Kołowrocki Krzysztof

Kuligowska Ewa

Soszyńska-Budny Joanna

Torbicki Mateusz
Gdynia Maritime University, Gdynia, Poland

Oil transport in port
Part 1
Port oil piping transportation system safety and resilience impacted by its operation process

Keywords

port oil piping transportation system, operation impacts, safety, resilience, cost analysis, cost optimization

Abstract

The paper is concerned with the model of critical infrastructure safety prediction with considering its operation process impacts. The general approach to the prediction of critical infrastructure safety and resilience is proposed and the safety and resilience indicators are defined for a critical infrastructure impacted by its operation process. Moreover, there is presented the model application for port oil piping transportation system safety and resilience prediction. Further, the cost analysis of critical infrastructure operation process is proposed and applied to the considered piping system.

1. Introduction

This paper is another part of the series of four papers proposed to comprehensive modelling and prediction of the safety and resilience of critical infrastructures with application to the port oil piping transportation system safety and resilience prediction in the scope of the EU-CIRCLE project Case Study 2, Storm and Sea Surge at Baltic Sea Port.

First, the critical infrastructure operation process is considered, its parameters are introduced and its main characteristics are found. Next, the notions of the safety analysis of critical infrastructure impacted by its operation are introduced, i.e. the critical infrastructure conditional and unconditional safety function and the critical infrastructure risk function are defined.

Moreover, the critical infrastructure and its assets main safety characteristics and indicators are determined, i.e. the mean lifetime and standard deviation in the safety state subset, the intensities of degradation (ageing) and the indicator of critical infrastructure resilience to operation process impact.

Further, the IMCIS Model 1 created in [EU-CIRCLE Report D3.3-Part3, 2017] is applied to the port oil piping transportation system. Safety and resilience indicators are determined for the port oil piping transportation system, the operation cost analysis is performed and optimization of piping operation process is presented.

2. Critical infrastructure safety model related to its operation process – IMCIS 1

In this section, we consider the critical infrastructure related to the operation process $Z(t)$, $t \in < 0,\infty)$, impacted in a various way at its operation states z_b, $b = 1,2,...,v.$ We assume that the changes of the operation states of the critical infrastructure operation process $Z(t)$ have an influence on and the critical infrastructure safety structure and on the safety of the critical infrastructure assets A_i, $i = 1,2,...,n,$ as well [Kołowrocki, Soszyńska-Budny, 2011].

The following critical infrastructure operation process parameters (OPP) can be identified either statistically using the data given at [GMU Safety Interactive Platform] and the methods given in [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D3.3-Part3, 2017] or evaluated approximately by experts:

- the number of operation states (OPP1) ν;
- the vector $[p_b(0)]_{1x\nu}$ of the initial probabilities (OPP2) of the critical infrastructure operation process $Z(t)$ staying at particular operation states z_b at the moment $t = 0$;
- the matrix $[p_{bl}]_{\nu x\nu}$ of probabilities of transition (OPP3) of the critical infrastructure operation process $Z(t)$ between the operation states z_b and z_l;
- the matrix $[M_{bl}]_{\nu x\nu}$ of mean values of conditional sojourn times (OPP4) of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation state z_b when the next state is z_l.

The following critical infrastructure operation process characteristics (OPC) can be either calculated analytically using the above parameters of the operation process or evaluated approximately by experts:

- the vector

$$[p_b]_{1x\nu} = [p_1, p_2, ..., p_\nu] \tag{1}$$

of limit values of transient probabilities (OPC1)

$$p_b(t) = P(Z(t) = z_b),\ t \in <0, +\infty),\ b = 1,2,...,\nu; \tag{2}$$

of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b (in the case of a periodic critical infrastructure operation process, the limit transient probabilities p_b, $b = 1,2,...,\nu$, at the operation states are the long term proportions of the critical infrastructure operation process $Z(t)$ sojourn times at the particular operation states z_b, $b = 1,2,...,\nu$);

- the vector

$$[\hat{M}_b]_{1x\nu} = [\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_\nu] \tag{3}$$

of the mean values of the total sojourn times (OPC2)

$$\hat{M}_b = E[\hat{\theta}_b] = p_b\theta,\ b = 1,2,...,\nu, \tag{4}$$

of the total sojourn times $\hat{\theta}_b$ of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,\nu$, during the fixed critical infrastructure opetation time θ.

2.1. Critical infrastructure safety indicators

We denote the critical infrastructure conditional lifetime in the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, while its operation process $Z(t)$, $t \in <0, \infty)$, is at the operation state z_b, $b = 1,2,...,\nu$, by $[T^1(u)]^{(b)}$, $u = 1,2,...,z$, and the conditional safety function of the critical infrastructure related to the operation process $Z(t)$, $t \in <0, \infty)$, by the vector [EU-CIRCLE Report D3.3-Part3, 2017]

$$[\boldsymbol{S}^1(t,\cdot)]^{(b)} = [1,\ [\boldsymbol{S}^1(t,1)]^{(b)},\ ...,\ [\boldsymbol{S}^1(t,z)]^{(b)}], \tag{5}$$

with the coordinates defined by

$$[\boldsymbol{S}^1(t,u)]^{(b)} = P([T^1(u)]^{(b)} > t | Z(t) = z_b) \tag{6}$$

for $t \in <0, \infty)$, $u = 1,2,...,z$, $b = 1,2,...,\nu$.

The safety function $[\boldsymbol{S}^1(t,u)]^{(b)}$, $u = 1,2,...,z$, is the conditional probability that the critical infrastructure related to the operation process $Z(t)$, $t \in <0, \infty)$, lifetime $[T^1(u)]^{(b)}$, $u = 1,2,...,z$, in the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, is greater than t, while the critical infrastructure operation process $Z(t)$, $t \in <0, \infty)$, is at the operation state z_b.

Next, we denote the critical infrastructure related to the operation process $Z(t)$, $t \in <0, \infty)$, unconditional lifetime in the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, by $T^1(u)$, $u = 1,2,...,z$, and the unconditional safety function (SafI1) of the critical infrastructure related to the operation process $Z(t)$, $t \in <0, \infty)$, by the vector

$$\boldsymbol{S}^1(t,\cdot) = [1,\ \boldsymbol{S}^1(t,1) ...,\ \boldsymbol{S}^1(t,z)], \tag{7}$$

with the coordinates defined by

$$\boldsymbol{S}^1(t,u) = P(T^1(u) > t) \tag{8}$$

for $t \in <0, \infty)$, $u = 1,2,...,z$.

In the case when the system operation time θ is large enough, the coordinates of the unconditional safety function of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, defined by (8), are given by

$$\boldsymbol{S}^1(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{S}^1(t,u)]^{(b)},\ t \ge 0\ ,u = 1,2,...,z, \quad (9)$$

where $[\boldsymbol{S}^1(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, are the coordinates of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, conditional safety functions defined by (5)-(6) and p_b, $b = 1,2,...,\nu$, are the critical infrastructure operation process $Z(t)$, $t \in <0,\infty)$, limit transient probabilities at the operation states z_b, $b = 1,2,...,\nu$, given by (1)-(2).

If r is the critical safety state, then the second safety indicator of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, the risk function (SafI2)

$$\boldsymbol{r}^1(t) = P(s(t) < r \mid s(0) = z) = P(T^1(r) \le t),$$
$$t \in <0,\infty), \quad (10)$$

is defined as a probability that the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the best safety state z at the moment $t = 0$ and given by [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D3.3-Part3, 2017]

$$\boldsymbol{r}^1(t) = 1 - \boldsymbol{S}^1(t,r),\ t \in <0,\infty), \quad (11)$$

where $\boldsymbol{S}^1(t,r)$ is the coordinate of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, unconditional safety function given by (9) for $u = r$.

The graph of the critical infrastructure risk function $\boldsymbol{r}^1(t)$, $t \in <0,\infty)$, defined by (11), is the safety indicator called the fragility curve (SafI3) of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$.

Other practically useful safety indicators of the critical infrastructure related to the operation process $Z(t)$, $t \in <0,\infty)$, are:

– the mean value of the critical infrastructure unconditional lifetime $T^1(r)$ up to exceeding critical safety state r (SafI4) given by

$$\boldsymbol{\mu}^1(r) = \int_0^\infty [\boldsymbol{S}^1(t,r)]dt \cong \sum_{b=1}^{\nu} p_b [\mu^1(r)]^{(b)}, \quad (12)$$

where $[\mu^1(r)]^{(b)}$ are the mean values of the critical infrastructure conditional lifetimes $[T^1(r)]^{(b)}$ in the safety state subset $\{r, r+1,...,z\}$ at the operation state z_b, $b = 1,2,...,\nu$, given by

$$[\mu^1(r)]^{(b)} = \int_0^\infty [\boldsymbol{S}^1(t,r)]^{(b)} dt,\ b = 1,2,...,\nu, \quad (13)$$

and $[\boldsymbol{S}(t,r)]^{(b)}$, $b = 1,2,...,\nu$, are defined by (5)-(6) and p_b are given by (2),

– the standard deviation of the critical infrastructure lifetime $T^1(r)$ up to the exceeding the critical safety state r (SafI5) given by

$$\boldsymbol{\sigma}^1(r) = \sqrt{n^1(r) - [\boldsymbol{\mu}^1(r)]^2}\ , \quad (14)$$

where

$$n^1(r) = 2\int_0^\infty t\, \boldsymbol{S}^1(t,r)dt, \quad (15)$$

and $\boldsymbol{S}^1(t,r)$ is defined by (8) for $u = r$ and $\boldsymbol{\mu}^1(r)$ is given by (12);

– the moment τ^1 of exceeding acceptable value of critical infrastructure risk function level δ (SafI6) given by

$$\tau^1 = \boldsymbol{r}^{1\,-1}(\delta), \quad (16)$$

where $\boldsymbol{r}^{1\,-1}(\delta)$ is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (10);

– the intensities of degradation of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, (SafI9), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^1(t,\cdot) = [0,\ \boldsymbol{\lambda}^1(t,1),\ \ldots,\ \boldsymbol{\lambda}^1(t,z)\ ],$$
$$t \in <0,+\infty), \quad (17)$$

where

$$\boldsymbol{\lambda}^1(t,u) = \frac{-\dfrac{d\boldsymbol{S}^1(t,u)}{dt}}{\boldsymbol{S}^1(t,u)},\ t \in <0,+\infty),$$
$$u = 1,2,...,z; \quad (18)$$

– the coefficients of operation process impact on the critical infrastructure intensities of degradation (the coefficients of operation process impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, ..., z\}$) (SafI10), i.e. the coordinates of the vector

$$\boldsymbol{\rho}^1(t,\cdot) = [0,\ \boldsymbol{\rho}^1(t,1), \ldots, \boldsymbol{\rho}^1(t,z)],$$

$$t \in <0,+\infty), \tag{19}$$

where

$$\boldsymbol{\lambda}^1(t,u) = \boldsymbol{\rho}^1(t,u) \cdot \boldsymbol{\lambda}^{\mathbf{0}}(t,u),\ t \in <0,+\infty),$$
$$u = 1,2,...,z, \tag{20}$$

i.e.

$$\boldsymbol{\rho}^1(t,u) = \frac{\boldsymbol{\lambda}^1(t,u)}{\boldsymbol{\lambda}^0(t,u)},\ t \in <0,+\infty),$$
$$u = 1,2,...,z, \tag{21}$$

and $\boldsymbol{\lambda}^{\mathbf{0}}(t,u)$, $t \in <0,+\infty)$, $u = 1,2,...,z,$ are the intensities of degradation of the critical infrastructure without of operation process impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^{\mathbf{0}}(t,\cdot) = [0,\ \boldsymbol{\lambda}^{\mathbf{0}}(t,1),, \ldots, \boldsymbol{\lambda}^{\mathbf{0}}(t,z)],$$
$$t \in <0,+\infty), \tag{22}$$

and $\boldsymbol{\lambda}^1(t,u)$, $t \in <0,+\infty)$, $u = 1,2,...,z,$ are the intensities of degradation of the critical infrastructure with of operation process impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^1(t,\cdot) = [0,\ \boldsymbol{\lambda}^1(t,1), \ldots, \boldsymbol{\lambda}^1(t,z)\ ],$$
$$t \in <0,+\infty), \tag{23}$$

– the indicator of critical infrastructure resilience to operation process impact (ResI1) defined by

$$\boldsymbol{RI}^1(t,r) = \frac{1}{\rho^1(t,r)},\ t \in <0,+\infty), \tag{24}$$

where $\boldsymbol{\rho}^1(t,r)$, $t \in <0,+\infty)$, is the coefficients of operation process impact on the critical infrastructure intensities of degradation given by (20) for $u = r$.

Further, we also will use the following critical infrastructure safety characteristics:

– the mean lifetime of the critical infrastructure in the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z,$ given by

$$\boldsymbol{\mu}^1(u) = \int_0^\infty [\boldsymbol{S}^1(t,u)]dt \cong \sum_{b=1}^{\nu} p_b [\mu^1(u)]^{(b)},$$
$$u = 1,2,...,z, \tag{25}$$

where $[\mu^1(u)]^{(b)}$ are the mean values of the critical infrastructure conditional lifetimes $[T^1(u)]^{(b)}$ in the safety state subset $\{u, u+1, ..., z\}$ at the operation state z_b, $b = 1,2,...,\nu,$ given by

$$[\mu^1(u)]^{(b)} = \int_0^\infty [\boldsymbol{S}^1(t,u)]^{(b)} dt,\ u = 1,2,...,z,$$
$$b = 1,2,...,\nu, \tag{26}$$

and $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,...,z,$ $b = 1,2,...,\nu,$ are defined by (5)-(6) and p_b are given by (1)-(2);

– the standard deviation of the critical infrastructure lifetime in the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z,$ given by

$$\boldsymbol{\sigma}^1(u) = \sqrt{n^1(u) - [\boldsymbol{\mu}^1(u)]^2},\ u = 1,2,...,z, \tag{27}$$

where

$$n^1(u) = 2\int_0^\infty t\boldsymbol{S}^1(t,u)dt,\ u = 1,2,...,z, \tag{28}$$

– the mean lifetimes $\bar{\boldsymbol{\mu}}^1(u)$, $u = 1,2,...,z,$ of the critical infrastructure in the particular safety states

$$\bar{\boldsymbol{\mu}}^1(u) = \boldsymbol{\mu}^1(u) - \boldsymbol{\mu}^1(u+1),\ u = 0,1,...,z-1,$$
$$\bar{\boldsymbol{\mu}}^1(z) = \boldsymbol{\mu}^1(z). \tag{29}$$

2.2. Critical infrastructure assets safety parameters

We denote the critical infrastructure asset A_i, $i = 1,2,...,n,$ conditional lifetime in the safety state subset $\{u, u+1, ..., z\}$ while the critical infrastructure is at the operation state z_b, $b = 1,2,...,\nu,$ by $[T_i^1(u)]^{(b)}$ and its conditional safety function (SafI1) by the vector [EU-CIRCLE Report D3.3-Part3, 2017]

$$[S_i^1(t,\cdot)]^{(b)} = [1,\ [S_i^1(t,1)]^{(b)}, \ldots,\ [S_i^1(t,z)]^{(b)}\ ],$$
$$t \in <0,\infty),\ \ b = 1,2,\ldots,v,\ \ i = 1,2,\ldots,n, \qquad (30)$$

with the coordinates defined by

$$[S_i^1(t,u)]^{(b)} = P([T_i^1(u)]^{(b)} > t | Z(t) = z_b) \qquad (31)$$

for $t \in <0,\infty)$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, $i = 1,2,\ldots,n$.
The safety function $[S_i^1(t,u)]^{(b)}$ is the conditional probability that the asset A_i lifetime $[T_i^1(u)]^{(b)}$ in the safety state subset $\{u,u+1,\ldots,z\}$ is greater than t, while the critical infrastructure is at the operation state z_b, $b = 1,2,\ldots,v$.
The conditional safety functions $[S_i^1(t,u)]^{(b)}$, $t \in <0,\infty)$, u = 1, 2, ..., z, $b = 1,2,\ldots,v$, i = 1,2,...,n, defined by (31) are called the coordinates of the asset A_i, i = 1,2,...,n, conditional safety function $[S_i^1(t,\cdot)]^{(b)}$, $t \in <0,\infty)$, $b = 1,2,\ldots,v$, i = 1,2,...,n, while the critical infrastructure operation process $Z(t)$ is at the operation state z_b, $b = 1,2,\ldots,v$, given by (30). Thus, the relationship between the conditional distribution function $[F_i^1(t,u)]^{(b)}$, $t \in <0,\infty)$, u = 1, 2, ..., z, $b = 1,2,\ldots,v$, i = 1,2,...,n, of the asset A_i, i = 1,2,...,n, lifetime $[T_i^1(u)]^{(b)}$, u = 1, 2, ..., z, $b = 1,2,\ldots,v$, i = 1,2,...,n, in the safety state subset $\{u,u+1,\ldots,z\}$, u = 1,2,...,z, and the coordinate $[S_i^1(t,u)]^{(b)}$, $t \in <0,\infty)$, u = 1, 2, ..., z, $b = 1,2,\ldots,v$, i = 1,2,...,n, of its conditional safety function is given by

$$\begin{aligned}[F_i^1(t,u)]^{(b)} &= P([T_i^1(u)]^{(b)} \le t), \\ &= 1 - P([T_i^1(u)]^{(b)} > t) \\ &= 1 - [S_i^1(t,u)]^{(b)},\ \ t \in <0,\infty),\end{aligned}$$
$$u = 1, 2, \ldots, z,\ b = 1,2,\ldots,v,\ i = 1,2,\ldots,n. \qquad (32)$$

Thus, the function

$$[r_i^1(t)]^{(b)} = 1 - [S_i^1(t,r)]^{(b)},\ \ t \in <0,\infty),\ \ b = 1,2,\ldots,v,$$
$$i = 1,2,\ldots,n, \qquad (33)$$

is the asset A_i, i = 1,2,...,n, the conditional risk function (SafI2) and its graph is the asset A_i, i = 1,2,...,n, fragility curve (SafI3) while the critical infrastructure is at the operation state z_b, $b = 1,2,\ldots,v$.
Moreover, the conditional mean lifetime of the asset A_i in the safety state subset $\{u,u+1,\ldots,z\}$, u = 1,2,...,z, while the critical infrastructure is at the operation state z_b, $b = 1,2,\ldots,v$, is given by

$$[\mu_i^1(u)]^{(b)} = \int_0^\infty [S_i^1(t,u)]^{(b)} dt,\ u = 1, 2, \ldots, z,$$
$$b = 1,2,\ldots,v,\ i = 1,2,\ldots,n. \qquad (34)$$

In the case, when the critical infrastructure assets A_i, $i = 1,2,\ldots,n$, at the critical infrastructure operation process $Z(t)$ states z_b, $b = 1,2,\ldots,v$, have the exponential safety functions, the coordinates (31) of the vector (30) are given by

$$\begin{aligned}[S_i^1(t,u)]^{(b)} &= P([T_i^1(u)]^{(b)} > t | Z(t) = z_b) \\ &= \exp[-[\lambda_i^1(u)]^{(b)} t],\ t \in <0,\infty),\end{aligned}$$
$$u = 1,2,\ldots,z,\ b = 1,2,\ldots,v,\ i = 1,2,\ldots,n. \qquad (35)$$

Existing in (35) the intensities of degradation of the critical infrastructure asset A_i, $i = 1,2,\ldots,n$, with the critical infrastructure operation process impact at the critical infrastructure operation states z_b, $b = 1,2,\ldots,v$, (SafI7), i.e. the coordinates of the vector

$$[\lambda_i^1(\cdot)]^{(b)} = [0,\ [\lambda_i^1(1)]^{(b)}, \ldots, [\lambda_i^1(z)]^{(b)}],\ t \in <0,+\infty),$$
$$b = 1,2,\ldots,v,\ i = 1,2,\ldots,n, \qquad (36)$$

are constant and given by

$$[\lambda_i^1(u)]^{(b)} = \frac{1}{[\mu_i^1(u)]^{(b)}},\ u = 1,2,\ldots,z,\ b = 1,2,\ldots,v,$$
$$i = 1,2,\ldots,n, \qquad (37)$$

and moreover

$$[\lambda_i^1(u)]^{(b)} = [\rho_i^1(u)]^{(b)} \cdot \lambda_i^0(u),\ u = 1,2,\ldots,z,$$
$$b = 1,2,\ldots,v,\ i = 1,2,\ldots,n, \qquad (38)$$

where $\lambda_i^0(u)$ are the intensities of degradation of the critical infrastructure asset A_i, $i = 1,2,\ldots,n$, without operation process impact (SafI7), i.e. the coordinate of the vector

$$\lambda_i^0(\cdot) = [0,\ \lambda_i^0(1), \ldots, \lambda_i^0(z)\ ],\ i = 1,2,\ldots,n, \qquad (39)$$

and $[\rho_i^1(u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, i = 1,2,...,n, are the coefficients of operation process impact on the critical infrastructure asset A_i, i = 1,2,...,n, intensities of degradation at the critical infrastructure operation states z_b, $b = 1,2,...,\nu$, (SafI8), i.e. the coordinate of the vector

$$[\rho_i^1(\cdot)]^{(b)} = [0,\ [\rho_i^1(1)]^{(b)},\ \ldots, [\rho_i^1(z)]^{(b)}\ ],$$
$$b = 1,2,...,\nu,\ \ i = 1,2,...,n. \quad (40)$$

where by (38)

$$[\rho_i^1(u)]^{(b)} = \frac{[\lambda_i^1(u)]^{(b)}}{\lambda_i^0(u)} = \frac{\mu_i^0(u)}{[\mu_i^1(u)]^{(b)}},\ \ u = 1,2,..., z,$$
$$b = 1,2,...,\nu,\ \ i = 1,2,...,n, \quad (41)$$

3. IMCIS 1 application to safety of port oil piping transportation system evaluation

In this section, we consider the port oil piping transportation system impacted by its operation process.

3.1. Parameters and characteristics of port oil piping transportation system operation process

On the basis of the statistical data and expert opinions, it is possible to fix and to evaluate the following unknown basic parameters of the port oil piping transportation system operation process [GMU Interactive Safety Platform]:

- the number of operation process states (OPP1) ν = 7 and the operation process states:
- the operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines of the subsystem S_3 illustrated in *Figure 1*;
- the operation state z_2 – transport of one kind of medium from the terminal part C to part B using two out of three pipelines of the subsystem S_3 illustrated in *Figure 1*;
- the operation state z_3 – transport of one kind of medium from the terminal part B through part A to pier using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 illustrated in *Figure 2*;
- the operation state z_4 – transport of one kind of medium from the pier through parts A and B to part C using one out of two pipelines of the subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines of the subsystem S_3 illustrated in *Figure 3*;
- the operation state z_5 – transport of one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 illustrated in *Figure 2*;
- the operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines of the subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 illustrated in *Figure 3*;
- the operation state z_7 – transport of one kind of medium from the terminal part B to C using one out of three pipelines of the subsystem S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines of the subsystem S_3 illustrated in *Figure 1*.

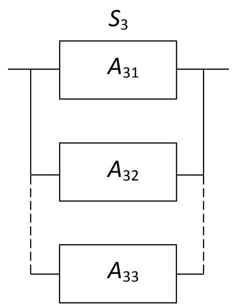

Figure 1. The scheme of the piping transportation system at the operation states z_1, z_2 and z_7

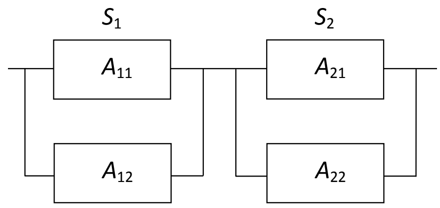

Figure 2. The scheme of piping transportation system at the operation state z_3 and z_5

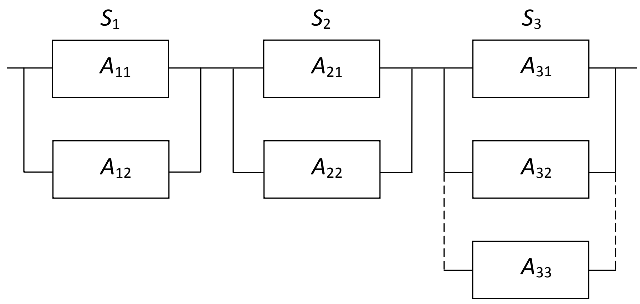

Figure 3. The scheme of the piping transportation system at the operation state z_4 and z_6

The port oil piping transportation system operation process $Z(t)$ characteristics, determined on the basis of the port oil piping transportation system operation process data given in [GMU Safety Interactive Platform], are:

– the limit values of transient probabilities (OPC1) of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,7$:

$$p_1 = 0.403,\ p_2 = 0.055,\ p_3 = 0.003,\ p_4 = 0.002,\ p_5 = 0.199,\ p_6 = 0.057,\ p_7 = 0.281; \quad (42)$$

– the expected values of the total sojourn times $\hat{\theta}_b$, $b = 1,2,...,7$, (OPC2) of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,7$, during the fixed operation time $\theta = 1$ year = 365 days:

$$\hat{M}_1 = E[\hat{\theta}_1] = 0.403 \text{ year} = 147.10 \text{ days},$$
$$\hat{M}_2 = E[\hat{\theta}_2] = 0.055 \text{ year} = 20.07 \text{ days},$$
$$\hat{M}_3 = E[\hat{\theta}_3] = 0.003 \text{ year} = 1.09 \text{ day},$$
$$\hat{M}_4 = E[\hat{\theta}_4] = 0.002 \text{ year} = 0.73 \text{ day},$$
$$\hat{M}_5 = E[\hat{\theta}_5] = 0.199 \text{ year} = 72.64 \text{ days},$$
$$\hat{M}_6 = E[\hat{\theta}_6] = 0.057 \text{ year} = 20.80 \text{ days},$$
$$\hat{M}_7 = E[\hat{\theta}_7] = 0.281 \text{ year} = 102.57 \text{ days}. \quad (43)$$

3.2. Parameters of operation process impact on port oil piping transportation system safety

The coefficients of the operation process impact on the port oil piping transportation system intensities of ageing at the operation states z_b, $b = 1,2,...,7$, are as follows [GMU Interactive Safety Platform] for the assets A_{ij}, $i = 1,2$, $j = 1,2$, $i = 3$, $j = 1,2,3$:

$$[\rho_{ij}^1(1)]^{(b)} = 1.00,\ [\rho_{ij}^1(2)]^{(b)} = 1.00,\ b = 1,2,7,\ i = 1,2,\ j = 1,2, \quad (44)$$

$$[\rho_{ij}^1(1)]^{(b)} = 1.20,\ [\rho_{ij}^1(2)]^{(b)} = 1.20,\ b = 3,4,5,6,\ i = 1,2,\ j = 1,2, \quad (45)$$

$$[\rho_{ij}^1(1)]^{(b)} = 1.00,\ [\rho_{ij}^1(2)]^{(b)} = 1.00,\ b = 3,5,\ i = 3,\ j = 1,2,3, \quad (46)$$

$$[\rho_{ij}^1(1)]^{(b)} = 1.20,\ [\rho_{ij}^1(2)]^{(b)} = 1.20,\ b = 1,2,4,6,7,\ i = 3,\ j = 1,2,3 \quad (47)$$

3.3. Safety parameters of port oil piping transportation system assets impacted by its operation process

Since according to (38), we have

$$[\lambda_{ij}^1(u)]^{(b)} = [\rho_{ij}^1(u)]^{(b)} \cdot \lambda_{ij}^0(u),\ u = 1,2,\ b = 1,2,...,7,\ i = 1,2,\ j = 1,2;\ i = 3,\ j = 1,2,3, \quad (48)$$

then applying the above formula to the parameters defined in [EU-CIRCLE Report for D6.4-Part 0, 2017] and (44)-(47), we get the intensities of ageing of the critical infrastructure assets A_{ij}, $i = 1,2$, $j = 1,2$, $i = 3$, $j = 1,2,3$, / the intensities of critical infrastructure assets A_{ij}, $i = 1,2$, $j = 1,2$, $i = 3$, $j = 1,2,3$, departure from the safety state subset $\{1,2\}$ and $\{2\}$ impacted by the port oil piping transportation system operation process, i.e. the coordinates of the vector

$$[\lambda_{ij}^1(\cdot)]^{(b)} = [0,\ [\lambda_{ij}^1(1)]^{(b)}, [\lambda_{ij}^1(2)]^{(b)}],\ i = 1,2,\ j = 1,2;\ i = 3,\ j = 1,2,3, \quad (49)$$

follows:

– the intensities of departure of the asset A_{11} and A_{12}
  - for safety state subset $\{1,2\}$

$$[\lambda_{11}^1(1)]^{(b)} = [\lambda_{12}^1(1)]^{(b)} = 0.00362,\ b = 1,2,7,$$
$$[\lambda_{11}^1(1)]^{(b)} = [\lambda_{12}^1(1)]^{(b)} = 0.004344,\ b = 3,4,5,6,$$

  - for safety state subset $\{2\}$

$$[\lambda_{11}^1(2)]^{(b)} = [\lambda_{12}^1(2)]^{(b)} = 0.00540,\ b = 1,2,7,$$
$$[\lambda_{11}^1(2)]^{(b)} = [\lambda_{12}^1(2)]^{(b)} = 0.00648,\ b = 3,4,5,6;$$

– the intensities of departure of the asset A_{21} and A_{22}
  - for safety state subset $\{1,2\}$

$$[\lambda_{21}^1(1)]^{(b)} = [\lambda_{22}^1(1)]^{(b)} = 0.01444,\ b = 1,2,7,$$
$$[\lambda_{21}^1(1)]^{(b)} = [\lambda_{22}^1(1)]^{(b)} = 0.017328,\ b = 3,4,5,6,$$

  - for safety state subset $\{2\}$

$$[\lambda_{21}^1(2)]^{(b)} = [\lambda_{22}^1(2)]^{(b)} = 0.02163,\ b = 1,2,7;$$
$$[\lambda_{21}^1(2)]^{(b)} = [\lambda_{22}^1(2)]^{(b)} = 0.025956,\ b = 3,4,5,6;$$

– the intensities of departure of the assets A_{31} and A_{32}
  - for safety state subset $\{1,2\}$

$[\lambda^1_{31}(1)]^{(b)} = [\lambda^1_{32}(1)]^{(b)} = 0.00730$, $b = 3,5$,
$[\lambda^1_{31}(1)]^{(b)} = [\lambda^1_{32}(1)]^{(b)} = 0.00876$,
$b = 1,2,4,6,7$,

- for safety state subset {2}

$[\lambda^0_{32}(2)]^{(b)} = [\lambda^1_{31}(2)]^{(b)} = 0.00912$, $b = 3,5$;
$[\lambda^1_{31}(2)]^{(b)} = [\lambda^0_{32}(2)]^{(b)} = 0.010944$,
$b = 1,2,4,6,7$;

– the intensities of departure of the asset A_{33}
  - for safety state subset {1,2}

$[\lambda^1_{33}(1)]^{(b)} = 0.00874$, $b = 3,5$,
$[\lambda^1_{33}(1)]^{(b)} = 0.010488$, $b = 1,2,4,6,7$,

- for safety state subset {2}

$[\lambda^1_{33}(2)]^{(b)} = 0.00984$, $b = 3,5$,
$[\lambda^1_{33}(2)]^{(b)} = 0.011808$, $b = 1,2,4,6,7$.

3.4. Characteristics of port oil piping transportation system safety impacted by its operation process

After applying formulae for the safety function of the "m_i out of l_i"-series critical infrastructure from [EU-CIRCLE Report D3.3-Part 3, 2017], we get the safety function of the port oil piping transportation system

$$\boldsymbol{S}^1(t,\cdot) = [1, \boldsymbol{S}^1(t,1), \boldsymbol{S}^1(t,2)], \; t \geq 0,$$

where

$$\begin{aligned}\boldsymbol{S}^1(t,1) = {} & 2.956\exp[-0.035630t] \\ & + 5.912\exp[-0.037394t] - 5.912\exp[-0.046154t] \\ & - 1.478\exp[-0.050120t] - 2.956\exp[-0.051884t] \\ & + 2.956\exp[-0.060644t] - 1.478\exp[-0.039250t] \\ & - 2.956\exp[-0.041014t] + 2.956\exp[-0.049774t] \\ & + 0.739\exp[-0.053740t] + 1.478\exp[-0.055504t] \\ & - 1.478\exp[-0.064264t] + 0.808\exp[-0.036332t] \\ & - 1.616\exp[-0.045102t] + 1.616\exp[-0.037802t] \\ & - 0.404\exp[-0.053720t] + 0.808\exp[-0.062490t] \\ & - 0.808\exp[-0.055190t] - 0.404\exp[-0.040676t] \\ & + 0.808\exp[-0.049446t] - 0.808\exp[-0.042146t] \\ & + 0.202\exp[-0.058064t] - 0.404\exp[-0.066834t] \\ & + 0.404\exp[-0.059534t] + 0.236\exp[-0.039252t] \\ & + 0.472\exp[-0.041016t] - 0.472\exp[-0.049776t] \\ & - 0.118\exp[-0.056640t] - 0.236\exp[-0.058404t] \\ & + 0.236\exp[-0.067164t] - 0.118\exp[-0.043596t] \\ & - 0.236\exp[-0.045360t] + 0.236\exp[-0.054120t] \\ & + 0.059\exp[-0.060984t] + 0.118\exp[-0.062748t] \\ & - 0.118\exp[-0.071508t], \; t \geq 0, \end{aligned} \tag{50}$$

$$\begin{aligned}\boldsymbol{S}^1(t,2) = {} & 2.956\exp[-0.048966t] \\ & + 5.912\exp[-0.049818t] - 5.912\exp[-0.060726t] \\ & - 1.478\exp[-0.070706t] - 2.956\exp[-0.071558t] \\ & + 2.956\exp[-0.082466t] - 1.478\exp[-0.054376t] \\ & - 2.956\exp[-0.055228t] + 2.956\exp[-0.066136t] \\ & + 0.739\exp[-0.076116t] + 1.478\exp[-0.076968t] \\ & - 1.478\exp[-0.087876t] + 0.808\exp[-0.050760t] \\ & - 1.616\exp[-0.060560t] + 1.616\exp[-0.051470t] \\ & - 0.404\exp[-0.076848t] + 0.808\exp[-0.086648t] \\ & - 0.808\exp[-0.077558t] - 0.404\exp[-0.057252t] \\ & + 0.808\exp[-0.067052t] - 0.808\exp[-0.057962t] \\ & + 0.202\exp[-0.083340t] - 0.404\exp[-0.093140t] \\ & + 0.404\exp[-0.084050t] + 0.236\exp[-0.054396t] \\ & + 0.472\exp[-0.055248t] - 0.472\exp[-0.066156t] \\ & - 0.118\exp[-0.080484t] - 0.236\exp[-0.081336t] \\ & + 0.236\exp[-0.092244t] - 0.118\exp[-0.060888t] \\ & - 0.236\exp[-0.061740t] + 0.236\exp[-0.072648t] \\ & + 0.059\exp[-0.086976t] + 0.118\exp[-0.087828t] \\ & - 0.118\exp[-0.098736t], \; t \geq 0. \end{aligned} \tag{51}$$

The graph of the safety function of the port oil piping transportation system is given in *Figure 4*.

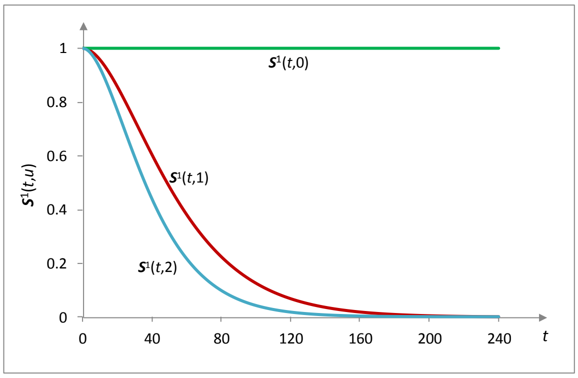

Figure 4. The graphs of the port oil piping transportation system safety function coordinates

According to (26), the conditional expected values of the port oil piping transportation system are:
– in the safety state subset {1,2}:

$$[\boldsymbol{\mu}^1(1)]^{(1)} = 57.229757, \; [\boldsymbol{\mu}^1(1)]^{(2)} = 57.229757,$$
$$[\boldsymbol{\mu}^1(1)]^{(3)} = 56.363181, \; [\boldsymbol{\mu}^1(1)]^{(4)} = 52.140982,$$
$$[\boldsymbol{\mu}^1(1)]^{(5)} = 56.363181, \; [\boldsymbol{\mu}^1(1)]^{(6)} = 52.140982,$$
$$[\boldsymbol{\mu}^1(1)]^{(7)} = 57.229757, \tag{52}$$

– in the safety state subset {2}:

$$[\boldsymbol{\mu}^1(2)]^{(1)} = 42.491358, \; [\boldsymbol{\mu}^1(2)]^{(2)} = 42.491358,$$
$$[\boldsymbol{\mu}^1(2)]^{(3)} = 40.728766, \; [\boldsymbol{\mu}^1(2)]^{(4)} = 38.183197,$$
$$[\boldsymbol{\mu}^1(2)]^{(5)} = 40.728766, \; [\boldsymbol{\mu}^1(2)]^{(6)} = 38.183197,$$
$$[\boldsymbol{\mu}^1(2)]^{(7)} = 42.491358. \tag{53}$$

After applying (25) and (13)-(15) to (42) and (52) and (53), the mean values and standard deviations of the unconditional lifetimes of the port oil piping transportation system are:

– in the safety state subset: {1,2}

$$\boldsymbol{\mu}^1(1) \cong \sum_{b=1}^{7} p_b[\boldsymbol{\mu}^1(1)]^{(b)} = 0.403 \cdot 57.229757$$
$$+ 0.055 \cdot 57.229757 + 0.003 \cdot 56.363181$$
$$+ 0.002 \cdot 52.140982 + 0.199 \cdot 56.363181$$
$$+ 0.057 \cdot 52.140982 + 0.281 \cdot 57.229757$$
$$= 56.7545 \text{ years}, \qquad (54)$$

$$\boldsymbol{\sigma}^1(1) = 38.0357 \text{ years},$$

– in the safety state subset {2}

$$\boldsymbol{\mu}^1(2) \cong \sum_{b=1}^{7} p_b[\boldsymbol{\mu}^1(2)]^{(b)} = 0.403 \cdot 42.491358$$
$$+ 0.055 \cdot 42.491358 + 0.003 \cdot 40.728766$$
$$+ 0.002 \cdot 38.183197 + 0.199 \cdot 40.728766$$
$$+ 0.057 \cdot 38.183197 + 0.281 \cdot 42.491358$$
$$= 41.8811 \text{ years}, \qquad (55)$$

$$\boldsymbol{\sigma}^1(2) = 28.1014 \text{ years}.$$

From (54)-(55), applying (16), the mean lifetimes $\bar{\mu}^1(u)$, $u = 1,2$, of the port oil piping transportation system in the particular safety states are:

$$\bar{\boldsymbol{\mu}}^1(1) = \boldsymbol{\mu}^1(1) - \boldsymbol{\mu}^1(2) = 14.8734 \text{ years},$$
$$\bar{\boldsymbol{\mu}}^1(2) = \boldsymbol{\mu}^1(2) = 41.8811 \text{ years}. \qquad (56)$$

As the critical safety state is $r = 1$, then by (4), the port oil Piping transportation system risk function is

$$\boldsymbol{r}^1(t) = 1 - \boldsymbol{S}^1(t, 1), \qquad (57)$$

where $\boldsymbol{S}^1(t, 1)$ is given by (50). By (8), the moment τ^1 of exceeding acceptable value of critical infrastructure risk function level $\delta = 0.05$ is

$$\tau^1 = (\boldsymbol{r}^1)^{-1}(0.05) = 10.9913 \text{ years}. \qquad (58)$$

The graph of the port oil piping transportation system risk function is presented in *Figure 5*.

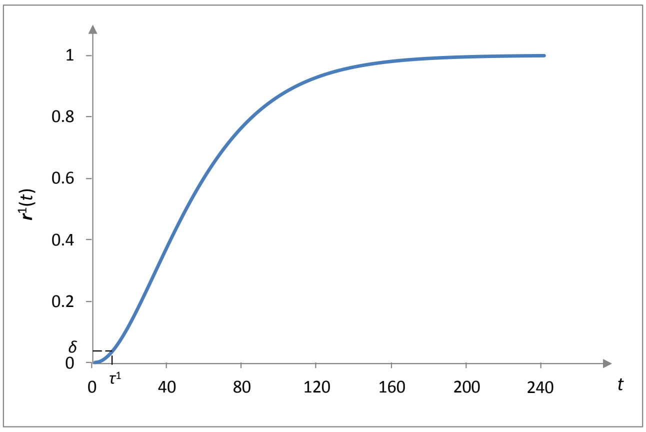

Figure 5. The graph of the port oil piping transportation system risk function

The intensities of degradation (ageing) of the port oil piping transportation system / the intensities the port oil piping transportation system departure from the safety state subset $\{1,2\}$, $\{2\}$, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^1(t,\cdot) = [0, \boldsymbol{\lambda}^1(t,1), \boldsymbol{\lambda}^0(t,2)], \ t \in <0,+\infty),$$
(59)

where

$$\boldsymbol{\lambda}^1(t,u) = \frac{-\frac{d\boldsymbol{S}^1(t,u)}{dt}}{\boldsymbol{S}^1(t,u)}, \ u = 1,2, \ t \in <0,+\infty), \qquad (60)$$

and $\boldsymbol{S}^1(t, u)$, $u = 1,2$, are given by (50)-(51)
The values of the intensities of degradation given by (60) stabilize for large time and approximately amounts

$$\boldsymbol{\lambda}^1(1) = \lim_{t \to +\infty} \boldsymbol{\lambda}^1(t,1) \cong 0.035630,$$
$$\boldsymbol{\lambda}^1(2) = \lim_{t \to +\infty} \boldsymbol{\lambda}^1(t,2) \cong 0.048966. \qquad (61)$$

The graphs of the intensities of degradation of the port oil piping transportation system are given in *Figure 6*.
According to (21) and (24), considering (4.42) from [EU-CIRCLE Report for D6.4-Part 0, 2017] and (61), the limit value of the indicator of critical infrastructure resilience to operation process impact is given by

$$\boldsymbol{RI}^1(1) = \lim_{t \to +\infty} \boldsymbol{RI}^1(t,1) = \lim_{t \to =\infty} \frac{\boldsymbol{\lambda}^0(t,1)}{\boldsymbol{\lambda}^1(t,1)}$$
$$\cong 0.03271/0.035630 \cong 0.92 = 92\%. \qquad (62)$$

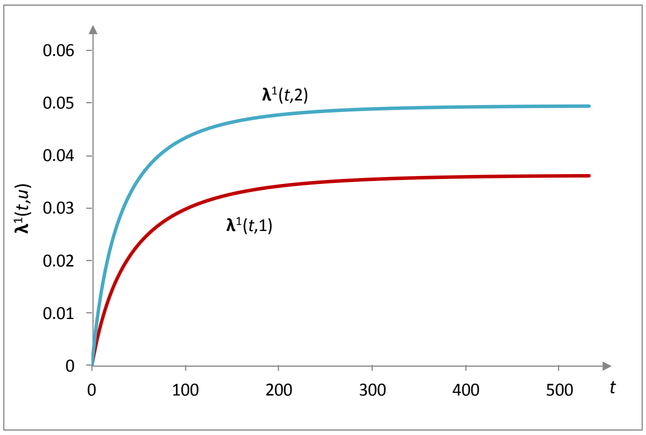

Figure 6. The graphs of the intensities of ageing of the port oil piping transportation system

If we replace in the above formula the intensities of degradation by the appropriate mean values, assuming

$$\boldsymbol{\lambda}^0(t,1) \cong 1/\boldsymbol{\mu}^0(1), \; \boldsymbol{\lambda}^1(t,1) \cong 1/\boldsymbol{\mu}^1(1), \tag{63}$$

then by (21), considering (4.36) from [EU-CIRCLE Report for D6.4 - Part 0, 2017] and (54), the approximate mean value of the indicator of critical infrastructure resilience to operation process impact is given by

$$\boldsymbol{RI}^1(1) \cong \frac{\boldsymbol{\mu}^1(1)}{\boldsymbol{\mu}^0(1)} \cong 56.7545/62.5692 \cong 0.91$$
$$= 91\%. \tag{64}$$

4. Cost analysis of critical infrastructure operation process

We consider the complex technical multistate system / the critical infrastructure consisted of n components and we assume that the operation costs of its single basic components at the operation state z_b, $b = 1,2,...,v$, during the system operation time θ, $\theta \geq 0$, amount

$$k_i^1(\theta,b), \; b = 1,2,...,v, \; i = 1,2,...,n.$$

First, we suppose that the system is non-repairable, i.e. the system during the operation has not exceeded the critical safety state r. In this case, the total cost of the non-repairable system during the operation time θ, $\theta \geq 0$, is given by

$$\boldsymbol{K}^1(\theta) = \sum_{b=1}^{v} p_b \sum_{i=1}^{n} k_i^1(\theta,b), \; \theta \geq 0, \tag{65}$$

where p_b, $b = 1,2,...,v$, are the transient probabilities defined by (1)-(2).

Further, we additionally assume that the system is repairable after exceeding the critical safety state r, its renovation time is ignored and the cost of the system singular renovation is k_{ig}^1.

Then, the approximate total operation cost of the repairable system with ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$\boldsymbol{K}_{ig}^1(\theta) \cong \sum_{b=1}^{v} p_b \sum_{i=1}^{n} k_i^1(\theta,b) + k_{ig}^1 H^1(\theta,r), \; \theta \geq 0, \tag{66}$$

where p_b, $b = 1,2,...,v$, are the transient probabilities defined by (1)-(2) and $H^1(\theta,r)$ is the mean value of the number of exceeding the critical reliability state r by the system operating at the variable conditions during the operation time θ defined by (3.58) in [Kołowrocki, Soszyńska-Budny, 2011].

Now, we assume that the system is repairable after exceeding the critical safety state r and its renewal time is non-ignored and have distribution function with the mean value $\mu_0^1(r)$ and the standard deviation $\sigma_0^1(r)$ and the cost of the system singular renovation is k_{nig}^1.

Then, the approximate total operation cost of the repairable system with non-ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$\boldsymbol{K}_{nig}^1(\theta) \cong \sum_{b=1}^{v} p_b \sum_{i=1}^{n} k_i^1(\theta,b) + k_{nig}^0 \; \overline{\overline{H}}{}^1(\theta,r), \; \theta \geq 0, \tag{67}$$

where p_b, $b = 1,2,...,v$, are the transient probabilities defined by (1)-(2) and $\overline{\overline{H}}{}^1(\theta,r)$ is the mean value of the number of renovations of the system operating at the variable conditions during the operation time θ defined by (3.92) in [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the mean values $H^1(\theta,r)$ and $\overline{\overline{H}}{}^1(\theta,r)$ for the repairable systems with ignored and non-ignored renovation times existing in the formulae (66) and (67), respectively defined by (3.58) and (3.92), are determined in Chapter 3 in [Kołowrocki, Soszyńska-Budny, 2011] for typical repairable critical infrastructures, i.e. for multistate series, parallel, "*m* out of *n*", consecutive "*m* out of *n*: F", series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-

series critical infrastructures operating at the variable operation conditions.

5. Cost analysis of port oil piping transportation system operation process

The port oil piping transportation system is composed of n = 2880 components and according to the information coming from experts, the approximate mean operation costs of its single basic components during the operation time is θ = 1 year, independently of the operation states z_b, $b = 1,2,...,7,$ amount

$$k_i^1(\theta, b) \cong 9.6 \text{ PLN}, \ b = 1,2,...,7, \ i = 1,2,...,2880.$$

Thus, according to (65), if the non-repairable port oil piping transportation system during the operation is θ = 1 year has not exceeded the critical safety state r = 1, then its total operation cost during the operation time θ = 1 year is approximately given by

$$\boldsymbol{K}^1(1) \cong \sum_{b=1}^{7} p_k \sum_{i=1}^{n} k_i^1(1) \cong 0.403 \cdot 1086 \cdot 9.6 + 0.055 \cdot 1086 \cdot 9.6 + 0.003 \cdot 1794 \cdot 9.6 + 0.002 \cdot 2880 \cdot 9.6 + 0.199 \cdot 1794 \cdot 9.6 + 0.057 \cdot 2880 \cdot 9.6 + 0.281 \cdot 1086 \cdot 9.6 = 12814.68 \text{ PLN}. \quad (68)$$

Further, we assume that the considered the port oil piping transportation system is repairable after exceeding the critical safety state r = 1, its renovation time is ignored and the approximate mean cost of the system singular renovation is

$$k_{ig}^1 = 88\,500 \text{ PLN}.$$

In this case, since the expected number of exceeding the critical reliability state r = 1, according to (3.58) in [Kołowrocki, Soszyńska-Budny, 2011], amounts

$$H^1(1,1) = 1/56.7545 = 0.01762,$$

the total operation cost of the repairable system with ignored its renovation time during the operation time θ = 1 year approximately amounts

$$\boldsymbol{K}_{ig}^1(1) \cong \sum_{b=1}^{7} p_b \sum_{i=1}^{n} k_i^1(1) + k_{ig}^1 H^1(1,1) = 12\,814.68 + 88\,500 \cdot 0.01762 = 12\,814.68 + 1\,559.37 \cong 14\,374 \text{ PLN}. \quad (69)$$

If the port oil piping transportation system is repairable after exceeding the critical safety state r = 1 and its renewal time is non-ignored and have distribution function with the mean value

$$\mu_0^1(1) = 0.2 \text{ year}$$

and the cost of the system singular renovation is

$$k_{nig}^1 = 90\,000 \text{ PLN}$$

then, since the number of exceeding the critical reliability state r = 1, according to (3.92) in [Kołowrocki, Soszyńska-Budny, 2011], amounts

$$\overline{\overline{H}}^1(1,1) = 1/(56.7545 + 0.2) = 0.01756,$$

the total operation cost of the repairable the port oil piping transportation system with non-ignored its renovation time during the operation time θ = 1 approximately amounts

$$\boldsymbol{K}_{nig}^1(1) \cong \sum_{b=1}^{7} p_b \sum_{i=1}^{n} k_i^1(1) + k_{nig}^1 \ \overline{\overline{H}}^1(1,1) = 12\,814.68 + 90\,000 \cdot 0.01756 = 12\,814.68 + 1580.4 \cong 14395 \text{ PLN}. \quad (70)$$

6. Optimization of operation and safety of port oil piping transportation system

6.1. Optimization problem formulation

Considering the equation (9), it is natural to assume that the critical infrastructure operation process has a significant influence on the critical infrastructure safety. This influence is also clearly expressed in the equation (25) for the mean values of the critical infrastructure unconditional lifetimes in the safety state subsets.

From the linear equation (25), we can see that the mean value of the critical infrastructure unconditional lifetime $\boldsymbol{\mu}^1(u)$, $u = 1,2,\ldots,z,$ is determined by the limit values of transient probabilities p_b, $b = 1,2,...,\nu,$ of the critical infrastructure operation process at the operation states and the mean values $[\boldsymbol{\mu}^1(u)]^{(b)}$, $b = 1,2,...,\nu,$ $u = 1,2,...,z,$ of the critical infrastructure conditional lifetimes in the safety state subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ given by (26). Therefore, the critical infrastructure lifetime optimization approach based on the linear programming [EU-CIRCLE Report D3.5-GMU, 2017] can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,...,\nu,$ of the transient probabilities p_b, $b = 1,2,...,\nu,$ of the critical infrastructure operation process at the operation states to maximize the mean value $\boldsymbol{\mu}^1(u)$ of the unconditional critical infrastructure lifetimes in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, under the assumption that the mean values $[\boldsymbol{\mu}^1(u)]^{(b)}$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, of the system conditional lifetimes in the safety state subsets are fixed. As a special and practically important case of the above formulated system lifetime optimization problem, we may look for the optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities p_b, $b = 1,2,...,\nu$, of the critical infrastructure operation process at the critical infrastructure operation states to maximize the mean value $\boldsymbol{\mu}^1(r)$ of the unconditional critical infrastructure lifetime in the critical infrastructure state subset $\{r, r+1, ..., z\}$ of the states not worse than the critical stare r, given by (12), under the assumption that the mean values $[\boldsymbol{\mu}^1(r)]^{(b)}$, $b = 1,2,...,\nu$, of the critical infrastructure conditional lifetimes in this safety state subset, given by (13), are fixed. More exactly, we may formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}^1(r) = \sum_{b=1}^{\nu} p_b [\boldsymbol{\mu}^1(r)]^{(b)} \qquad (71)$$

for a fixed $r \in \{1,2,...,z\}$ and with the following bound constraints

$$\breve{p}_b \le p_b \le \hat{p}_b, \; b = 1,2,...,\nu, \; \sum_{b=1}^{\nu} p_b = 1, \qquad (72)$$

where $[\boldsymbol{\mu}^1(r)]^{(b)}$, $[\boldsymbol{\mu}^1(r)]^{(b)} \ge 0$, $b = 1,2,...,\nu$, are fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{r, r+1, ..., z\}$ and

$$\breve{p}_b, \; 0 \le \breve{p}_b \le 1 \text{ and } \hat{p}_b, \; 0 \le \hat{p}_b \le 1, \; \breve{p}_b \le \hat{p}_b, \; b = 1,2,...,\nu, \qquad (73)$$

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,...,\nu$, respectively.

The procedure of finding the optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities p_b, $b = 1,2,...,\nu$, that will be applied in the next section can be found in [EU-CIRCLE Report D3.5-GMU, 2017].

6.2. Optimization of port oil piping transportation system operation process

The objective function defined by (71), in this case, as the port oil piping transportation system critical state is $r = 1$, considering (45)-(51) takes the form

$$\boldsymbol{\mu}^1(1) \cong \sum_{b=1}^{7} p_b [\boldsymbol{\mu}^1(1)]^{(b)} = p_1 \cdot 57.229757 + p_2 \cdot 57.229757 + p_3 \cdot 56.363181 + p_4 \cdot 52.140982 + p_5 \cdot 56.363181 + p_6 \cdot 52.140982 + p_7 \cdot 57.229757. \qquad (74)$$

The lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown transient probabilities p_b, $b = 1,2,...,7$, coming from experts respectively are:

$$\breve{p}_1 = 0.31, \; \breve{p}_2 = 0.04, \; \breve{p}_3 = 0.002, \; \breve{p}_4 = 0.001, \; \breve{p}_5 = 0.15, \; \breve{p}_6 = 0.04, \; \breve{p}_7 = 0.25;$$
$$\hat{p}_1 = 0.46, \; \hat{p}_2 = 0.08, \; \hat{p}_3 = 0.006, \; \hat{p}_4 = 0.004, \; \hat{p}_5 = 0.26, \; \hat{p}_6 = 0.08, \; \hat{p}_7 = 0.40. \qquad (75)$$

Therefore, according to (72)-(73) and (75), we assume the following bound constraints

$$0.31 \le p_1 \le 0.46, \; 0.04 \le p_2 \le 0.08, \; 0.002 \le p_3 \le 0.006, \; 0.001 \le p_4 \le 0.004, \; 0.15 \le p_5 \le 0.26, \; 0.04 \le p_6 \le 0.08, \; 0.25 \le p_7 \le 0.40, \; \sum_{b=1}^{7} p_b = 1. \qquad (76)$$

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,...,7$, that maximize the objective function (74), w arrange the system conditional lifetime mean values $[\boldsymbol{\mu}^1(1)]^{(b)}$, $b = 1,2,...,7$, in non-increasing order

$$[\boldsymbol{\mu}^1(1)]^{(1)} \ge [\boldsymbol{\mu}^1(1)]^{(2)} \ge [\boldsymbol{\mu}^1(1)]^{(7)} \ge [\boldsymbol{\mu}^1(1)]^{(3)} \ge [\boldsymbol{\mu}^1(1)]^{(5)} \ge [\boldsymbol{\mu}^1(1)]^{(4)} \ge [\boldsymbol{\mu}^1(1)]^{(6)}.$$

Next, according to procedure given in [EU-CIRCLE Report D3.5-GMU, 2017], and considering (75), we substitute

$$x_1 = p_1, \; x_2 = p_2, \; x_3 = p_7, \; x_4 = p_3, \; x_5 = p_5, \; x_6 = p_4, \; x_7 = p_6, \qquad (77)$$

and

$$\breve{x}_1 = \breve{p}_1 = 0.31, \; \breve{x}_2 = \breve{p}_2 = 0.04, \; \breve{x}_3 = \breve{p}_7 = 0.25,$$

$$\breve{x}_4 = \breve{p}_3 = 0.002,\ \ \breve{x}_5 = \breve{p}_5 = 0.15,\ \ \breve{x}_6 = \breve{p}_4 = 0.001,$$
$$\breve{x}_7 = \breve{p}_6 = 0.04,$$

$$\hat{x}_1 = \hat{p}_1 = 0.46,\ \ \hat{x}_2 = \hat{p}_2 = 0.08,\ \ \hat{x}_3 = \hat{p}_7 = 0.40,$$
$$\hat{x}_4 = \hat{p}_3 = 0.006,\ \ \hat{x}_5 = \hat{p}_5 = 0.26,\ \hat{x}_6 = \hat{p}_4 = 0.004,$$
$$\hat{x}_7 = \hat{p}_6 = 0.08, \tag{78}$$

and we maximize with respect to x_i, $i = 1,2,...,7$, the linear form (74) that according to (77)-(78) takes the form

$$\begin{aligned}\boldsymbol{\mu}^1(1) = {} & x_1 \cdot 57.229757 + x_2 \cdot 57.229757 \\ & + x_3 \cdot 57.229757 + p_4 \cdot 56.363181 \\ & + x_5 \cdot 56.363181 + x_6 \cdot 52.140982 \\ & + x_7 \cdot 52.140982.\end{aligned} \tag{79}$$

with the following bound constraints

$$0.31 \le x_1 \le 0.46,\ 0.04 \le x_2 \le 0.08,\ \ 0.25 \le x_3 \le 0.40,$$
$$0.002 \le x_4 \le 0.006,\ \ 0.15 \le x_5 \le 0.26,$$
$$0.001 \le x_6 \le 0.004,\ \ 0.04 \le x_7 \le 0.08,\ \ \sum_{i=1}^{7} x_i = 1. \tag{80}$$

According to the procedure given in [EU-CIRCLE Report D3.5-GMU,2017], we calculate

$$\breve{x} = \sum_{i=1}^{7} \breve{x}_i = 0.793,$$
$$\hat{y} = 1 - \breve{x} = 1 - 0.793 = 0.207 \tag{81}$$

and we find

$$\breve{x}^0 = 0,\ \ \hat{x}^0 = 0,\ \ \hat{x}^0 - \breve{x}^0 = 0,$$
$$\breve{x}^1 = 0.31,\ \hat{x}^1 = 0.46,\ \hat{x}^1 - \breve{x}^1 = 0.15,$$
$$\breve{x}^2 = 0.35,\ \ \hat{x}^2 = 0.54,\ \hat{x}^2 - \breve{x}^2 = 0.19,$$
$$\breve{x}^3 = 0.60,\ \ \hat{x}^3 = 0.94,\ \hat{x}^3 - \breve{x}^3 = 0.34,$$
$$\dots$$
$$\breve{x}^7 = 0.793\ \ \hat{x}^7 = 1.29,\ \ \hat{x}^7 - \breve{x}^7 = 0.497. \tag{82}$$

From the above, as according to (81), the appropriate inequality from [EU-CIRCLE Report D3.5-GMU, 2017] takes the form

$$\hat{x}^I - \breve{x}^I < 0.207, \tag{83}$$

then it follows that the largest value $I \in \{0,1,...,7\}$ such that this inequality holds is $I = 2$.
Therefore, we fix the optimal solution that maximize linear function (79) according to the appropriate rule from [EU-CIRCLE Report D3.5-GMU, 2017]. Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.46,\ \ \dot{x}_2 = \hat{x}_2 = 0.08,$$
$$\dot{x}_3 = \hat{y} - \breve{x}^2 + \breve{x}^2 + \breve{x}_3 = 0.267,\ \ \dot{x}_4 = \breve{x}_4 = 0.002,$$
$$\dot{x}_5 = \breve{x}_5 = 0.15,\ \dot{x}_6 = \breve{x}_6 = 0.001,\ \ \dot{x}_7 = \breve{x}_7 = 0.04. \tag{84}$$

Finally, after making the substitution inverse to (74), we get the optimal transient probabilities

$$\dot{p}_1 = \dot{x}_1 = 0.46,\ \ \dot{p}_2 = \dot{x}_2 = 0.08,\ \ \dot{p}_7 = \dot{x}_3 = 0.267,$$
$$\dot{p}_3 = \dot{x}_4 = 0.002,\ \ \dot{p}_5 = \dot{x}_5 = 0.15,\ \ \dot{p}_4 = \dot{x}_6 = 0.001,$$
$$\dot{p}_6 = \dot{x}_7 = 0.04, \tag{85}$$

that maximize the pipeline system mean lifetime $\boldsymbol{\mu}^1(1)$ in the safety state subset $\{1,2\}$ expressed by the linear form (74).
Considering (85), and assuming as in Section 3.2 the system operation time $\theta = 1$ year = 365 days, after appropriate formula from [EU-CIRCLE Report D3.5-GMU, 2017] , we get the optimal mean values of the total sojourn times at the particular operation states during this operation time:

$$\dot{\hat{M}}_1 = \dot{E}[\hat{\theta}_1] = \dot{p}_1\theta = 0.46 \cdot 365 = 167.9,$$
$$\dot{\hat{M}}_2 = \dot{E}[\hat{\theta}_2] = \dot{p}_2\theta = 0.08 \cdot 365 = 29.2,$$
$$\dot{\hat{M}}_3 = \dot{E}[\hat{\theta}_3] = \dot{p}_3\theta = 0.002 \cdot 365 = 0.73,$$
$$\dot{\hat{M}}_4 = \dot{E}[\hat{\theta}_4] = \dot{p}_4\theta = 0.001 \cdot 365 = 0.365,$$
$$\dot{\hat{M}}_5 = \dot{E}[\hat{\theta}_5] = \dot{p}_5\theta = 0.15 \cdot 365 = 54.75,$$
$$\dot{\hat{M}}_6 = \dot{E}[\hat{\theta}_6] = \dot{p}_6\theta = 0.04 \cdot 365 = 14.6,$$
$$\dot{\hat{M}}_7 = \dot{E}[\hat{\theta}_7] = \dot{p}_7\theta = 0.267 \cdot 365 = 97.455. \tag{86}$$

6.3. Optimal safety characteristics of port oil piping transportation system

Thus, as a result of Section 7.2 analysis, the optimal value of the port oil piping transportation system $\boldsymbol{\mu}^1(1)$ in the safety state subset $\{1,2\}$, according to (74) and (85), is

$$\begin{aligned}\dot{\mu}(1) = {} & \dot{p}_1 \cdot 57.229757 + \dot{p}_2 \cdot 57.229757 \\ & + \dot{p}_3 \cdot 56.363181 + \dot{p}_4 \cdot 52.140982 \\ & + \dot{p}_5 \cdot 56.363181 + \dot{p}_6 \cdot 52.140982 \\ & + \dot{p}_7 \cdot 57.229757 \\ = {} & 0.46 \cdot 57.229757 + 0.08 \cdot 57.229757 \\ & + 0.002 \cdot 56.363181 + 0.001 \cdot 52.140982 \\ & + 0.15 \cdot 56.363181 + 0.04 \cdot 52.140982 \\ & + 0.267 \cdot 57.229757 \cong 56.8894.\end{aligned} \tag{87}$$

Further, substituting the optimal solution (85) into the formula (79), we obtain the optimal solution for the mean value of the port oil piping transportation system unconditional lifetime in the safety state subset {2}

$$\begin{aligned}\dot{\mu}(2) = {} & \dot{p}_1 \cdot 42.491358 + \dot{p}_2 \cdot 42.491358 \\ & + \dot{p}_3 \cdot 40.728766 + \dot{p}_4 \cdot 38.183197 \\ & + \dot{p}_5 \cdot 40.728766 + \dot{p}_6 \cdot 38.183197 \\ & + \dot{p}_7 \cdot 42.491358 \\ = {} & 0.46 \cdot 42.491358 + 0.08 \cdot 42.491358 \\ & + 0.002 \cdot 40.728766 + 0.001 \cdot 38.183197 \\ & + 0.15 \cdot 40.728766 + 0.04 \cdot 38.183197 \\ & + 0.267 \cdot 42.491358 \cong 42.0468, \end{aligned} \tag{88}$$

and according to (6.23) in [Kołowrocki, Soszyńska-Budny, 2011], the optimal values of the mean values of the port oil piping transportation system unconditional lifetimes in the particular safety states 1 and 2, respectively are

$$\dot{\bar{\mu}}(1) = \dot{\mu}(1) - \dot{\mu}(2) = 14.8426$$

$$\dot{\bar{\mu}}(2) = \dot{\mu}(2) = 42.0468 \tag{89}$$

Moreover, according to (6.20)-(6.21) from [Kołowrocki, Soszyńska-Budny, 2011], considering the intensities of departure of the assets from Section 3.3, the corresponding optimal unconditional multistate safety function of the port oil piping transportation system (Saf1) is of the form

$$\dot{\boldsymbol{S}}(t,\cdot) = [1,\ \dot{\boldsymbol{S}}(t,1),\ \dot{\boldsymbol{S}}(t,2)\ ], \tag{90}$$

with the coordinates given by

$$\begin{aligned}\dot{\boldsymbol{S}}(t,1) = {} & 3.228\exp[-0.03563t] \\ & + 6.456\exp[-0.037394t] - 6.456\exp[-0.046154t] \\ & - 1.614\exp[-0.05012t] - 3.228\exp[-0.051884t] \\ & + 3.228\exp[-0.060644t] - 1.614\exp[-0.03925t] \\ & - 3.228\exp[-0.041014t] + 3.228\exp[-0.049774t] \\ & + 0.807\exp[-0.05374t] + 1.614\exp[-0.055504t] \\ & - 1.614\exp[-0.064264t] + 0.608\exp[-0.036332t] \\ & - 1.216\exp[-0.045102t] + 1.216\exp[-0.037802t] \\ & - 0.304\exp[-0.05372t] + 0.608\exp[-0.06249t] \\ & - 0.608\exp[-0.05519t] - 0.304\exp[-0.040676t] \\ & + 0.608\exp[-0.049446t] - 0.608\exp[-0.042146t] \\ & + 0.152\exp[-0.058064t] - 0.304\exp[-0.066834t] \\ & + 0.304\exp[-0.059534t] + 0.164\exp[-0.039252t] \\ & + 0.328\exp[-0.041016t] - 0.328\exp[-0.049776t] \\ & - 0.082\exp[-0.05664t] - 0.164\exp[-0.058404t] \\ & + 0.164\exp[-0.067164t] - 0.082\exp[-0.043596t] \\ & - 0.164\exp[-0.04536t] + 0.164\exp[-0.05412t] \\ & + 0.041\exp[-0.060984t] + 0.082\exp[-0.062748t] \\ & - 0.082\exp[-0.071508t],\ t \geq 0, \end{aligned} \tag{91}$$

$$\begin{aligned}\dot{\boldsymbol{S}}(t,2) = {} & 3.228\exp[-0.048966t] \\ & + 6.456\exp[-0.049818t] - 6.456\exp[-0.060726t] \\ & - 1.614\exp[-0.070706t] - 3.228\exp[-0.071558t] \\ & + 3.228\exp[-0.082466t] - 1.614\exp[-0.054376t] \\ & - 3.228\exp[-0.055228t] + 3.228\exp[-0.066136t] \\ & + 0.807\exp[-0.076116t] + 1.614\exp[-0.076968t] \\ & - 1.614\exp[-0.087876t] + 0.608\exp[-0.05076t] \\ & - 1.216\exp[-0.06056t] + 1.216\exp[-0.05147t] \\ & - 0.304\exp[-0.076848t] + 0.608\exp[-0.086648t] \\ & - 0.608\exp[-0.077558t] - 0.304\exp[-0.057252t] \\ & + 0.608\exp[-0.067052t] - 0.608\exp[-0.057962t] \\ & + 0.152\exp[-0.08334t] - 0.304\exp[-0.09314t] \\ & + 0.304\exp[-0.08405t] + 0.164\exp[-0.054396t] \\ & + 0.328\exp[-0.055248t] - 0.328\exp[-0.066156t] \\ & - 0.082\exp[-0.080484t] - 0.164\exp[-0.081336t] \\ & + 0.164\exp[-0.092244t] - 0.082\exp[-0.060888t] \\ & - 0.164\exp[-0.06174t] + 0.164\exp[-0.072648t] \\ & + 0.041\exp[-0.086976t] + 0.082\exp[-0.087828t] \\ & - 0.082\exp[-0.098736t],\ t \geq 0, \end{aligned} \tag{92}$$

Further, by (6.22) from [Kołowrocki, Soszyńska-Budny, 2011], considering (87)-(88) and (91)-(92), the corresponding optimal standard deviations of the port oil piping transportation system unconditional lifetime in the state subsets are

$$\dot{\sigma}(1) \cong 38.1159, \tag{93}$$

$$\dot{\sigma}(2) \cong 28.1949. \tag{94}$$

As the port oil piping transportation system critical safety state is r =1, then its optimal system risk function, according to (6.24) in [Kołowrocki, Soszyńska-Budny, 2011], considering (91), is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{S}}(t,1),\ t \geq 0, \tag{95}$$

where $\dot{\boldsymbol{S}}(t,1)$ is given by (95). Hence, and considering (6.25) in [Kołowrocki, Soszyńska-Budny, 2011], the moment when the optimal system risk function exceeds a permitted level (SafI6), for instance δ = 0.05, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 11.0174 \text{ year}. \tag{96}$$

By (87) and (93), the port oil piping transportation system optimal mean lifetime up to exceeding critical safety state r = 1 (SafI4) is

$$\dot{\mu}(1) \cong 56.8894 \text{ years}, \tag{97}$$

and the optimal standard deviation of the port oil piping transportation system lifetime up to exceeding critical safety state r = 1 (SafI5) is

$$\dot{\sigma}(1) \cong 38.1159. \tag{98}$$

By (91)-(92), applying (60), the port oil piping transportation system optimal intensities of ageing (SafI7) are:

$$\dot{\boldsymbol{\lambda}}(t,1) \cong 0.035630 \text{ for large } t, \tag{99}$$

$$\dot{\boldsymbol{\lambda}}(t,2) \cong 0.048966 \text{ for large } t. \tag{100}$$

Considering (99)-(100) and the values of the port oil piping transportation system intensities of ageing without of operation impact from [EU-CIRCLE Report for D6.4-Part 0, 2017] and applying (41), the optimal coefficients of the operation process impact on the port oil piping transportation system intensities of ageing (SafI8) are:

$$\dot{\boldsymbol{\rho}}(t,1) = \frac{\dot{\boldsymbol{\lambda}}(t,1)}{\boldsymbol{\lambda}^0(t,1)} = \frac{0.035630}{0.032710} \cong 1.089, \tag{101}$$

$$\dot{\boldsymbol{\rho}}(t,2) = \frac{\dot{\boldsymbol{\lambda}}(t,2)}{\boldsymbol{\lambda}^0(t,2)} = \frac{0.048966}{0.045330} \cong 1.080. \tag{102}$$

Finally, by (62) and (101), the optimal port oil piping transportation system resilience indicator (RI1), i.e. the coefficient of the port oil piping transportation system resilience to operation process impact, is

$$R\dot{I}(t) = 1/\dot{\boldsymbol{\rho}}(t,1) \cong 0.918 \cong 92\%,\ t \in <0,+\infty). \tag{103}$$

6.4. Port oil piping transportation system operation strategy

The knowledge of optimal transient probabilities $\dot{p}_b$, $b=1,2,...,7,$ at the particular operation states given by (85), may be the basis to improving the port oil piping transportation system safety indicators before its operation process optimization determined in Section 3.4 to that determined after its operation process optimization determined in Section 6.3. This justifies the sensibility of the performed operation process optimization, and some suggestions on new strategy of the port oil piping transportation system operation process organizing should be proposed.

The first suggestion is to organize intuitively the operation process in the way that makes the transient probabilities p_b, $b=1,2,...,7,$ at the particular operation states before the optimization, given by (42), approximately convergent to their optimal values $\dot{p}_b$, $b=1,2,...,7,$ given by (85).

The easiest way of the port oil piping transportation system operation process reorganizing is that leading to the approaching the values of its total sojourn times $\hat{M}_b$, $b=1,2,...,7,$ at the particular operation states during the fixed operation time for instance θ = 1 year, before the optimization given by (43) to the values of its optimal total sojourn times $\dot{\hat{M}}_b$, $b=1,2,...,7,$ after the operation process optimization given by (86).

More complicated way of the complex system operation process reorganization after its optimization is proposed in [Kołowrocki, Soszyńska-Budny, 2011].

7. Critical infrastructure operation cost optimization

7.1. Critical infrastructure optimal operation cost after its operation optimization with respect to its safety maximization

After the optimization of the critical infrastructure operation process and safety, the critical infrastructure total operation costs given by (68)-(70) assume their optimal values expressed by the appropriate formulae given in this section.

The total optimal cost of the non-repairable critical infrastructure during the operation time θ, $\theta \geq 0$, is given by

$$\dot{\boldsymbol{K}}^1(\theta) = \sum_{b=1}^{v} \dot{p}_b \sum_{i=1}^{n} k_i^1(\theta,b),\ \ \theta \geq 0, \tag{104}$$

where $\dot{p}_b$, $b=1,2,...,v,$ are the optimal transient probabilities.

The optimal total operation cost of the repairable system with ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$\dot{\boldsymbol{K}}_{ig}^1(\theta) \cong \sum_{b=1}^{v} \dot{p}_b \sum_{i=1}^{n} k_i^1(\theta,b) + k_{ig}^1 \dot{H}^1(\theta,r),\ \ \theta \geq 0, \tag{105}$$

where $\dot{p}_b$, $b=1,2,...,v,$ are the optimal transient probabilities and $\dot{H}^1(\theta,r)$ is the mean value of the optimal number of exceeding the critical reliability state r by the system operating at the variable conditions during the operation time θ defined by (6.29) in [Kołowrocki, Soszyńska-Budny, 2011].

The optimal total operation cost of the repairable system with non-ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$\dot{K}^1_{nig}(\theta) \cong \sum_{b=1}^{v} \dot{p}_b \sum_{i=1}^{n} k_i^1(\theta,b) + k^0_{nig}\ \dot{\bar{\bar{H}}}^1(\theta,r),\ \theta \geq 0, \quad (106)$$

where $\dot{p}_b$, $b = 1,2,...,v,$ are the optimal transient probabilities and $\dot{\bar{\bar{H}}}^1(\theta,r)$ is the mean value of the optimal number of renovations of the system operating at the variable conditions during the operation time θ defined by (6.37) in [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the mean values $\dot{H}^1(\theta,r)$ and $\dot{\bar{\bar{H}}}^1(\theta,r)$ for the repairable systems with ignored and non-ignored renovation times existing in the formulae (105) and (106), respectively defined by (6.29) and (6.37), are determined in Chapter 6 in [Kołowrocki, Soszyńska-Budny, 2011] for typical repairable critical infrastructures, i.e. for multistate series, parallel, "*m* out of *n*", consecutive "*m* out of *n*: F", series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-series critical infrastructures operating at the variable operation conditions.

7.2. Port oil piping transportation system operation cost optimization

7.2.1. Port oil piping transportation system optimal operation cost after its operation optimization with respect to its safety maximization

In this section, we will analyze the port oil piping transportation system operation cost after its operation process optimization.

Thus, according to (104), if the non-repairable port oil piping transportation system during the operation is $\theta = 1$ year has not exceeded the critical safety state $r = 1$, then its optimal total operation cost during the operation time $\theta = 1$ year is approximately given by

$$\dot{\boldsymbol{K}}^1(1) \cong \sum_{b=1}^{7} \dot{p}_k \sum_{i=1}^{n} k_i^1(1) \cong 0.46 \cdot 1086 \cdot 9.6$$
$$+ 0.08 \cdot 1086 \cdot 9.6 + 0.002 \cdot 1794 \cdot 9.6$$
$$+ 0.001 \cdot 2880 \cdot 9.6 + 0.15 \cdot 1794 \cdot 9.6$$
$$+ 0.04 \cdot 2880 \cdot 9.6 + 0.267 \cdot 1086 \cdot 9.6$$
$$= 12\ 164.83 \text{ PLN.} \quad (107)$$

Further, as the expected optimal number of exceeding the critical reliability state $r = 1$ amounts

$$\dot{H}^1(1,1) = 1/56.8894 = 0.01758,$$

then according to (105), the optimal total operation cost of the repairable system with ignored its renovation time during the operation time $\theta = 1$ year approximately amounts

$$\dot{\boldsymbol{K}}^1_{ig}(1) \cong \sum_{b=1}^{7} \dot{p}_b \sum_{i=1}^{n} k_i^1(1) + k^1_{ig}\dot{H}^1(1,1)$$
$$= 12164.83 + 88\ 500 \cdot 0.01758$$
$$= 12164.83 + 1555.83 \cong 13\ 721 \text{ PLN.} \quad (108)$$

Since the expected optimal number of exceeding the critical reliability state $r = 1$ amounts

$$\dot{\bar{\bar{H}}}^1(1,1) = 1/(56.8894 + 0.2) = 0.01752,$$

the total optimal operation cost of the repairable the port oil piping transportation system with non-ignored its renovation time during the operation time $\theta = 1$ approximately amounts

$$\dot{\boldsymbol{K}}^1_{nig}(1) \cong \sum_{b=1}^{7} \dot{p}_b \sum_{i=1}^{n} k_i^1(1) + k^1_{nig}\ \dot{\bar{\bar{H}}}^1(1,1)$$
$$= 12164.83 + 90\ 000 \cdot 0.01752$$
$$= 12164.83 + 1576.8 \cong 13\ 742 \text{ PLN.} \quad (109)$$

8. Conclusions

The proposed in [EU-CIRCLE Report D3.3-Part 3, 2017] Model 1 of critical infrastructure safety was applied to safety and resilience analysis of the port oil piping transportation system impacted by its operation process. The application of this model is supported by suitable computer software that is placed at the GMU Safety Interactive Platform http://gmu.safety.am.gdynia.pl/.

The results of this application will be generalized and applied to the safety and resilience analysis of port oil piping transportation system impacted by its operation process and climate-weather change process, in the next parts of the series of 4 papers concerned with the EU-CIRCLE project Case Study 2, Storm and Sea Surge at Baltic Sea Port.

Acknowledgments

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/.

References

[1] Blokus-Roszkowska, A., Bogalecka, M., Kołowrocki, K., Kuligowska, E., Soszyńska-Budny, J., Torbicki, M., *Inventory of Critical Infrastructure Impact Assessment Models for Climate Hazards, Task 3.4 Inventory of Critical Infrastructure Impact Assessment Models for Climate Hazards, Task 3.5, Holistic Risk Assessment Propagation Model*, EU-CIRCLE Report D3.3-Part3-Critical Infrastructure Safety and Resilience Indicators-V1.0, 2017.

[2] Blokus-Roszkowska, A., Dziula, P., Guze, S., Kołowrocki, K., Soszyńska-Budny, J., *Holistic Critical Infrastructure Climate Hazard Risk Assessment Framework-Critical Infrastructure Operation and Safety Optimization, Task 3.5 Holistic Risk Assessment Propagation Model*, EU-CIRCLE Report D3.5-GMU-V1.0-2017.

[3] EU-CIRCLE Report D3.5-GMU, 2017.

[4] Kołowrocki, K., Kuligowska, E., Soszyńska-Budny, J., Torbicki, M., *Oil Transport in Port – Safety and Resilience of Port Oil Piping Transportation System Related to Climate-Weather Change, Part 0. Port Oil Piping Transportation System Safety Without Outside Impacts*, EU-CIRCLE Report for D6.4, GMU-V1.0, 2017.

[5] Kołowrocki, K., Soszyńska-Budny, J., Reliability and *Safety of Complex Technical Systems and Processes: Modeling – Identification – Prediction – Optimization*, 1st ed. London: Springer-Verlag, 2011.

[6] GMU Safety Interactive Platform http://gmu.safety.am.gdynia.pl/