Diffraction of sound pulses on a spherical target in an oceanic waveguide covered with ice

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This paper is devoted to modeling of the pulse scattering by a spherical target immersed in a homogeneous waveguide covered with ice. For calculating the echo signal in the frequency domain we have followed Hackman and Sammelmann's general approach. The arising scattering coefficients of a sphere were evaluated with the use of the normal mode method. The amount of normal modes forming the backscattered field is determined by the given directivity of the source. The emitted signal is a pulse with a Gaussian envelope. Computational results are obtained in a wide frequency range 8 - 12 kHz for water depths equal to several hundreds of m, and distances between the source/receiver and a target from 1 km up to 10 km. The target is assumed to be acoustically rigid or fluid. In particular, the properties of the ice cover and a scatterer may coincide.

Keywords: diffraction of acoustic waves, ice-covered homogeneous waveguide, normal modes of the waveguide, a pulse with a Gaussian envelope, surface waves: Franz type, Whispering Gallery type

1. Introduction

The problem of acoustic pulse scattering by a target embedded in an oceanic waveguide covered with ice, is of undoubted importance. This paper is devoted to modeling of the backscattered field from a target immersed in a homogeneous waveguide, over a homogeneous, fluid half-space. The water layer is covered with ice, which is assumed to be a fluid half-space as well. A spherical scatterer of a radius *a* is acoustically rigid or fluid. In particular, the properties of the ice cover and a target may coincide. The source and receiver are located at the point M (0, y, z), y>0 of the waveguide $-b \le z \le d$ (see Fig.1). First, we will assume that the point source emits a spherical incident wave with a cyclical frequency ω. The range *r* of interest is from 1 km up to 10 km, and the frequency band of interest is 8 – 12 kHz

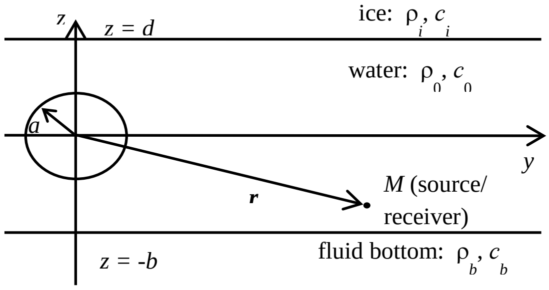

Fig. 1. The scattering geometry.

The normal mode evaluation is applied to calculate the scattering coefficients of a sphere in the frequency domain. The amount of normal modes forming the backscattered field is defined by a given directivity of the source. The emitted signal is a pulse with a Gaussian envelope. For different distances *r*, two echo signals are compared: for an acoustically rigid scatterer, and for an ice scatterer. Different parts of the echo signal are interpreted.

2. Theory

Solving the scattering problem formulated above, we will follow Hackman and Sammelmann's general approach [1] which was applied to the waveguide covered with ice in [2], where the radiated signal was assumed to be harmonic. In [1, 2] the acoustic potential of the backscattered field from a target is represented in the form

$$\Phi = -\frac{i}{k_0}\sum_{l=0}^{\infty} T_l \sum_{m=0}^{l} A_{ml}(\mathbf{r}) C_{ml}(\mathbf{r}) . \tag{1}$$

In (1) $k_0 = \omega / c_0$ is the wave number in water, T_l are elements of the free-field T-matrix. The coefficients $A_{ml}(\mathbf{r})$ in (1) are called the scattering coefficients of a sphere. They depend on the reflection coefficients for the upper surface and the lower surface, depth of the water layer, frequency, and distance between the target and source/receiver. In this paper we will use the single-scatter approximation when $C_{ml}(\mathbf{r}) = A_{ml}(\mathbf{r})$. In more realistic medium models it will be necessary to take into account the roughness of the undersurface of the ice. Comparison of the effect of multiple scattering behaviour of the solution (1), and the effect of the roughness of the undersurface of the ice is an interesting and important problem, but it is out of the scope of present paper.

The truncation level $l_{\max}$ is set by the rule suggested by Kargl and Marston [3]

$$l_{\max} = [k_0 a + 4.05 (k_0 a)^{1/3}] + 3 , \tag{2}$$

where [x] is the integer part of x. For $a = 2$ *m* and $f = 10$ *kHz* Eq. (2) gives $l_{\max} = 104$. Thus, computing the backscattered field (1), it will be necessary to sum up more than 5000 summands.

The integral representation of the scattering coefficients $A_{ml}(\mathbf{r})$ obtained in [1] is valid for arbitrary frequencies, distances between the source and target, and waveguide depths. At frequencies of interest, and distances r from 1 *km* up to 10 *km*, the integrand of these integrals is rapidly oscillating and slowly decreasing; that makes the straightforward calculation of scattering coefficients extremely time-consuming. To speed up the computation of integrals $A_{ml}(\mathbf{r})$ we will evaluate them by using the normal mode method, as was proposed in [2]. The dispersion equation for finding the eigenvalues $\xi = \xi_j$ can be written in the form

$$\mathrm{tg}[\mathrm{k}_0\mu_0(\mathrm{b+d})] = -i\frac{\mu_0\mu_b\rho_i + \mu_0\mu_i\rho_b}{\mu_0^2\rho_i + \mu_i\mu_b\rho_0}, \tag{3}$$

where $\mu_0 = \sqrt{1-\xi^2}$, $\mu_i = \sqrt{n_i^2 - \xi^2}$, $\mu_b = \sqrt{n_b^2 - \xi^2}$, $n_i = c_0 / c_i$, $n_b = c_0 / c_b$, $n_i < n_b < 1$.

To the propagating normal modes of the waveguide, the roots ξ_j of the dispersion equation (3) belonging to the interval (n_b, 1) correspond. If the radiation takes place inside a cone, having the angular width equal to α_a, it is necessary to take into account only propagating normal modes with ξ_j satisfying the inequality $\xi_j \geq \cos(\alpha_a / 2)$. Solving of Eq. (3) was discussed in details in [2].

Let the source emit a signal with a central frequency of f_c = 10 kHz and a Gaussian envelope

$$\varphi(\omega) = \exp(-0.5((\omega - \omega_c)/\sigma_\omega)^2) = \exp(-0.5((f - f_c)/\sigma_f)^2). \tag{4}$$

Here $\omega_c = 2\pi f_c$, $\sigma_f = 9\cdot10^2$; $\sigma_\omega = 2\pi\sigma_f$. In the time domain, we will compare signals received at the observation point M in the free water space

$$\hat{\Phi}^{(f)}(t) = \frac{1}{2\pi\sigma_\omega}\mathrm{Re}\{\int_{\omega_c-\Delta\omega}^{\omega_c+\Delta\omega}\Phi^{(f)}(\omega)\varphi(\omega)\exp(-i\omega t)d\omega\}, \tag{5}$$

where $\Delta_f = 2kHz, \Delta_\omega = 2\pi\Delta_f$,

$$\Phi^{(f)} = ik_0/(4\pi)\sum_{l=0}^{\infty}(2l+1)[h_l^{(1)}(k_0 r)]^2 T_l, \tag{6}$$

$h_l^{(1)}(x)$ is the spherical Hankel function of the first kind, and in the case when a scatter is located in a waveguide (see (1))

$$\hat{\Phi}(t) = (1/2\pi\sigma_\omega)\mathrm{Re}\{\int_{\omega_c-\Delta\omega}^{\omega_c+\Delta\omega}\Phi(\omega)\varphi(\omega)\exp(-i\omega t)d\omega\}. \tag{7}$$

3. Computational results

Let us start from the case of a free water space. The radius *a* of the scatterer is 2m. The sound speed in water c_0 = 1500 *m/s*, the water density $\rho_0 = 10^3$ kg/m^3. We will compare the echo signals scattered by two targets: impenetrable (acoustically rigid), and an ice sphere with the sound speed *c* and density ρ. If the ice sphere is attenuating $c = c_i$ = (3500 - 22.53*i*) *m/s*. In the case of the non-attenuating ice sphere *c* = 3500 *m/s*; ρ = 900 kg/m^3.

Figures 2 and 3 show the normalized echo signal (see (5) and (6))

$$F^{(f)}(t) = \frac{8\pi r^2}{a}\hat{\Phi}^{(f)}(t), \tag{8}$$

reflected from the impenetrable (acoustically rigid) sphere (Fig. 2) and the ice attenuating sphere (Fig. 3) at *r* = 1 *km*.

If the scatterer is acoustically rigid, there is only one reflected pulse, marked by A. In the case of the attenuating ice sphere (see Fig. 3) we get also the pulse B which is reflected from the back surface of the sphere and the pulse C corresponding to the superposition of the circumferential surface waves of the Whispering Gallery type (propagating internally (see, for example, [4] and Fig. 4)).

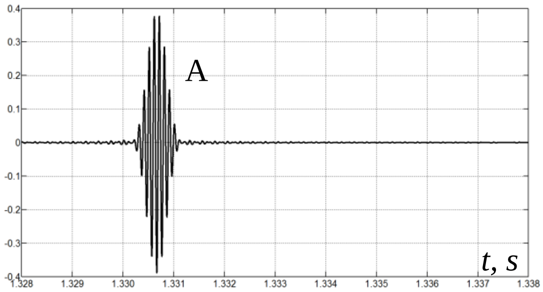

Fig. 2. The normalized echo signal (8) reflected from the acoustically rigid sphere of a radius a = 2 m for the free water space at r = 1 km.

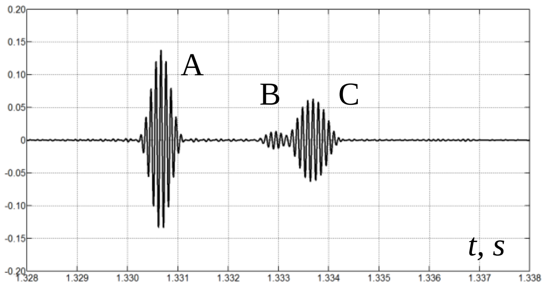

Fig. 3. The normalized echo signal (8) reflected from the attenuating ice sphere of a radius a = 2 m for the free water space at r = 1 km.

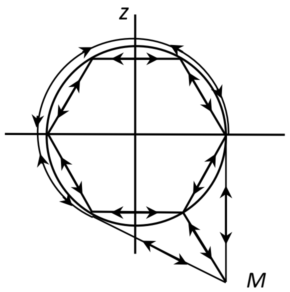

Fig. 4. Schematic illustration of circumferential surface waves: Franz type, propagating externally and the Whispering Gallery type, propagating internally.

The pulse corresponding to the Franz waves is not visible in Figs. 2 and 3 because of the strong attenuation of these waves in the considered frequency band. Their amplitude does not exceed $1 \cdot 10^{-3}$ and calculated arrival time is 1.3375 *s*.

In Fig. 5 the normalized echo signal is calculated for the ice target when we do not take into account the attenuation in ice. In this case, the pulse A reflected from the front part of the sphere changes insignificantly, as opposed to pulses B and C that increase essentially. One more pulse, D, appears. It corresponds to circumferential internal surface waves which enveloped the sphere twice. The pulse, E, corresponding to the external circumferential surface waves of the Franz type becomes visible.

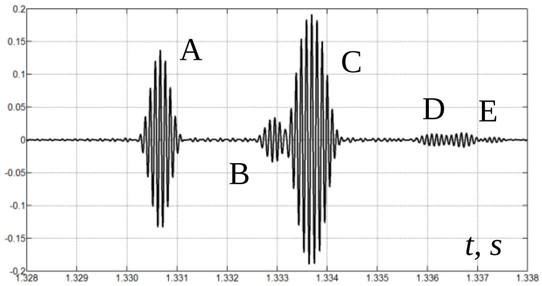

Fig. 5. The normalized echo signal (8) reflected from the non-attenuating ice sphere of a radius a = 2 m for free water space at r = 1 km.

Now let us consider a medium model with a waveguide of 200 m depth. The bottom of the waveguide is assumed to be sandy. In order to take into account the attenuation in it, the sound speed in the bottom c_b is considered complex c_b = (1730 – 24.74i) m/s. The density of the sandy bottom is ρ_b = 2050 kg / m^3. The distance d from the center of the sphere to the upper boundary of the waveguide is 30 m, z = - 20 m, r = 4.5 km.

Figures 6 and 7 show the normalized echo signals

$$F(t) = \frac{8\pi r^2}{a}\hat{\Phi}(t) \quad (9)$$

(see also Eqs. (1) and (7)), scattered by a sphere of a radius a = 2 m. In Fig. 6 the scatterer is acoustically rigid, and in Fig. 7 it is the non-attenuating sphere. In this case, the first pulse A is separated into three parts: the first one $A^{(f)}$ corresponds to the pulse, reflected from the front part of the sphere (its propagation time is $2(r - a)/c_0$ = 5.9973 s). The third part $A^{(d)}$ corresponds to the pulse reflected from the ice/water interface, put on the sphere, reflected from it and going back to the point M along the same way (its propagation time is $2(r_d - a)/c_0$ = 5.9982 s, where $r_d = (y^2 + (-z + 2d)^2)^{1/2}$). The second part $A^{(f,d)}$ corresponds to the pulse reflected from the interface, then put on the sphere, reflected from it, and going back to the point M, or in the opposite order: the point M → sphere → interface → point M. The propagation time of these two pulses is 5.99775 s.

In the case of the non-attenuating ice sphere (see Fig. 7) we observe also the pulse B reflected from the back surface of the sphere, and the pulse corresponding to the superposition of the circumferential internal surface waves of the Whispering Gallery type, which separates into three parts $C^{(f)}$, $C^{(d)}$ and $C^{(f,d)}$ in accordance with the way they arrive at the point M. The pulses B′, $D^{(f)}$, $D^{(f,d)}$ and $D^{(d)}$ correspond to the pulse which reflected twice from the back surface of the sphere and to the circumferential internal surface waves that enveloped the sphere twice. The pulse E corresponds to the external circumferential surface waves of the Franz type.

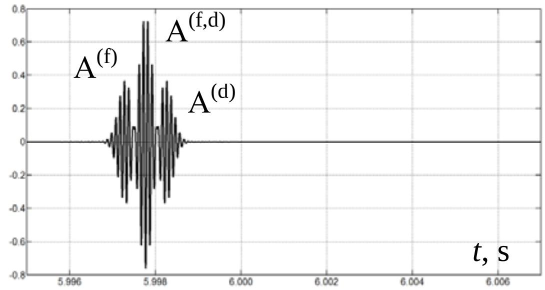

Fig. 6. The normalized echo signal (9) scattered by the acoustically rigid sphere of a radius a = 2 m located in a waveguide covered with ice of 200 m depth at d = 30 m, z = - 20 m, r = 4.5 km

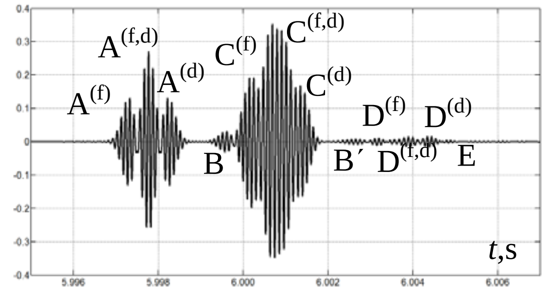

Fig. 7. The normalized echo signal (9) scattered by the non-attenuating ice sphere of a radius a = 2 m located in a waveguide covered with ice of 200 m depth at d = 30 m, z = -20 m, r = 4.5 km.

At r = 4.5 km and at a given directivity of the source, the target is not illuminated by rays reflected from the bottom. Now, let us consider the distance r = 5 km, when the target is illuminated by rays reflected from the bottom; d = 30 m, z = -20 m, b + d = 200 m.

Figure 8 shows the normalized echo signal (see (9)), scattered by the non-attenuating ice sphere of a radius a = 2 m. The first arriving pulses in Fig. 8 are interpreted similarly to the case of r = 4.5 km. The pulses arriving later 6.*68 s* become difficult to separate and interpret.

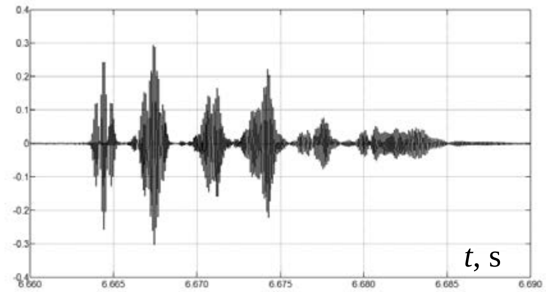

Fig. 8. The normalized echo signal (9) scattered by the non-attenuating ice sphere of a radius a = 2 m located in a waveguide covered with ice of 200 m depth at d = 30 m, z = - 20 m, r = 5 km.

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