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Management decision making based on Markov reward models for refrigeration system

Keywords

markov reward models, reliability measures, average availability, MTTF, refrigeration system

Abstract

This paper presents a method for calculation the reliability measures of multi-state supermarket refrigeration system for decision making of system structure, where the system and its components can have different performance levels ranging from perfect functioning to complete failure. The suggested approach presents the Markov reward models for computation of average availability, total number of system's elements failures and mean time to system failure for multi-state system. Corresponding procedures for reward matrix definition is suggested. A numerical example is presented in order to illustrate the approach.

1. Introduction

Supermarkets suffer serious financial losses owing to problems with their refrigeration systems. A typical supermarket may contain more than one hundred individual refrigerated cabinets, cold store rooms and items of plant machinery which interact as part of a complex integrated refrigeration system within the store. Things very often go wrong with individual units (icing up of components, electrical or mechanical failure, and so forth…) or with components which serve a network of units (coolant tanks, pumps, compressors, and so on).

In almost all supermarkets, refrigerated cabinets, cold store rooms and coldspaces are attached to a network of piping through which refrigerant is pumped. Heat from the coldspaces is absorbed by evaporating refrigerant which is then compressed and pumped to condensing units outside the store where the heat is expelled. Due to the system's highly integrated nature, a fault in a single unit or item of machinery can't have detrimental effects on the entire store, else only decrease of system cool capacity. The most commonly used refrigeration system for supermarkets today is the multiplex direct expansion system [1], [3]. All display cases and cold store rooms use direct expansion air-refrigerant coils that are connected to the system compressors in a remote machine room located in the back or on the roof of the store. Heat rejection is usually done with air-cooled condensers with simultaneously working axial blowers mounted outside. Evaporative condensers can be used as well and will reduce condensing temperature and system energy consumption.

Figure 1 shows the major elements of a multiplex refrigeration system. Multiple compressors operating at the same saturated suction temperature are mounted on a skid, or rack, and are piped with common suction and discharge refrigeration lines. Using multiple compressors in parallel provides a means of capacity control, since the compressors can be selected and cycled as needed to meet the refrigeration load.

Due to the system's highly integrated nature, a fault in a single unit or item of machinery can't have detrimental effects on the entire store, only decrease of system cool capacity. Failure of compressor or axial condenser blower leads to partial system failure (degradation of output cooling capacity) as well as to complete failures of the system. We treat refrigeration system as multi-state system (MSS), where components and systems have an arbitrary finite number of states. According to the generic MSS model [6], the system can have different states corresponding to the system's performance rates. The performance rate of the system at any instant is a discrete-state continuous-time stochastic process.

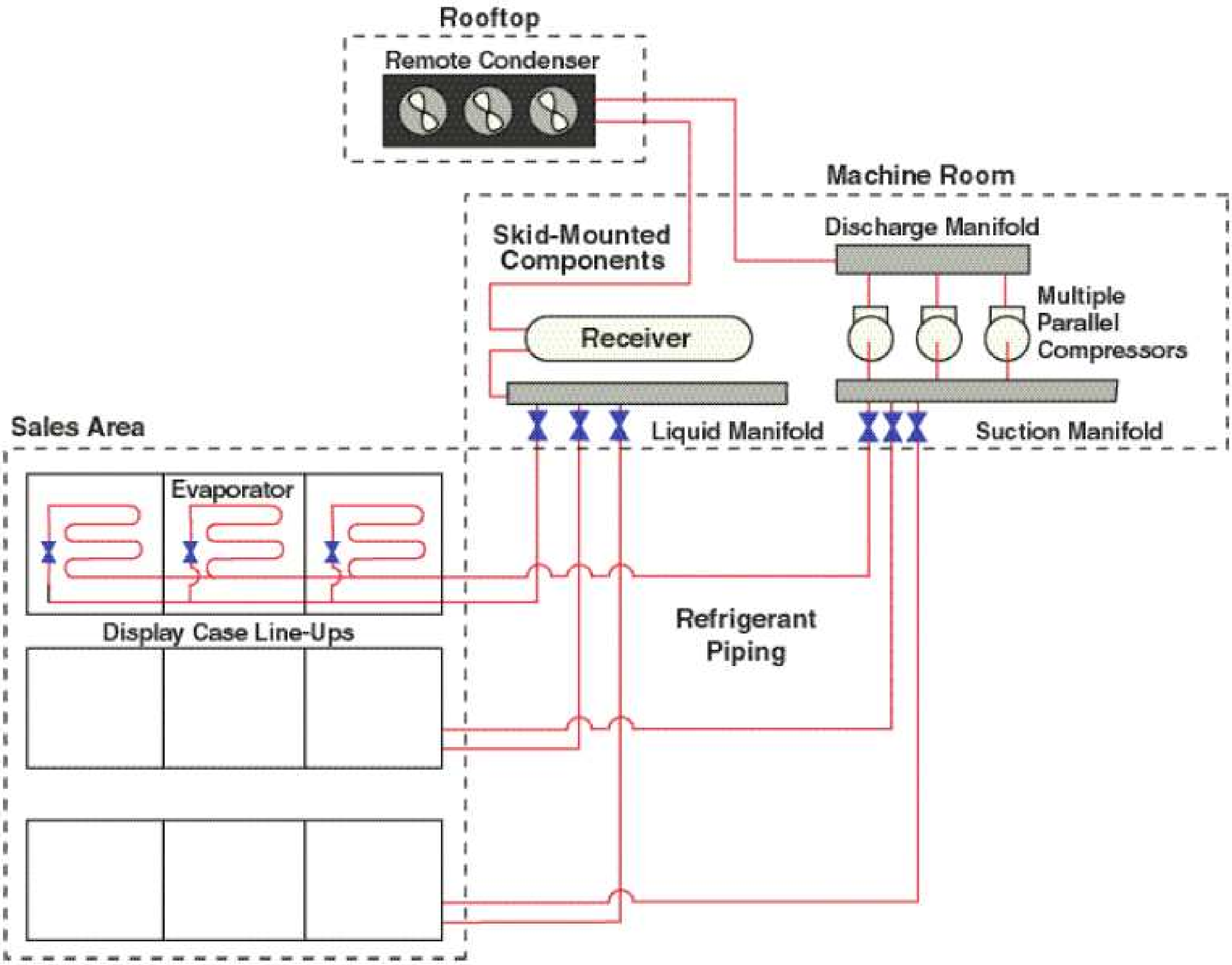

Figure 1. Multiplex refrigeration system

In this paper, a generalized approach for calculation the reliability measures for decision making of multi-state supermarket refrigeration system structure is suggested. The approach is based on the application of the Markov Reward Model. The MSS reliability measures can be found by corresponding rewards definitions for this model and then by using a standard procedure for finding an expected accumulated reward during the time interval [0,*t*] as a solution of the system of differential equations.

2. Model description

Traditional binary-state reliability models allow for a system and its components only two possible states: perfect functionality (Up) and complete failure (Down). However, many real-world systems are composed of multi-state components, which have different performance levels and for which one cannot formulate an "all or nothing" type of failure criterion. Failures of some system elements lead in these cases only to performance degradation. Such systems are called Multi-state Systems (MSS). The traditional reliability theory, which is based on a binary approach, has recently been extended by allowing components and systems to have an arbitrary finite number of states. According to the generic Multi-state System model [6], any system element $j \in \{1,2,\ldots n\}$ can have k_j different states corresponding to the performance rates, represented by the set $\mathbf{g_j}=\{g_{j1}, g_{j2},\ldots, g_{jk_j}\}$, where g_{ji} is the performance rate of element *j* in state *i*, $i \in \{1,2,\ldots k_j\}$. The performance rate *Gj*(*t*) of element *j* at any instant $t \geq 0$ is a discrete-state continuous-time stochastic process that takes its values from $\mathbf{g_j}$: $G(t) \in \mathbf{g_j}$. The system structure function $G(t) = \phi(G_1(t), \ldots, G_n(t))$ produces the stochastic process corresponding to the output performance of the entire MSS. In practice, a desired level of system performance (demand) also can be represented by a discrete-state continuous-time stochastic process $W(t)$. The relation between the MSS output performance and the demand represented by two corresponding stochastic processes should be studied in order to define the performance deficiency for the entire MSS.

The General Markov Reward Model considers the continuous time Markov chain with a set of states $\{1,\ldots,k\}$ and transition intensity matrix $\boldsymbol{a} = |a_{ij}|,\ i, j = 1,...,k$. It is assumed that while the process is in any state i during any time unit, some money r_{ii} should be paid. It is also assumed that if there is a transition from state i to state j, the amount r_{ij} will be paid. The amounts r_{ii} and r_{ij} are called rewards. They can be negative while representing loss or penalty. The main problem is to find a total expected reward, accumulated up to time instant T under specific initial conditions. Let $V_i(t)$ be the total expected reward accumulated up to time t at state i. According to [2], the following system of differential equations must be solved under initial conditions in order to find the total expected reward:

$$\frac{dV_i(t)}{dt} = r_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{k} a_{ij} r_{ij} + \sum_{j=1}^{k} a_{ij} V_j(t), \quad i = 1,...,k \qquad (1)$$

For the reliability measures computation, we partition the set of states $\boldsymbol{g}$, into $\boldsymbol{g}_0$, the set of operational or acceptable system states, and $\boldsymbol{g}_f$, the set of unacceptable states. The system states acceptability depends on the relation between the MSS output performance and the desired level of this performance – demand, which is determined outside the system. In general case demand $W(t)$ is also a random process that can take discrete values from the set $\boldsymbol{w}=\{w_1,\ldots,w_M\}$. The desired relation between the system performance and the demand at any time instant t can be expressed by the acceptability function $\Phi(G(t), W(t))$ [6]. The acceptable system states correspond to $\Phi(G(t), W(t)) \geq 0$ and the unacceptable states correspond to $\Phi(G(t), W(t)) < 0$. The last inequality defines the MSS performance deficiency criterion. In many practical cases, the MSS performance should be equal to or exceed the demand. Therefore, in such cases the acceptability function takes the following form:

$$\Phi(G(t), W(t)) = G(t) - W(t) \qquad (2)$$

and the criterion of state performance deficiency can be expressed as

$$\Phi(G(t), W(t)) = G(t) - W(t) < 0 \qquad (3)$$

Here without loss of generality we assume that required demand level is constant $W(t) \equiv w$ and all system states with performance greater than or equal to w corresponds to the set of acceptable states and all system states with performance lower than w correspond to the set of unacceptable states.

The MSS average availability $\bar{A}(T)$ is defined as mean fraction of time, when the system resides in the set of acceptable states during time interval $[0,T]$. In order to assess $\bar{A}(T)$ for MSS, the rewards in matrix r for MSS model should be determined by the following manner:

- The rewards associated with all acceptable states should be defined as 1 and (2)
- The rewards associated with all unacceptable states should be zeroed as well as all rewards associated with transitions.

The mean reward $V_K(T)$ accumulated during interval $[0, T]$ will define a part of time that MSS will be in the set of acceptable states in the case when state K is the initial state . This reward should be found as a solution of system (1). After solving (1) and finding $V_K(T)$, MSS instantaneous availability can be obtained as $\bar{A}(T) = V_K(T)/T$.

Mean number $N_f(T)$ of blowers' failures during time interval $[0, T]$. This measure can be treated as a mean number of MSS transitions in cause of blowers' failures during time interval $[0, T]$. For its computation, the rewards associated with each such transition should be defined as 1. All other rewards should be zeroed. In this case, a mean accumulated reward $V_K(T)$ will define a mean number blowers' failures during a time interval $[0,T]$: $N_f(T) = V_K(T)$.

Mean Time To Failure (MTTF) is the mean time up to the instant when the MSS enters the subset of unacceptable states for the first time. For its computation the multi-state model should be transformed - all transitions that return MSS from unacceptable states should be forbidden, as in this case all unacceptable states should be treated as absorbing states.

In order to assess MTTF for MSS, the rewards in matrix $\boldsymbol{r}_{MTTF}$ should be determined as follows:

- The rewards associated with all acceptable states should be defined as 1.
- The reward associated with unacceptable (absorbing) states should be zeroed as well as all rewards associated with transitions.

In this case, the mean accumulated reward $V_i(t)$ defines the mean time accumulated up to the first entrance into the subset of unacceptable states or MTTF, if the state i is the initial state.

3. Numerical example

Consider the refrigeration system used in one of the Israel supermarkets. The system consists of 4 compressors, situated in the machine room and 2 main axial condenser blowers. It is possible to add one reserve blower. The reserve blower begins to work only when one of the main blowers has failed. Compressor failure rate is one per year and axial condenser blower failure rate is 10 per year. The mean repair time for the compressor is one month and for blower is 24 hours. The state-space diagram for the system without reserved blower is presented in *Figures 2*.

All transition intensities are shown in the *Figure 2*. The transition intensity matrix (5) is shown below.

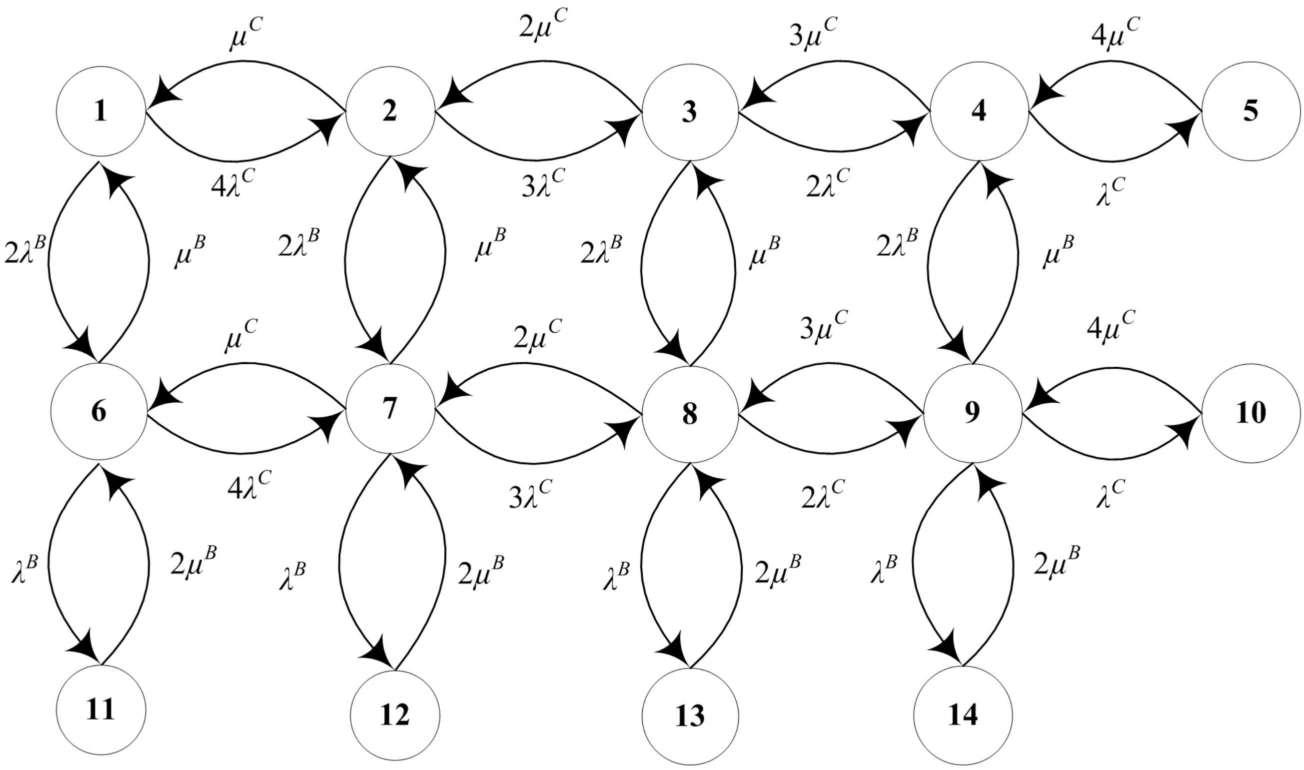

Figure 2. The state-space diagram for the refrigeration system without reserved blower

In state 1 the refrigeration system has full performance 10.5•10^9 BTU per year. In state 2 the refrigeration system has performance 7.9•10^9 BTU per year. The refrigeration system performance in states 3, 6, 7 and 8 is 5.2•10^9 BTU per year and in states 4 and 9 is 2.6•10^9 BTU per year.

The required cool capacity demand is 5•10^9 BTU per year, so only states 1, 2, 3, 6, 7 and 8 are acceptable states.

In order to find the MSS average availability $A(t)$ we should present the corresponding reward matrix $\boldsymbol{a}$ in the following form (4):

$$\mathbf{r}^A = \begin{Bmatrix} r_{11} = r_{22} = r_{33} = r_{66} = r_{77} = r_{88} = 1, \\ \text{all other elements are zero} \end{Bmatrix} \quad (4)$$

The system of differential equations (7) can be written in order to find to order the expected total rewards $V_i(t), i = 1,2,...14$. The initial conditions are $V_i(t) = 0,\ i = 1,2,...,14$.

By solving the systems of differential equations (7) with transition intensity matrix $\boldsymbol{a}$ and reward matrix $\mathbf{r}^A$ we can obtain an MSS average availability. The results of calculation are presented in *Figure 6*.

$$\boldsymbol{a}=\begin{pmatrix}
\boldsymbol{a}_{11} & 4\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\mu^{C} & \boldsymbol{a}_{22} & 3\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 2\mu^{C} & \boldsymbol{a}_{33} & 2\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 3\mu^{C} & \boldsymbol{a}_{44} & \lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 4\mu^{C} & \boldsymbol{a}_{55} & 0 & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0\\
\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{66} & 4\lambda^{C} & 0 & 0 & 0 & \lambda^{B} & 0 & 0 & 0\\
0 & \mu^{B} & 0 & 0 & 0 & \mu^{C} & \boldsymbol{a}_{77} & 3\lambda^{C} & 0 & 0 & 0 & \lambda^{B} & 0 & 0\\
0 & 0 & \mu^{B} & 0 & 0 & 0 & 2\mu^{C} & \boldsymbol{a}_{88} & 2\lambda^{C} & 0 & 0 & 0 & \lambda^{B} & 0\\
0 & 0 & 0 & \mu^{B} & 0 & 0 & 0 & 3\mu^{C} & \boldsymbol{a}_{99} & \lambda^{C} & 0 & 0 & 0 & \lambda^{B}\\
0 & 0 & 0 & 0 & \mu^{B} & 0 & 0 & 0 & 4\mu^{C} & \boldsymbol{a}_{10,10} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 2\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{11,11} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 2\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{12,12} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{13,13} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{14,14}
\end{pmatrix} \quad (5)$$

where

$$a_{11}=4\lambda^{C}+2\lambda^{B} \qquad a_{66}=4\lambda^{C}+\lambda^{B}+\mu^{B} \qquad a_{11,11}=2\mu^{B}$$

$$a_{22}=3\lambda^{C}+2\lambda^{B}+\mu^{C} \qquad a_{77}=3\lambda^{C}+\lambda^{B}+\mu^{C}+\mu^{B} \qquad a_{12,12}=2\mu^{B}$$

$$a_{33}=2\lambda^{C}+2\lambda^{B}+2\mu^{C} \qquad a_{88}=2\lambda^{C}+\lambda^{B}+2\mu^{C}+\mu^{B} \qquad a_{13,13}=2\mu^{B}$$

$$a_{44}=\lambda^{C}+2\lambda^{B}+3\mu^{C} \qquad a_{99}=\lambda^{C}+\lambda^{B}+3\mu^{C}+\mu^{B} \qquad a_{14,14}=2\mu^{B}$$

$$a_{55}=2\lambda^{B}+4\mu^{C} \qquad a_{10,10}=4\mu^{C}+\mu^{B}$$

In order to find the mean total number of blowers' failures $N_f(t)$ we should present the corresponding reward matrix in the following form:

$$\mathbf{r}_{Nf}=\left\{\begin{matrix} r_{16}=r_{27}=r_{38}=r_{49}= \\ r_{6,11}=r_{7,12}=r_{8,13}=r_{9,14}=1, \\ \text{all other elements are zero}\end{matrix}\right\} \qquad (6)$$

By solving the systems of differential equations (1) with transition intensity matrix $\boldsymbol{a}$ and reward matrix $\mathbf{r}_{Nf}$ we can obtain mean total number of blowers' failures during time period [0, T]. The results of calculation are presented in Figure 7.

For computation of the Mean Time To Failure during the time interval the state space diagram of generated system should be transformed – all transitions that return system from unacceptable states should be forbidden and all unacceptable states should be treated as absorbing state. The state space diagram is presented on *Figure 3*.

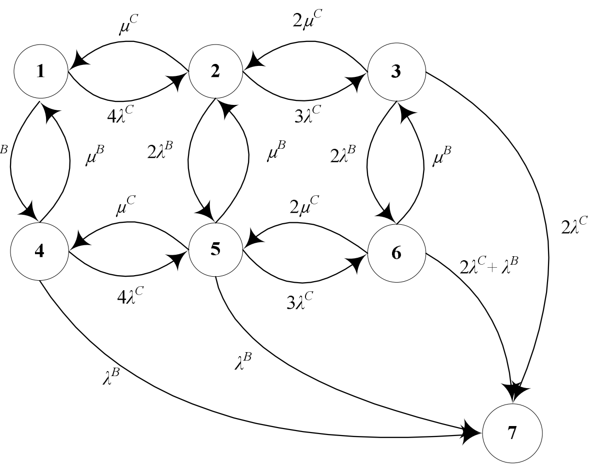

Figure 3. The state-space diagram for the refrigeration system without reserved blower with absorbing state

$$
\begin{aligned}
\frac{dV_1(t)}{dt} &= 1-(4\lambda^C+2\lambda^B)\cdot V_1(t)+4\lambda^C\cdot V_2(t)+2\lambda^B\cdot V_6(t)\\
\frac{dV_2(t)}{dt} &= 1+\mu^C\cdot V_1(t)-(3\lambda^C+2\lambda^B+\mu^C)\cdot V_2(t)+3\lambda^C\cdot V_3(t)+2\lambda^B\cdot V_7(t)\\
\frac{dV_3(t)}{dt} &= 1+2\mu^C\cdot V_2(t)-(2\lambda^C+2\lambda^B+2\mu^C)\cdot V_3(t)+2\lambda^C\cdot V_4(t)+2\lambda^B\cdot V_8(t)\\
\frac{dV_4(t)}{dt} &= 3\mu^C\cdot V_3(t)-(\lambda^C+2\lambda^B+3\mu^C)\cdot V_4(t)+\lambda^C\cdot V_5(t)+2\lambda^B\cdot V_9(t)\\
\frac{dV_5(t)}{dt} &= 4\mu^C\cdot V_4(t)-(2\lambda^B+4\mu^C)\cdot V_5(t)+2\lambda^B\cdot V_{10}(t)\\
\frac{dV_6(t)}{dt} &= 1+\mu^B\cdot V_1(t)-(4\lambda^C+\lambda^B+\mu^B)\cdot V_6(t)+4\lambda^C\cdot V_7(t)+\lambda^B\cdot V_{11}(t)\\
\frac{dV_7(t)}{dt} &= 1+\mu^B\cdot V_1(t)-(4\lambda^C+\lambda^B+\mu^B)\cdot V_7(t)+4\lambda^C\cdot V_7(t)+\lambda^B\cdot V_{11}(t)\\
\frac{dV_8(t)}{dt} &= 1+2\mu^C\cdot V_7(t)-(2\lambda^C+\lambda^B+\mu^B+2\mu^C)\cdot V_8(t)+2\lambda^C\cdot V_9(t)+\lambda^B\cdot V_{13}(t)\\
\frac{dV_9(t)}{dt} &= 2\mu^B\cdot V_4(t)+3\mu^C\cdot V_8(t)-(\lambda^C+\lambda^B+\mu^B+3\mu^C)\cdot V_9(t)+\lambda^C\cdot V_{10}(t)+\lambda^B\cdot V_{14}(t)\\
\frac{dV_{10}(t)}{dt} &= \mu^B\cdot V_5(t)+4\mu^C\cdot V_9(t)-(\mu^B+4\mu^C)\cdot V_{10}(t)\\
\frac{dV_{11}(t)}{dt} &= 2\mu^B\cdot V_6(t)-2\mu^B\cdot V_{11}(t)\\
\frac{dV_{12}(t)}{dt} &= 2\mu^B\cdot V_7(t)-2\mu^B\cdot V_{12}(t)\\
\frac{dV_{13}(t)}{dt} &= 2\mu^B\cdot V_8(t)-2\mu^B\cdot V_{13}(t)\\
\frac{dV_{14}(t)}{dt} &= 2\mu^B\cdot V_9(t)-2\mu^B\cdot V_{14}(t)
\end{aligned}
\qquad (7)
$$

According to this state space diagram transition intensity matrix a can be presented as follows (8):

$$
\boldsymbol{a}=\begin{pmatrix}
\boldsymbol{a}_{11} & 4\lambda^C & 0 & 2\lambda^B & 0 & 0 & 0\\
\mu^C & \boldsymbol{a}_{22} & 3\lambda^C & 0 & 2\lambda^B & 0 & 0\\
0 & 2\mu^C & \boldsymbol{a}_{33} & 0 & 0 & 2\lambda^B & 2\lambda^C\\
\mu^B & 0 & 0 & \boldsymbol{a}_{44} & 4\lambda^C & 0 & \lambda^B\\
0 & \mu^B & 0 & \mu^C & \boldsymbol{a}_{55} & 3\lambda^C & \lambda^B\\
0 & 0 & \mu^B & 0 & 2\mu^C & \boldsymbol{a}_{66} & 2\lambda^C+\lambda^B\\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \quad (8)
$$

where $a_{11}, a_{22}, \ldots, a_{66}$ are the same like in previous case.

In order to find *Mean Time To Failure* we should present the reward matrixes $\boldsymbol{r}_{MTTF}$ in the form (9), shown below.

$$
\boldsymbol{r}_{MTTF}=\begin{vmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{vmatrix} \quad (9)
$$

The system of differential equations (10) can be written in order to find to order the expected total rewards $V_i(t),\ i=1,2,...,14$. The initial conditions are $V_i(t)=0,\ i=1,2,...,14$.

By solving the systems of differential equations (10) with transition intensity matrix $\boldsymbol{a}$ and reward matrix $\boldsymbol{r}_{MTTF}$ we can obtain the Mean Time To Failure. The results of calculation are presented in *Figure 8.*

$$
\begin{aligned}
\frac{dV_1(t)}{dt} &= 1-(4\lambda^C+2\lambda^B)\cdot V_1(t)+4\lambda^C\cdot V_2(t)+2\lambda^B\cdot V_4(t)\\
\frac{dV_2(t)}{dt} &= 1+\mu^C\cdot V_1(t)-(3\lambda^C+2\lambda^B+\mu^C)\cdot V_2(t)+3\lambda^C\cdot V_3(t)+2\lambda^B\cdot V_5(t)\\
\frac{dV_3(t)}{dt} &= 1+2\mu^C\cdot V_2(t)-(2\lambda^C+2\lambda^B+2\mu^C)\cdot V_3(t)+2\lambda^C\cdot V_6(t)+2\lambda^B\cdot V_7(t)\\
\frac{dV_4(t)}{dt} &= 1+\mu^B\cdot V_1(t)-(4\lambda^C+\lambda^B+\mu^B)\cdot V_4(t)+4\lambda^C\cdot 5_7(t)+\lambda^B\cdot V_7(t)\\
\frac{dV_5(t)}{dt} &= 1+\mu^B\cdot V_2(t)+\mu^C\cdot V_4(t)-(3\lambda^C+\lambda^B+\mu^B)\cdot V_5(t)+3\lambda^C\cdot V_6(t)+\lambda^B\cdot V_7(t)\\
\frac{dV_6(t)}{dt} &= 1+2\mu^B\cdot V_3(t)+2\mu^C\cdot V_5(t)-(2\lambda^C+\lambda^B+\mu^B+2\mu^C)\cdot V_6(t)+(2\lambda^C+\lambda^B)\cdot V_7(t)\\
\frac{dV_7(t)}{dt} &= 0
\end{aligned}
\qquad (10)
$$

In case with reserved blower, the state-space diagram for the system is presented in *Figures 4*.

There are 19 states. In states 1, 6, 11, 16 – all 4 compressors are on-line, in states 2, 7, 12, 17 – 3 compressors are on-line, in states 3, 8, 13, 18 – 2 compressors are on-line, in states 4, 9, 14, 19 – only one compressor is on-line, states 5, 10, 15 – failure of all 4 compressors. In states 1–5 two axial condenser blowers are on-line, in states 6–10 one main blower and reserved blower are on line, in states 11–14 only one blower is on line and in states 16–19 failure of all blowers.

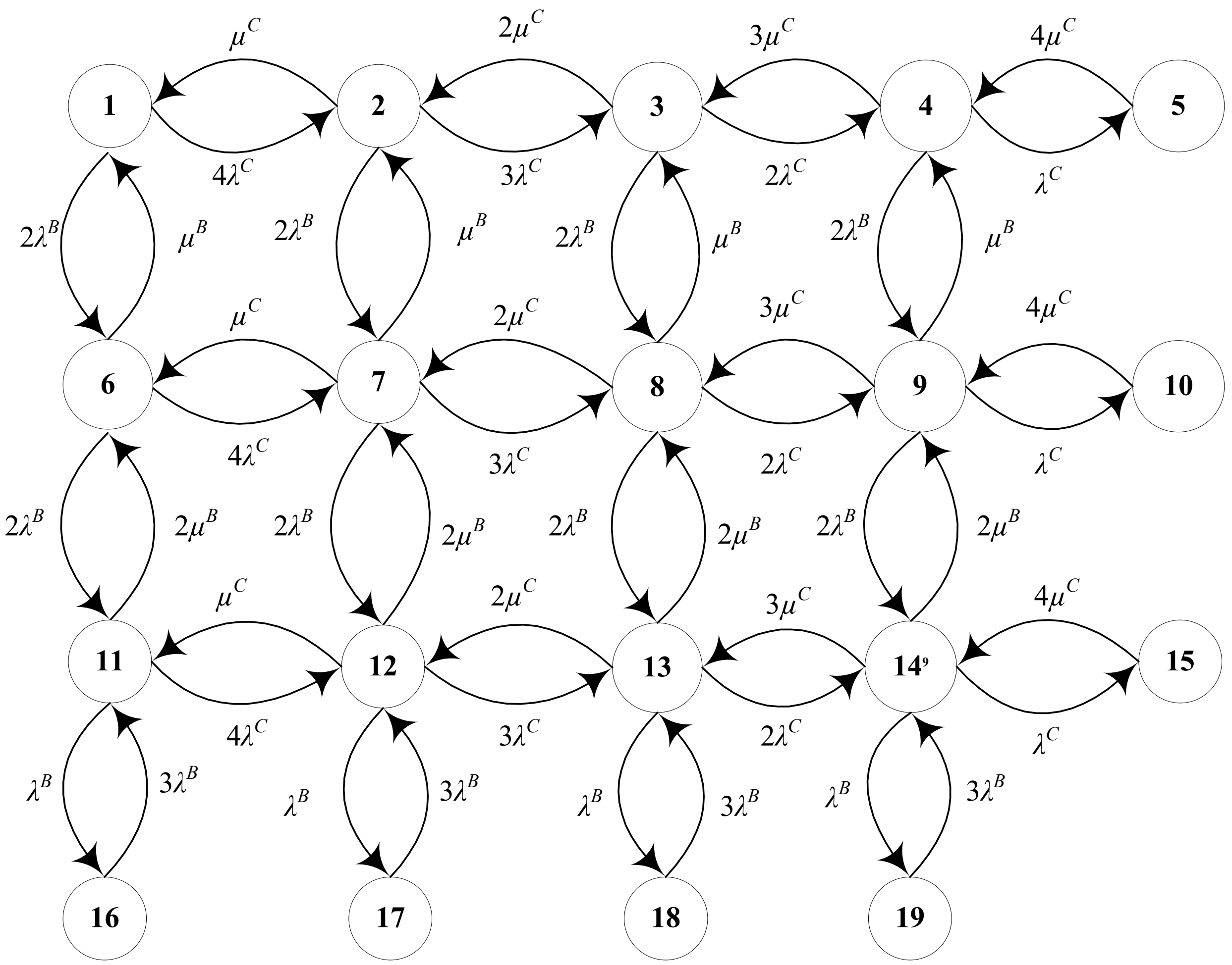

Figure 4. The state-space diagram for the refrigeration system with reserved blower

In states 1 and 6 the refrigeration system has full performance 10.5•10[9] BTU per year. In states 2 and 7 the refrigeration system has performance 7.9•10[9] BTU per year. The refrigeration system performance in states 3, 8, 11–13 is 5.2•10[9] BTU per year and in states 4, 9 and 14 is 2.6•10[9] BTU per year.

The required cool capacity demand as in previous case is 5•10[9] BTU per year, so only states 1, 2, 3, 6, 7, 8, 11, 12 and 13 are acceptable states and states 4, 5, 9, 10, 14, 15, 16–19 are unacceptable states.

The transition intensity matrix may be presented in the following form.

In order to find the MSS average availability $A(t)$ we should present the corresponding reward matrix in the following form:

$$\mathbf{r}^A = \begin{Bmatrix} r_{11} = r_{22} = r_{33} = r_{66} = r_{77} = r_{88} = \\ r_{11,11} = r_{12,12} = r_{13,13} = 1, \\ \text{all other elements are zero} \end{Bmatrix} \qquad (12)$$

By solving the systems of differential equations (1) with transition intensity matrix (11) and reward matrix $\mathbf{r}^A$ (12) we can obtain an MSS a average availability. The results of calculation are presented in *Figure 6*.

$$\boldsymbol{a} = \begin{pmatrix}
\boldsymbol{a}_{11} & 4\lambda^C & 0 & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu^C & \boldsymbol{a}_{22} & 3\lambda^C & 0 & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2\mu^C & \boldsymbol{a}_{33} & 2\lambda^C & 0 & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3\mu^C & \boldsymbol{a}_{44} & \lambda^C & 0 & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4\mu^C & \boldsymbol{a}_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu^B & 0 & 0 & 0 & 0 & \boldsymbol{a}_{66} & 4\lambda^C & 0 & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu^B & 0 & 0 & 0 & \mu^C & \boldsymbol{a}_{77} & 3\lambda^C & 0 & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu^B & 0 & 0 & 0 & 2\mu^C & \boldsymbol{a}_{88} & 2\lambda^C & 0 & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu^B & 0 & 0 & 0 & 3\mu^C & \boldsymbol{a}_{99} & \lambda^C & 0 & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4\mu^C & \boldsymbol{a}_{10,10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2\mu^B & 0 & 0 & 0 & 0 & \boldsymbol{a}_{11,11} & 4\lambda^C & 0 & 0 & 0 & \lambda^B & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\mu^B & 0 & 0 & 0 & \mu^C & \boldsymbol{a}_{12,12} & 3\lambda^C & 0 & 0 & 0 & \lambda^B & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu^B & 0 & 0 & 0 & 2\mu^C & \boldsymbol{a}_{13,13} & 2\lambda^C & 0 & 0 & 0 & \lambda^B & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu^B & 0 & 0 & 0 & 3\mu^C & \boldsymbol{a}_{14,14} & \lambda^C & 0 & 0 & 0 & \lambda^B \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4\mu^C & \boldsymbol{a}_{15,15} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\mu^B & 0 & 0 & 0 & 0 & \boldsymbol{a}_{16,16} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\mu^B & 0 & 0 & 0 & 0 & \boldsymbol{a}_{17,17} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\mu^B & 0 & 0 & 0 & 0 & \boldsymbol{a}_{18,18} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\mu^B & 0 & 0 & 0 & 0 & \boldsymbol{a}_{19,19}
\end{pmatrix} \qquad (11)$$

where

$$a_{11} = \lambda_{12} + \lambda_{16} \qquad a_{77} = \lambda_{78} + \lambda_{7,12} + \mu_{72} + \mu_{76} \qquad a_{13,13} = \lambda_{13,14} + \lambda_{13,18} + \mu_{13,8} + \mu_{13,12}$$

$$a_{22} = \lambda_{23} + \lambda_{27} + \mu_{21} \qquad a_{88} = \lambda_{89} + \lambda_{8,13} + \mu_{83} + \mu_{87} \qquad a_{14,14} = \lambda_{14,15} + \lambda_{14,19} + \mu_{14,9} + \mu_{14,13}$$

$$a_{33} = \lambda_{34} + \lambda_{38} + \mu_{32} \qquad a_{99} = \lambda_{89} + \lambda_{8,13} + \mu_{94} + \mu_{98} \qquad a_{15,15} = \mu_{15,14}$$

$$a_{44} = \lambda_{45} + \lambda_{49} + \mu_{43} \qquad a_{10,10} = \mu_{10,9} \qquad a_{16,16} = \mu_{16,11}$$

$$a_{55} = \mu_{54} \qquad a_{11,11} = \lambda_{11,12} + \lambda_{11,16} + \mu_{11,6} \qquad a_{17,17} = \mu_{17,12}$$

$$a_{66} = \lambda_{6,11} + \lambda_{67} + \mu_{61} \qquad a_{12,12} = \lambda_{12,13} + \lambda_{12,17} + \mu_{12,7} + \mu_{12,11} \qquad a_{18,18} = \mu_{18,13},\ a_{19,19} = \mu_{19,14}$$

In order to find the mean total number of blowers failures $N_f(t)$ we should present the corresponding reward matrix in the following form (13):

$$\mathbf{r}_{Nf} = \begin{Bmatrix} r_{16} = r_{27} = r_{38} = r_{49} = \\ r_{6,11} = r_{7,12} = r_{8,13} = r_{9,14} = \\ r_{11,16} = r_{12,17} = r_{13,18} = r_{14,19} = 1, \\ \text{all other elements are zero} \end{Bmatrix} \qquad (13)$$

By solving the systems of differential equations (1) with transition intensity matrix (11) and reward matrix $\mathbf{r}_{Nf}$ (13) we can obtain an MSS mean total number of blowers failures during time period [0, *T*], where *T*=1 year. The results of calculation are presented in *Figure 7*.

For computation of the Mean Time To Failure during the time interval the state space diagram of generated system like in previous case should be transformed - all transitions that return system from unacceptable states should be forbidden and all unacceptable states should be treated as absorbing state. The state space diagram is presented on *Figure 5*.

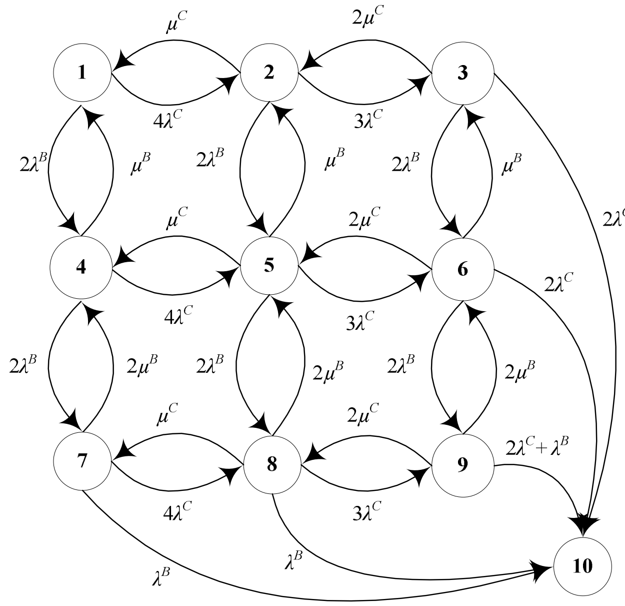

Figure 5. The state-space diagram for the refrigeration system with reserved blower with absorbing state

According to this state space diagram transition intensity matrix a can be presented as follows (14):

$$\boldsymbol{a}=\begin{pmatrix} \boldsymbol{a}_{11} & 4\lambda^C & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu^C & \boldsymbol{a}_{22} & 3\lambda^C & 0 & 2\lambda^B & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\mu^C & \boldsymbol{a}_{33} & 0 & 0 & 2\lambda^B & 0 & 0 & 0 & 2\lambda^C \\ \mu^B & 0 & 0 & \boldsymbol{a}_{44} & 4\lambda^C & 0 & \lambda^B & 0 & 0 & 0 \\ 0 & \mu^B & 0 & \mu^C & \boldsymbol{a}_{55} & 3\lambda^C & 0 & 2\lambda^B & 0 & 0 \\ 0 & 0 & \mu^B & 0 & 2\mu^C & \boldsymbol{a}_{66} & 0 & 0 & 2\lambda^B & 2\lambda^C \\ 0 & 0 & 0 & 2\mu^B & 0 & 0 & \boldsymbol{a}_{77} & 4\lambda^C & 0 & \lambda^B \\ 0 & 0 & 0 & 0 & 2\mu^B & 0 & \mu^C & \boldsymbol{a}_{88} & 3\lambda^C & \lambda^B \\ 0 & 0 & 0 & 0 & 0 & \mu^B & 0 & 2\mu^C & \boldsymbol{a}_{99} & \lambda^B \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (14)$$

where $a_{11}, a_{22}, ..., a_{99}$ are the same like in previous case. In order to find *Mean Time To Failure* we should present the reward matrixes *r* in the following form (15):

$$\boldsymbol{r}_{MTTF}=\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \quad (15)$$

By solving the systems of differential equations (1) with transition intensity matrix (14) and reward matrix r_{MTTF} (15) we can obtain an MTTF during time period.

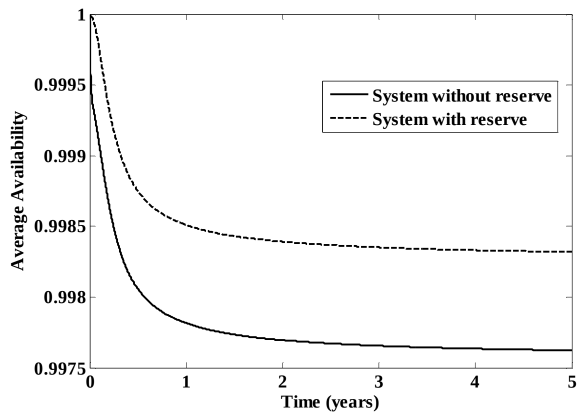

Figure 6. MSS average availability for different types of systems

Curves in *Figures 6* support the engineering decision-making and determine the areas where required performance deficiency level of the refrigeration system can be provided by configuration "with reserve" or by configuration "without reserve". For example, from the *Figure 6* one can conclude that the configuration "without reserve" cannot provide the required average availability if it is greater than 0.988.

Figure 7 presents mean total number of blowers' failures for different types of systems and gives logistics information for decision on spare parts supply, because long delay may occur if spares are not to hand when needed and holding spares costs money. From this figure one can conclude that mean total number of blowers' failures is not different for reserved and not reserved systems.

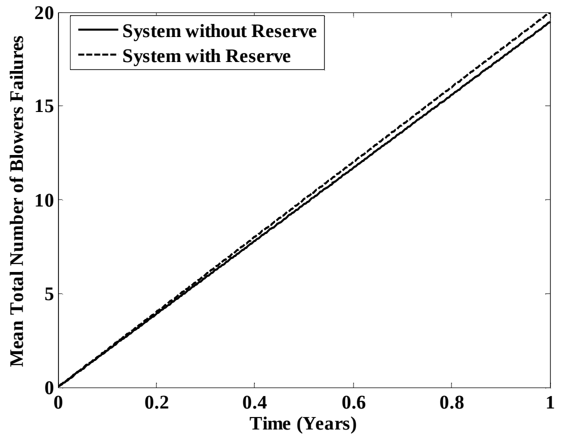

Figure 7. MSS mean total number of blowers' failures for different types of systems

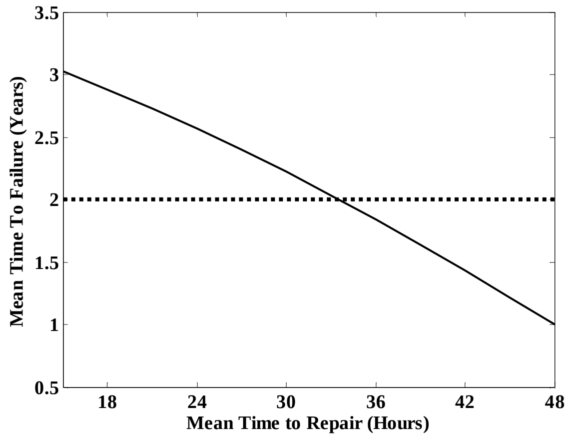

Figure 8. MSS Mean Time to Failure

From *Figure 8* one can see dependence of mean time to failure (MTTF) on mean time to repair for not reserved system, provided by different repair teams. Comparison with required level of MTTF, established for the Israel Supermarkets, shows that only repair teams with MTTR grater then 36 hours provide this required level. For reserved system mean time to system failure growth seven times and reach 14.75 years.

4. Conclusion

- The universal method was suggested to compute MSS reliability measures: average availability, total number of blowers' failures and MTTF. The method is based on different reward matrix determinations for an MSS model that is interpreted as a Markov Reward Model.
- The approach suggested is well formalized and suitable for practical application in reliability engineering. It supports the engineering decision-making and determines different system structures providing a required reliability/availability level of MSS.
- The numerical example is presented in order to illustrate the suggested approach.

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