mm	$D_{out} = 150$ mm	$U_{tip} = 103$ m/s
$b_1 = 38.9$ mm	$L_p = 154$ mm	$R_\theta = 5 - 150$ mm	$f_{nom} = 120$ Hz
$D_2 = 330$ mm	$A_{in} = 626$ mm^2	$f_{rot} = 100$ Hz	$\dot{m}_{nom} = 0.8$
$b_2 = 14.5$ mm	$A_{out} = 823$ mm^2	$\dot{m} = 0.75$ kg/s	$\pi_{nom} = 1.12$

2.2 Signal gathering and analysis

Static pressure measurements were made at inlet, p_{s-in}, and outlet, p_{s-out}, of the compressor (subscript s corresponds to static pressure) with two pressure dynamic subminiature Kulite transducers, placed as it was shown in Fig. 5. A 100 kHz sampling rate was employed for data acquisition with a total acquisition time

20.97 s. An Iotech Wavebook 516/E acquisition system was used to capture the data. Every measurements consists 2097152 collected signal samples. hence total acquisition time – signal time corresponds to above 2000 rotor full revolutions.

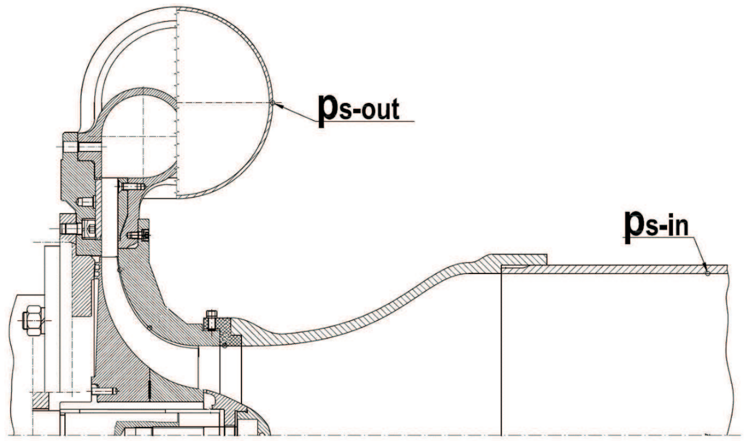

Figure 5: Cross section of centrifugal blower DP1.12 with pressure gauges positions.

The pressure signal was gathered for different operation points both stable and unstable. It was achieved by decreasing the outlet area which was expressed in TOA (throttle opening area) parameter, varying between 0 and 100% (fully closed and fully open respectively). This parameter is proposed in [33] where also whole mass flow calculation is described. In this study also the relationship between TOA and mass flow, $\dot{m} = f(\mathrm{TOA})$ is implemented according to Fig. 6 with equation of this curve presented on the right bottom corner of Fig. 6.

Based on this function the mass flow was designated and transformed to dimensionless parameter ϕ. Parameter A_c was set as passage outlet area and was equal to 676 mm^2 and air density $\rho_a = 1.168$ kg/m^3. Also pressure rise was transformed into dimensionless value ψ. Fragment of pressure signal from TOA= 4.24% is presented in Fig. 7. This signal was measured in the lowest mass flow conditions, so it is deepest surge condition obtained in the experiment, and its spectral analysis is presented. The frequency of strongest observed oscillations signal is 10.87 Hz, which gives $\omega = 68.31$ rad/s. Amplitude of signal obtained by spectral analysis is equal 0.6292 but as one can see the value of amplitude of real signal is bigger. It can be accounted to a fact that in real machine many factors have their impact on amplitude. Despite the fact that real signal seems to have bigger oscillations, the surge amplitude about 0.6292 was assumed. Surge frequency can be also detected when surge amplitude is not biggest from all components

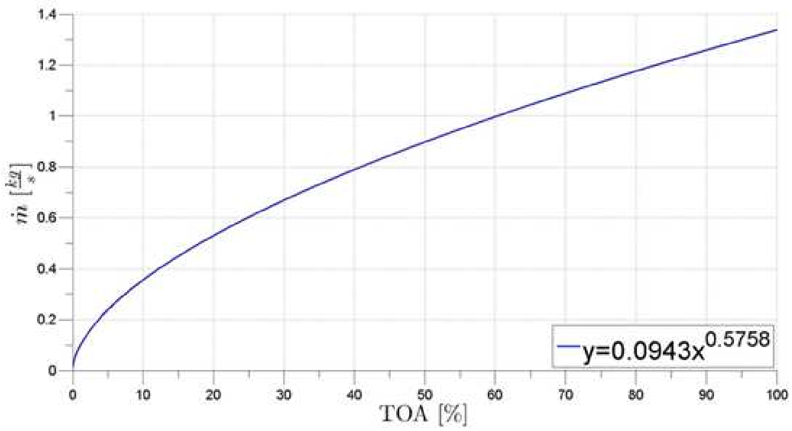

Figure 6: Curve used in mass flow calculation in TOA function.

in frequency spectrum. In such situation the highest peak near predicted surge frequency is picked. It is noticeable that this oscillations have limit cycle character and are strongly noised. Mean value of signal is 0.9761. Same analysis and information was extracted for other operating points.

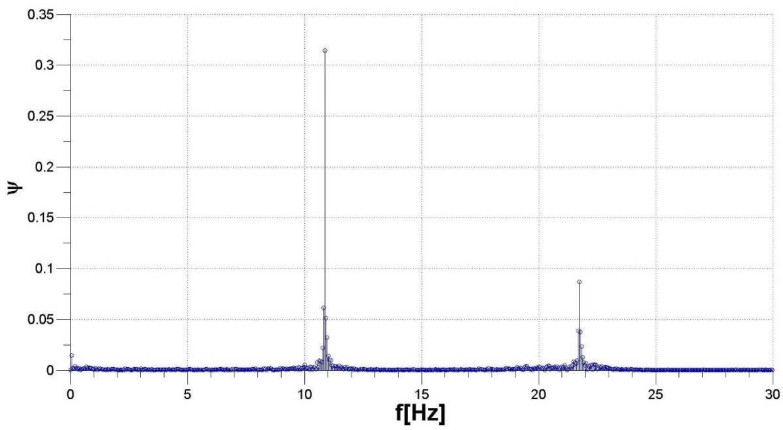

Figure 7: Dimensionless pressure signal and its frequency spectrum-TOA 4.24%.

2.3 Compressor performance curve approximation

First step in modeling compressor is to approximate performance curve based on signals from different operation points. It is easy to plot points from stable work conditions because pressure signal is almost stable but different situation is when surge occurs and pressure signal is highly unstable. In this research in both situations mass flow rate was taken form curve presented in Fig. 6 and pressure rise was designated to be mean value of signal. In bigger machines assumption that in surge pressure oscillates around operating point can be a big mistake because in that kind of machines pressure mean value is above operating point. However in this research this assumption is acceptable. Experimental points from stable and unstable operation are shown in Fig. 8.

The most common method of compressor performance curve approximation is third order polynomial [6,7,11,24,35]. In this research it was impossible to fit all experimental points with this method, so two curves have been chosen. First one fits stable work points and this curve is called ‘right-sided’ approximation

$$\Psi_c(\Phi_c) = 1.06 + 0.0992 \left[1 + \frac{3}{2}\left(\frac{\Phi_c}{0.6736} - 1\right) - \frac{1}{2}\left(\frac{\Phi_c}{0.6736} - 1\right)^3 \right] , \tag{9}$$

and second fits unstable points and is called ‘left-sided’ approximation

$$\Psi_c(\Phi_c) = 0.9 + 0.1792 \left[1 + \frac{3}{2}\left(\frac{\Phi_c}{0.6736} - 1\right) - \frac{1}{2}\left(\frac{\Phi_c}{0.6736} - 1\right)^3 \right] , \tag{10}$$

Because none of those two curves fit all points hence new approximation is proposed, new is the fourth order polynomial which fits all points

$$\Psi_c(\Phi_c) = 0.1116{\Phi_c}^4 - 0.6654{\phi_c}^3 + 0.9218{\Phi_c}^2 + 0.8524 . \tag{11}$$

All three approximations are presented in Fig. 8.

2.4 Compressor modeling

The result of compressors operation simulation, using two equation Greitzer model, is signal similar to sinusoid. Three main quantities describing sinusoid are mean value, amplitude and frequency. Therefore to model compressor, simulation have to be adjust so as to match its quantities with experimental signal. This quantities are called criteria of simulation. The modeling process is presented by simulation one operation point for TOA = 4.24%.

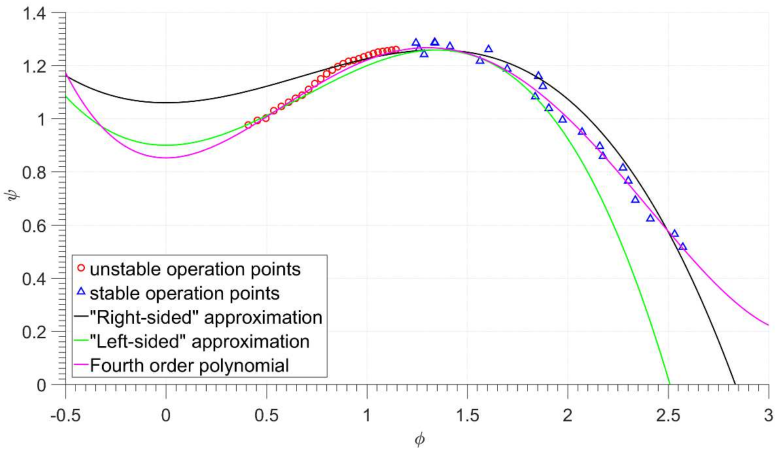

Figure 8: Experimental points and performance curve approximation.

Mean value criteria can be fulfilled by choosing proper compressor performance curve because of assumption that in surge pressure and mass flow oscillations are around operation point . Therefore the 'right-sided' approximation is neglected since its pressure value in operation point is much higher than in experimental signal. 'Left-sided' approximation has similar pressure to experimental signal but due to its deviations in stable operating region it is not correct also. Therefore fourth order polynomial is chosen as a compressor performance curve because it is most adequate to approximate experimental points.

Two other criteria, frequency and amplitude, can be received from spectral analysis done by fast Fourier transform (FFT). Frequency of highest amplitude obtained by simulation is considered as surge frequency and its value is received from spectral analysis. Amplitudes gained from FFT are sometimes understated due to spectral leakage but its real value can be estimated in way descripted later in this paper.

Criteria which determinate simulation agreement with experimental signal for TOA $= 4.34\%$ are summed up in Tab. 3.

In this paper centrifugal blower DP1.12 has been modeled using two equation Greitzer model, where two parameters need to be determined: parameter B and Helmholtz frequency, ω_h. Firstly, the impact of Helmholtz frequency and B parameter on Greitzer model has been analyzed. Simulation for two different ω_h and for B varied from 0.001 to 3.5 with 0.01 step has been made. In mild surge, which occurs in experiment, surge frequency of pressure oscillations is slightly

Table 3: Summary of simulation criteria.

Criteria	Criteria value
Mean value	0.9761
Frequency	$\omega = 68.31$ rad/s
Amplitude	$A = 0.6292$

less than the Helmholtz frequency. That is why first analyzed frequency was $\omega_h = 68.31$ rad/s (signal frequency). Second frequency was much higher to obtain significant differences between results and was 100 rad/s. After this analysis new optimized method of modeling has been developed and whole compressor in its unstable region has been modeled. However, to obtain simulations of different operation points throttle characteristic had to be determined, and in accordance with literature, it is mostly done by quadratic function $\Psi_t = k{\Phi_t}^2$ [7]. Because operation point is in cross of compressor performance curve with throttle characteristic some parameter k has been introduced to achieve desired operation point. For presented example of TOA = 4.24% parameter k is equal to 5.57. Simulation time was equal to experimental signal time, but sampling time was 10 kHz and the same sampling time has FFT.

3 Results

Figures 9 and 10 present results of two equation Greitzer model simulation. Analyzed variables in simulation was B parameter and two Helmholtz frequencies. These figures presents impact of parameter B and Helmholtz frequency, ω_h on Greitzer model simulation. In figures two criteria which were mentioned before are shown by horizontal lines, amplitude and frequency. It visualize at which value of B parameter criteria is fulfilled. Points on beginning of Fig. 9 looks very chaotic. Amplitude connected with surge amplitude is very small (surge is just beginning) so FFT picked different frequency. Therefore region from 0 to critical value of B parameter is neglected [28,33]. Critical B parameter at which surge begins is signed as vertical line on both figures. Fluctuations of amplitude in Fig. 10 are because of spectral leakage which was mentioned before. In fact, real amplitude of simulation is obtained by connecting peaks of those fluctuations (marked as big points on fluctuations peaks).

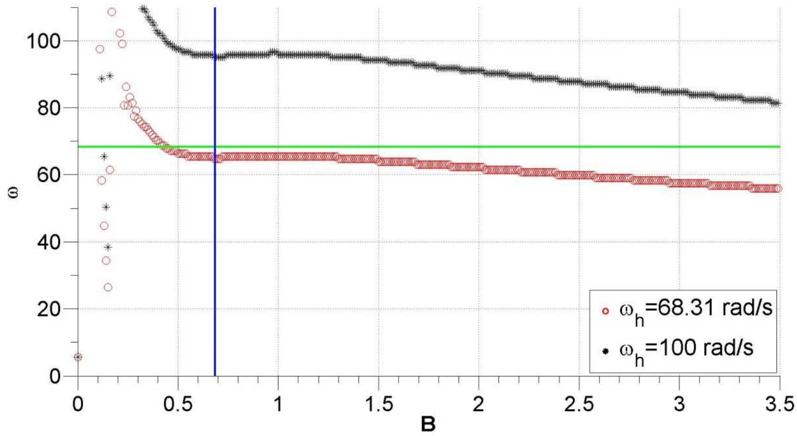

Figure 9: Influence of B and ω_h on simulation frequency.

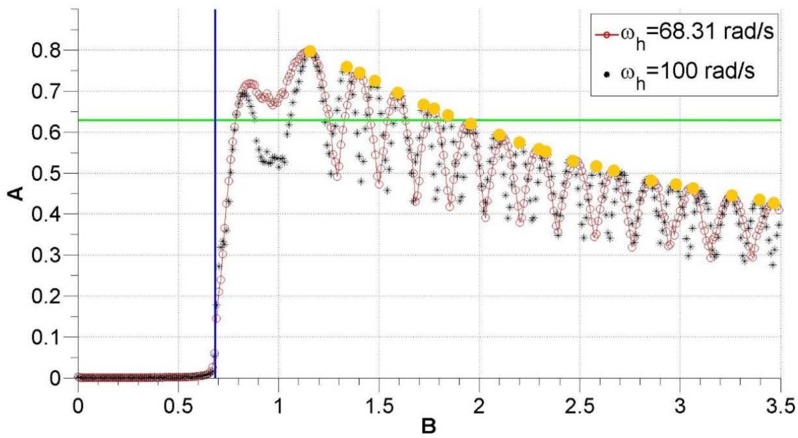

Figure 10: Influence of B and ω_h on simulation amplitude.

4 Discussion

4.1 The impact of Helmholtz frequency

Impact of Helmholtz frequency on Greitzer model simulation is easily noticeable in Figs 9 and 10. It seems that Helmholtz frequency has impact on simulation frequency, moreover, one can see that amplitude is equal for both frequencies. It is noticeable that curve for $\omega_h = 100$ rad/s oscillation, generated by spectral leakage, are almost two times more frequent than for $\omega_h = 68.31$ rad/s. This results from almost two times higher of this frequancy then $\omega_h = 68.31$ rad/s. Fact that Helmholtz frequency does not have impact on amplitude is very important observation for compressor modeling. Basing on this observation Helmholtz frequency now can be used to adjust simulation frequency without changing amplitude. Next two conclusions are based on results shown in Fig. 10. In this figure one can see that the higher Helmholtz frequency is the higher frequencies in the simulations. Last conclusion is that simulation never reaches assumed Helmholtz frequency no matter what parameter B is approach, because compressor system is same as Helmholtz resonator only when throttle is fully closed (TOA = 0%). When it is even slightly open the frequency would decrease. In Fig. 9 both curves has step shape and that is the influence of FFT sampling.

In literature there is no information about the influence of Helmholtz frequency on Greitzer model simulation, but only values of it were presented [11,24,37]. Only in [26] it was mentioned to control the first resonance mode of surge. Conclusion that simulation never reaches assumed Helmholtz frequency is confirmed also in [4,5,8,9]. As one can see neither of analyzed Helmholtz frequencies are chosen correctly but after observations of Helmholtz frequency impact it could be expected that the assumed Helmholtz frequency should be slightly higher than 68.31 rad/s.

4.2 The impact of B parameter

Parameter B was variable in simulation of both applied Helmholtz frequencies. Its influence on Greitzer model simulation is clearly visible in Figs. 9 and 10. Influence of B parameter on amplitude is presented in Fig. 10. In the beginning of surge with rise of B parameter amplitude is sharply rising up to its maximum, and then amplitude start slightly decreasing. Situation is similar in Fig. 9. At the beginning frequency grows a little with B, but after reaching maximum it slightly decreases. It is important B parameter has impact on both, amplitude and frequency, hence is most influential parameter and most important in compressor modeling.

In literature impact of B parameter is said to be well known but only in [38] it is said that B has relevant impact on amplitude, while influence on frequency is neglected. There is no value of parameter B in which frequency criteria is fulfilled but for amplitude criteria B which satisfy it is 0.7813 or approximately 1.93 but in this second point frequency is much different than required.

4.3 Method of compressor modeling

New method of compressor modeling has been formulated on the base of conclusion made in previous section. First step is to approximate the compressor performance curve and throttle characteristic for current operation point. This operation will fulfill mean value criteria if we assume that surge oscillations fluctuates around operation point. Because Helmholtz frequency does not change amplitude so at the beginning it can be set up as equal to experimental signal frequency. To fulfill amplitude criteria B parameter must be determined, therefore it is set up to be equal to 0 or equal to critical value to minimize number of iterations. Then by iterations B is increased until amplitude of simulation is equal to signal amplitude. After that by increasing Helmholtz frequency, frequency of simulation is adjust until it fulfill frequency criteria. After this procedure one operation point of compressor is modeled, however, to model another operation point throttle constant, k, should be changed. This method can be summing up in few consecutive steps:

- Approximate compressor performance curve $\Psi_c\,(\Psi_c)$ and throttle characteristic $\Psi_t\,(\Psi_t)$.
- Determine Helmholtz frequency, ω_h, to be equal to experimental signal frequency.
- Set B parameter to be equal to 0 or critical value of B parameter.
- Increase B until amplitude of simulation is equal to experimental signal amplitude.
- Increase ω_h until frequency of simulation is equal to experimental signal frequency.
- Change throttle constant, k, and model another operation point.

By this method 16 points of unstable operation have been modeled. Results of this modeling are presented below. Precision of the method was measured as relative errors of amplitude and frequency. In Fig. 11 errors for ten points of unstable operation are shown. One can see that errors are very small and that leads to conclusion that presented method is precise and can be used in compressor modeling. In Fig. 12 parameters B and Helmholtz frequencies obtained for different TOA (different operation points) are shown. Values of B and Helmholtz frequency does not vary in wide range and the differences between next values are small. It suggests that is possible that B is constant what corresponds well with theory [11,15].

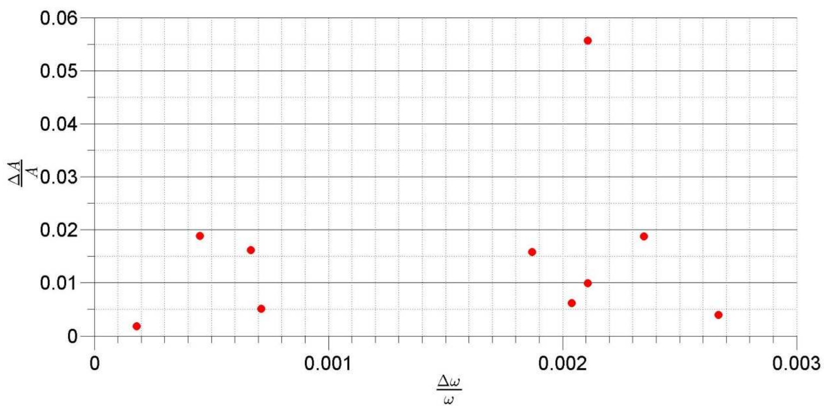

Figure 11: Relative errors of modeled compressor for different operation points. On the abscissa relative error of frequency and on the ordinate relative error of amplitude.

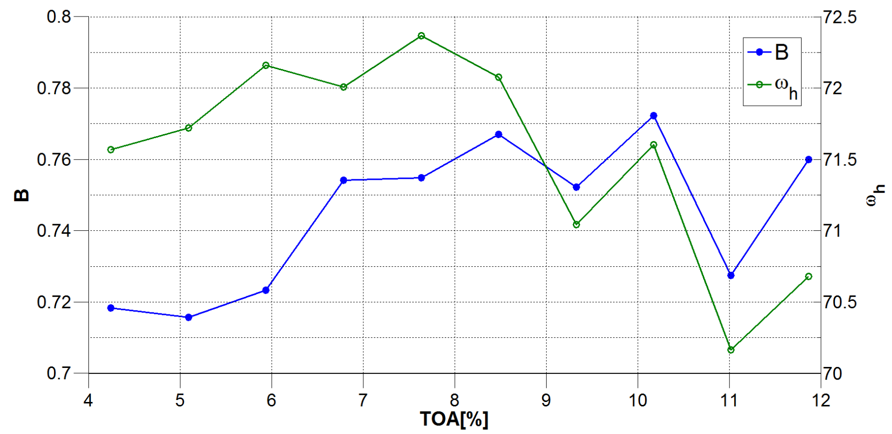

Figure 12: Obtained parameters B and Helmholtz frequencies for different TOA.

5 Conclusions

The influence of parameter B and Helmholtz frequency on two equation Greitzer model was presented. After analysis of this impact, a new model of compressor modeling based on experimental signal of pressure oscillations was proposed and verified.

Two different Helmholtz frequencies were assumed and simulation performed for B parameter varying from 0.001 to 3.5. After this simulations following conclusions were learned Helmholtz frequency impact:

- Helmholtz frequency has no impact on simulation amplitude;
- the higher Helmholtz frequency leads to the higher simulation frequency is;
- simulation frequency is always lower than Helmholtz frequency.

and B parameter impact:

- B parameter has impact on both, frequency and amplitude;
- with increasing parameter B, amplitude sharply rise until it reach peak and then slightly falls. It has the same impact on simulation frequency;
- parameter B is stability parameter and after crossing critical value surge appears.

Based on this conclusions new method of compressor modeling was proposed and verified. Value of biggest error was approximately 5.5% for amplitude and 0.26% for frequency. Therefore this method is very accurate and can be used to model a compressor. It is also much simpler than traditional modeling with Greitzer model. In this method only A_c (pipe across area) parameter is needed but it is ease to designate by outlet surface from rotor. In future it can be used in designing anti surge systems what is general purpose of understanding surge. It is hard to predict where is the limit of this method.

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References

[1] Mirsky S., Moines D., Jacobson W.: *Development and design of antisurge and performance control systems*. In: Proc. 42nd Turbomachinery Symp., Houston, Oct. 1-3, 2012, 2013.

[2] Wu X., Li Y.: *Computationally Efficient Data-Driven Surge Map Modeling for Centrifugal Air Compressors*. In: Proc. American Control Conf. Jul. 2007, 810–815.

[3] Wu X., Li Y.: *Self-learning based centrifugal compressor surge mapping with computationally efficient adaptive asymmetric support vector machine*. J. Dyn. Sys., Meas., Control-T ASME **134**(2012), 5, 051008.

[4] Jager, de, B.: *Rotating stall and surge control: A survey*. In: Decision and Control. Proc. 34th IEEE Conf., Dec. 1995, 1857–1862.

[5] Willems F., Jager, de, B.: *Modeling and Control of Rotating Stall and Surge? An Overview*. Control Applications. In: Proc. IEEE Int. Conf., Sept. 1998, 331–335.

[6] Gravdahl J.T., Egeland O.: *Compressorsurge and rotating stall: Modeling and control*. Springer Verlag, 2011.

[7] Horodko L.: *Application of time-frequency analysis to analyze non stationary operation of the centrifugal blower*. Zeszyty Naukowe. Rozprawy Naukowe Politechniki Łódzkiej, Łódź 2006, 4–114 (in Polish).

[8] Pinsley J.E., Guenette G.R., Epstein A.H., Greitzer E.M.: *Active Stabilization of Centrifugal Compressor Surge*. J. Turbomach. **113**(1991), 4, 723.

[9] Willems F., Jager, de, B.: *Modeling and control of compressor flow instabilities.* IEEE Contr. Sys. Mag. **19**(1999), 5, 8–18.

[10] Mickers R.W.: *Comparative Parameter Studies of Dynamic Models for Centrifugal Compression Systems.* Technische Universiteit Eindhoven, 2006.

[11] Willems F.: *Modeling and Bounded Feedback Stabilization of Centrifugal Compressor Surge.* Technische Universiteit Eindhoven, 2000.

[12] Emmons H.W., Pearson C.E., Grant H.P.: *Compressor surge and stall propagation.* Trans. ASME **77**(1955), 4, 455–469.

[13] Badmus O.O., Chowdhury S., Nett C.N.: *Nonlinear control of surge in axial compression systems.* Automatica **32**(1996), 1, 59–70.

[14] Botros K.K.: *Transient phenomena in compressor stations during surge.* In: Proc. ASME 1992 Int. Gas Turbine and Aeroengine Cong. Expo. 1992, V003T07A003–V003T07A003.

[15] Greitzer E.: *Surge and rotating stall in axial flow compressors—Part I: Theoretical compression system model.* J. Eng. Gas Turbines Power **98**(1976), 190–198.

[16] Hansen K.E., Jorgensen P., Larsen P.S.: *Experimental and theoretical study of surge in a small centrifugal compressor.* J. Fluids Eng. **103**(1981), 3, 391–395.

[17] Fink D.A., Cumpsty N.A., Greitzer E.: *Surge dynamics in a free-spool centrifugal compressor system.* J. Turbomach. **114**(1992), 2, 321–332.

[18] Greitzer E.M., Moore F.K.: *A theory of post-stall transients in axial compression systems. Part IŮDevelopment of equations.* J. Eng. Gas Turbines Power **108**(1986), 1, 68–76.

[19] Haynes J.M., Hendricks G.J., Epstein, A.H.: *Active stabilization of rotating stall in a three-stage axial compressor.* In: Proc. ASME 1993 Int. Gas Turbine and Aeroengine Cong. Expo., 1993, V03CT17A007–V03CT17A007.

[20] Greitzer E.: *Surge and rotating stall in axial flow compressors—Part II: Experimental results and comparison with theory.* J. Eng. Gas Turbines Power **98**(1976), 199–211.

[21] Meuleman C.: *Measurement and Unsteady Flow Modeling of Centrifugal Compressor Surge.* Technische Universiteit Eindhoven, 2002.

[22] Helvoirt, van, J., Jager, de, B.: *Dynamic model including piping acoustics of a centrifugal compression system.* J. Sound Vib. **302**(2007), 1–2, 361–378.

[23] Willems F., De Jager B., de Jager B.: *Active compressor surge control using a one-sided controlled bleed/recycle valve.* In: Proc. IEEE Conf. on Decision and Control **3**(1998), 2546–2551.

[24] Yoon S.Y., Lin Z., Goyne C., Allaire P.E.: *An enhanced Greitzer compressor model including pipeline dynamics and surge.* J. Vib. Acoustics **133**(2011), 5, 051005.

[25] Grapow F.: *Influence of Greitzer parameter on the theoretical centrifugal compressor's work.* 33rd Int. Seminar of the Students' In: Associations Proc., 2014.

[26] Goyne C., Allaire P.E.: *An enhanced Greitzer compressor model with pipeline dynamics included.* In: Proc. American Control Conf. 2011, 4731–4736.

[27] Meuleman C., Willems F., Lange, de, R., Jager,de, B.: *Surge in a low-speed radial compressor.* Int. Gas Turbine and Aeroengine Congress, 1998.

[28] Helvoirt, van, J., Jager, de, B., Steinbuch M., Smeulers J.: *Stability parameter identification for a centrifugal compression system.* In: Proc. 43rd IEEE Conf. on Decision and Control (CDC) (IEEE Cat. No.04CH37601), **4**(2004), 3400–3405.

[29] Helvoirt, van, J., Jager, de, B., Steinbuch M., Smeulers J.: *Modeling and identification of centrifugal compressor dynamics with approximate realizations.* In: Proc. IEEE Conf. on Control Applications, 2005. CCA 2005, 1441–1447.

[30] Liśkiewicz G., Horodko L., Stickland M., Kryłłowicz W.: *Identification of phenomena preceding blower surge by means of pressure spectral maps.* Exp. Thermal Fluid Sci. **54**(2014), 267–278.

[31] Garcia D., Stickland M., Liśkiewicz G.: *Dynamical system analysis of unstable* Open Eng. **5**(2015), 1, 332–342.

[32] Liśkiewicz G., Horodko L.: *Time-frequency analysis of the surge onset in the centrifugal blower.* Open Eng. **5**(2015), 1, 299–306.

[33] Liśkiewicz G.: *Numerical model of the flow phenomena preceding surge in the centrifugal blower and assessment of its applicability in designing anti-surge devices.* Politechnika Łódzka, 2014.

[34] Kuz'min V.A., Khazhuev V.N.: *Measurement of liquid or gas flow (flow velocity) using convergent channels with a witoszynski profile.* Meas. Tech. **36**(1993), 3, 288–296.

[35] Gravdahl J.T., Egeland O.: *A Moore-Greitzer axial compressor model with spool dynamics.* Proc. 36th IEEE Conf. Decision and Control **5**(1997), 4714–4719.

[36] Braembussche, van den, R.: *Centrifugal compressors analysis and design.* **192**(2014).

[37] Hagino N., Uda K., Kashiwabara Y.: *Prediction and Active Control of Surge Inception of a Centrifugal Compressor* 2003.

[38] Ng E., Liu N., Tan S.: *Parametric study of Greitzer's instability flow model through compressor system using the Taguchi method.* Int. J. Rotating Machinery **10**(2004), 2, 91–97.