AGH DRILLING, OIL, GAS • Vol. 30 • No. 1 • 2013

http://dx.doi.org/10.7494/drill.2013.30.1.19

**K.A. Alibayeva*, A.B. Kuljabekov*, M.S. Tungatorova*, A. Kaltayev***

# STUDY OF USAGE EFFICIENCY OF MULTISTAGE FILTERS ON MINERAL LEACHING PROCESS

## 1. INTRODUCTION

During the considering the problem in the bottomhole formation zone wellbore is considered as a surface with a constant reduced pressure. In work [12] it's shown that such assumption does not show a qualitative picture of the fluid flow in the bottomhole zone.

To construct an accurate mathematical model it's necessary to use Navier-Stokes equation for the interior of a vertical wellbore, and the filtration law for modeling the filtration in the reservoir. Strictly speaking, it would have had to sew two laws on the contact surface of a rock and filter. Such review requires enormous computing, as far as computational grid must be sufficiently thick to cover the interior of the wellbore [1].

Therefore, the fluid flow in the wellbore is approximately regarded as filtering in a fictitious porous medium with an apparent permeability $k_1$, which allows using filtration law at the bottomhole zone instead of the Navier-Stokes equation. In practice, the value of $k_1$ is determined by many factors (type of construction of the filter, its porosity, the shape of the perforations, etc., Fig. 1).

## 2. MATHEMATICAL FORMULATION OF THE PROBLEM

The law of fluid motion in the wellbore is defined by following relation

$$\frac{dP}{dz} = -f(u) \tag{1}$$

where $f(u)$ – given function, satisfying the condition $f(0) = 0$. For the function $f(u)$ it's possible to use linear, binomial or power-law motion according to the intensity of well work [1].

* Al-Farabi Kazakh National University, Almaty, Kazakhstan

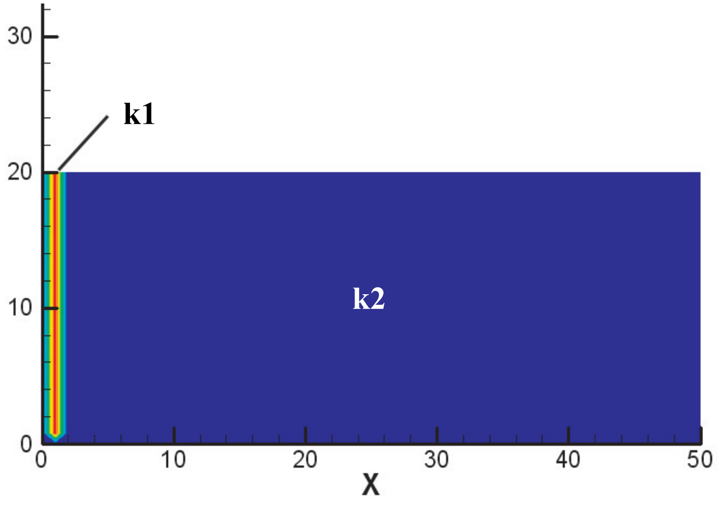


**Fig. 1.** Bottomhole zone with permeability of $k_2$.
Producing well with permeability of $k_1$ noted by red color

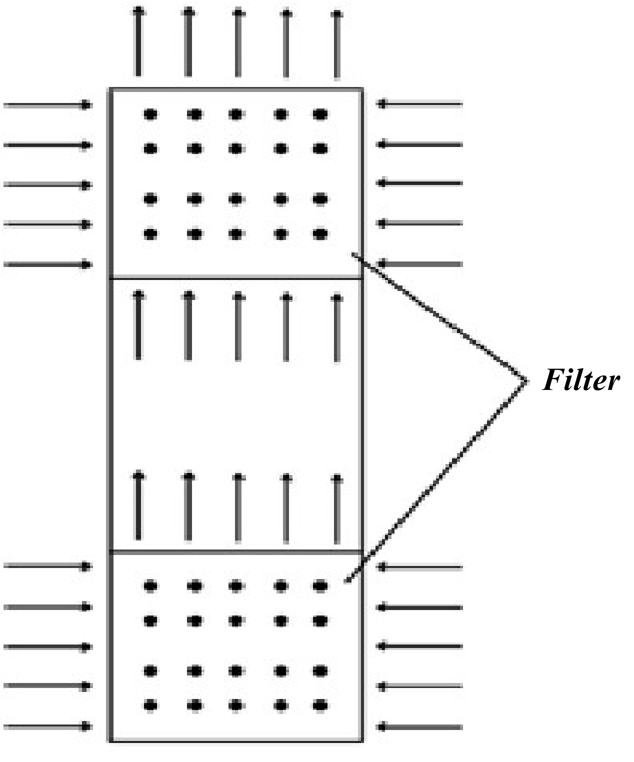


**Fig. 2.** Multistage scheme of filters setting

Let's consider the fluid flow to the well with multistage scheme of filters setting (Fig. 2). The law of motion in the wellbore will be taken on the law (1), and it is considered that the movement in the wellbore obeys a linear law and

$$f(u) = \frac{\mu u}{k_2} \tag{2}$$

where:

$\mu$ – liquid viscosity,
$k_2$ – fictitious permeability in the wellbore,
$u$ – is vertical speed in the wellbore.

Radial inflow of fluid to the well filter is defined by Dupuit formula.

$$Q = \frac{2\pi k_1 h}{\mu}\left(\frac{P_0 - P_w}{\ln\left(\frac{R}{r_c}\right)}\right) \tag{3}$$

where:

$k_1$ – rock permeability,
$P_0$ – pressure on external boundary (reservoir pressure),
$P_w$ – pressure on the well,
$R$ – radius of external boundary,
$r_c$ – well radius,
$h$ – seam thickness.

The flow in the bore between the planes $z$ and $z + dz$ is considering. From (3) it's obtain the fluid flow to the lateral surface of the well (Fig. 3).

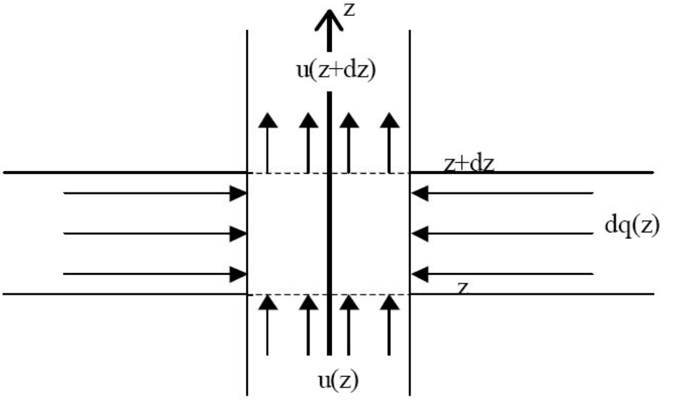


**Fig. 3.** Scheme of fluid flow inside the wellbore in a zone with filter

$$dq(z) = \frac{2\pi k_1 dz}{\mu}\left(\frac{P_0 - P(z)}{\ln\left(\frac{R}{r_c}\right)}\right) \tag{4}$$

here *P*(*z*)– reduced pressure along the well altitude. From the mass conservation law it's clear that the amount of flow passing through the lateral surface of the volume and through a section of z should be equal to the flow through the cross section $z + dz$. In case of the well zones, where no filters, we will not consider the radial flow to the well, but only flow within the channel is considered, i.e. set $dq = 0$ (Fig. 4).

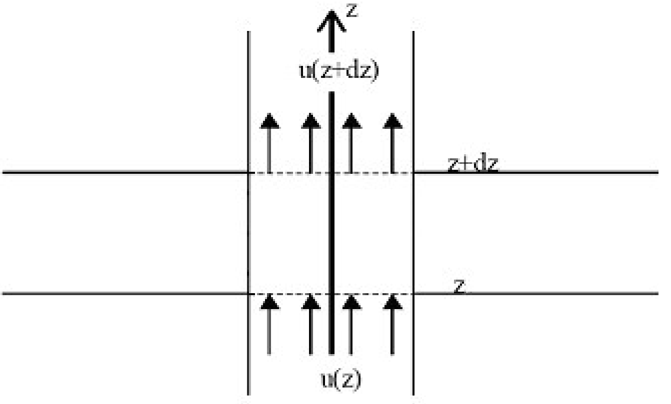


**Fig. 4.** Scheme of fluid flow inside the wellbore in a zone without filter

From the mass conservation law it's obtained

$$\begin{cases} \pi r_c^2 u(z+dz) = dq(z) + \pi r_c^2 u(z), & \text{for a zone with filter,} \\ \pi r_c^2 u(z+dz) = \pi r_c^2 u(z), & \text{for a zone without filter,} \end{cases} \tag{5}$$

or

$$\begin{cases} u(z+dz) - u(z) = \dfrac{dq(z)}{\pi r_c^2}, & \text{for a zone with filter,} \\ u(z+dz) - u(z) = 0, & \text{for a zone without filter.} \end{cases} \tag{6}$$

Since the law of motion in the wellbore is defined through the relation (1),

$$\frac{dP(z+dz)}{dz} = -f(u(z+dz)) \tag{7}$$

Subtracting from (7) the relevant parts of (1), it's obtained

$$\frac{dP(z+dz)}{dz} - \frac{dP(z)}{dz} = -\left[f(u(z+dz)) - fu(z+dz)\right] \tag{8}$$

The equation (8) is rewrote in the form

$$\frac{dP(z+dz)}{dz} - \frac{dP(z)}{dz} = -\left[u(z+dz) - u(z+dz)\right]\frac{\left[f(u(z+dz)) - fu(z+dz)\right]}{u(z+dz) - u(z+dz)} \tag{9}$$

applying Lagrange's theorem, and $dz \to 0$ then we get

$$\begin{cases} \dfrac{d^2P(z)}{dz^2} = -\left[\dfrac{q(z)}{\pi r_c^2}\right] f'(u), & \text{for a zone with filter,} \\ \dfrac{d^2P(z)}{dz^2} = 0, & \text{for a zone without filter.} \end{cases} \tag{10}$$

Here, the derivatives on the right side of the equation are taken with respect to $u$. Equation (10) describes the motion of the fluid along the bore, with the assumption that the radial flow inside the bore can be neglected. If we define the function $f(u)$ in the form (2), then from (10) and (4) it's obtained

$$\begin{cases} \dfrac{d^2P(z)}{dz^2} = -\dfrac{2k_1}{r_c^2 k_2}\left(\dfrac{P_0 - P(z)}{\ln\left(\dfrac{R}{r_c}\right)}\right), & \text{for a zone with filter,} \end{cases} \tag{11}$$

$$\frac{d^2P(z)}{dz^2} = 0, \qquad \text{for a zone without filter.}$$

The system equation (11) takes into account the motion of the fluid within the wellbore, the radial flow to the well, the pressure distribution within the well on height and heterogeneity on setting the filters. In addition, the solution of this problem can be applied as a boundary condition for the 3D case, as far as the inflow to the well taken into account by the Dupuit formula. The $k_2$ – can be varied in section of the well, where the filters is located. In this case, we take $k_2$ = const over the entire height of the well.

The boundary conditions for (11) will be a condition on the reservoir roof $z = b$ and on the bottom $z = 0$, and we write

$$P\big|_{z=b} = P_c \quad \text{и} \quad \left.\frac{dP}{dz}\right|_{z=0} = 0 \tag{12}$$

The solution of (11), (12) can be obtained using software for the solution of systems of ordinary differential equations or directly numerically solving.

The flow to the well with multi-stage planted filter in the reservoir with a seam thickness $b = 28m$ is considered. Filters are defined on the heights $z \in [5m;\, 11m]$ and $z \in [17m;\, 21m]$, and the coordinate $z$ directed vertically upwards. In this case the system (11) as follows

$$\begin{cases} \dfrac{d^2P(z)}{dz^2} = -\dfrac{2k_1}{r_c^2 k_2}\left(\dfrac{P_0 - P(z)}{\ln\left(\dfrac{R}{r_c}\right)}\right), & \text{at } z \in [5m;11m] \text{ or } z \in [17m;21m], \\ \dfrac{d^2P(z)}{dz^2} = 0, & \text{at } z \notin [5m;11m] \text{ or } z \notin [17m;21m]. \end{cases} \tag{13}$$

with boundary conditions $P\big|_{z=b} = P_c$ and $\left.\dfrac{dP}{dz}\right|_{z=0} = 0.$

After solving the problem (13) the pressure distribution on well height is obtained, then using (4) flow rate can be find at each level of $z$.

Results of the solution of (13) is presented at different ratios and coefficients $k_1$ and $k_2$.

The graphics are shown for the flow rate $q(z)$ for the case when filters are set through the production zone and for the multilevel setting at various ratios of filtration coefficients (Fig. 5–8).

Figure 9 shows a qualitative comparison of the calculation results with the data from the industrial experiment. Green curve shows the flow rate $q(z)$ at multi-stage filter set, red curve for fully filter along the height of $z$.

Problem has been treated in a cylindrical coordinate system. Like the previous problem well is modeled as a medium with an apparent permeability, depending on the porosity of the filter.

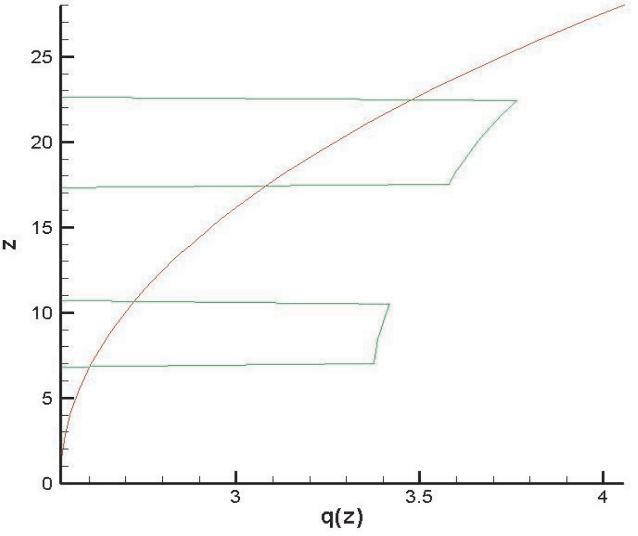


**Fig. 5.** Flow rate of liquid throughout the well. The green curve shows the flow rate $q(z)$ at multilevel setting of filters for the problem (13), red curve for the full filter along the height of $z$

$$\frac{k_1}{k_2} = 0.0001, \frac{P_0}{P_c} = 1.3$$

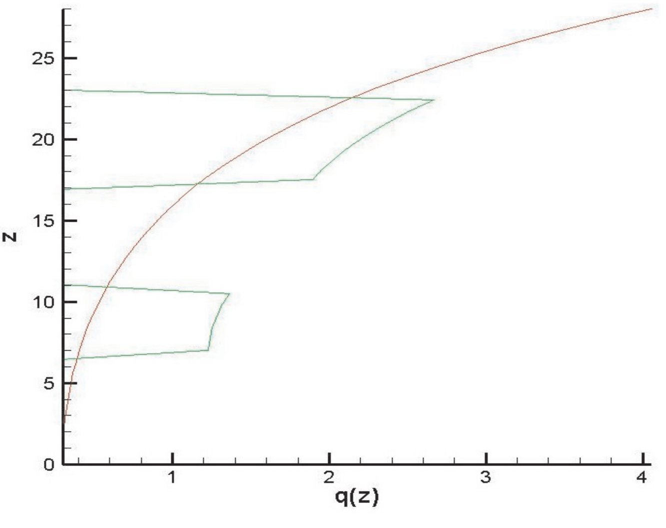


**Fig. 6.** Flow rate of liquid throughout the well. The green curve shows the flow rate $q(z)$ at multilevel setting of filters for the problem (13), red curve for the full filter along the height of $z$

$$\frac{k_1}{k_2} = 0.001, \frac{P_0}{P_c} = 1.3$$

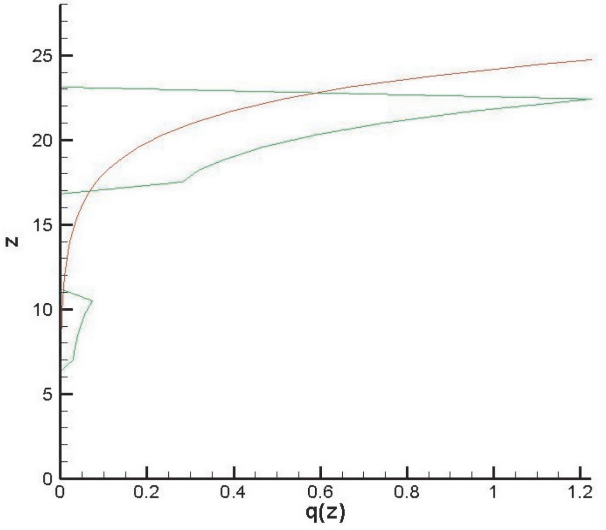


**Fig. 7.** Flow rate of liquid throughout the well. The green curve shows the flow rate $q(z)$ at multilevel setting of filters for the problem (13), red curve for the full filter along the height of $z$

$$\frac{k_1}{k_2} = 0.01, \frac{P_0}{P_c} = 1.3$$

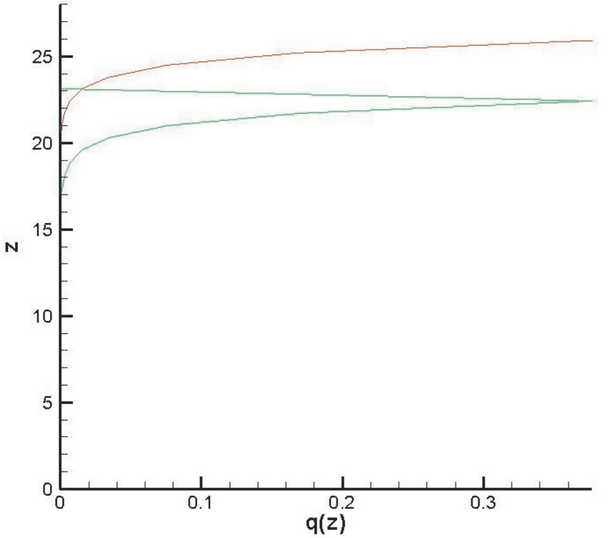


**Fig. 8.** Flow rate of liquid throughout the well. The green curve shows the flow rate $q(z)$ at multilevel setting of filters for the problem (13), red curve for the full filter along the height of $z$

$$\frac{k_1}{k_2} = 0.1, \frac{P_0}{P_c} = 1.3$$

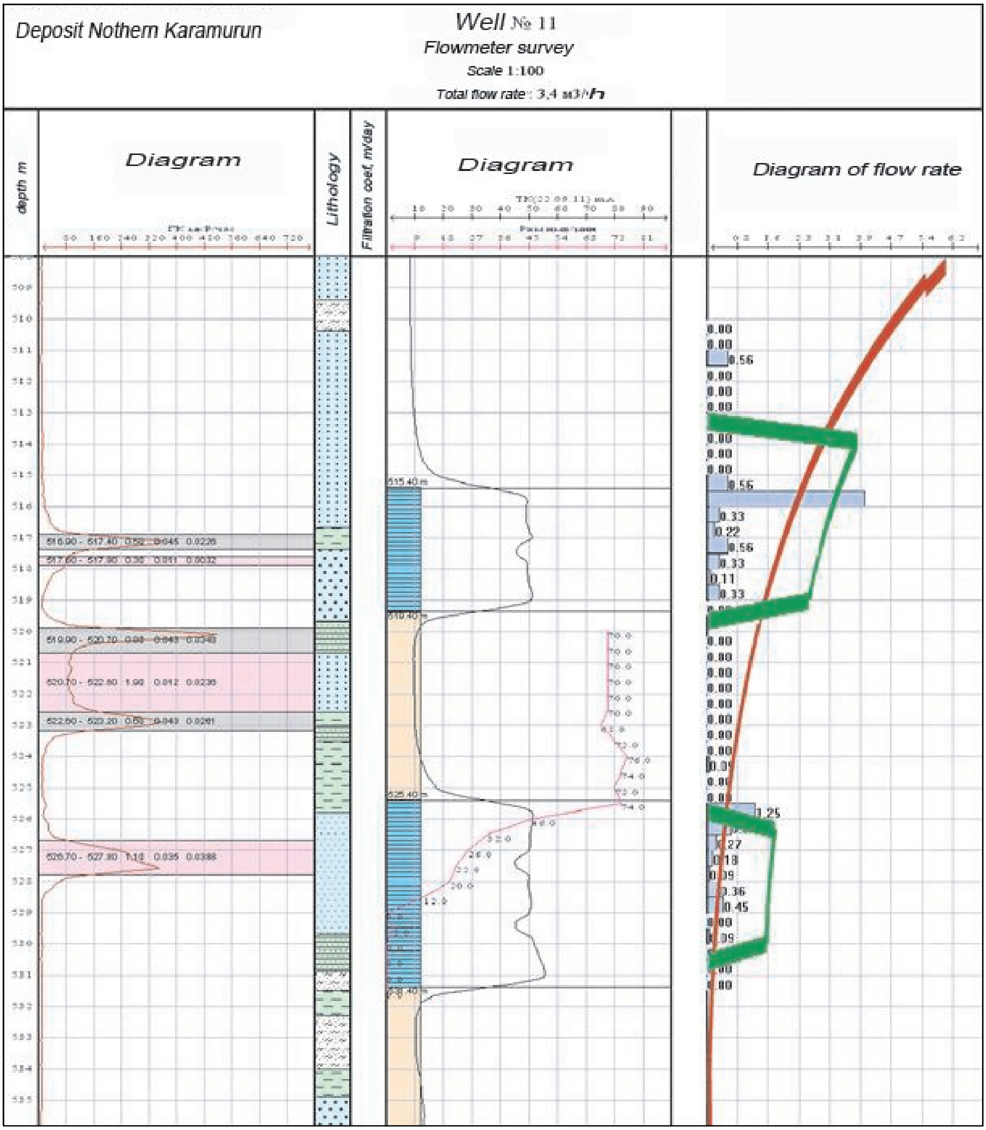


**Fig. 9.** Qualitative comparison of the calculation results with the data from the industrial experiment. The green curve shows the flow rate $q(z)$ at multi-stage setting filters for the problem (13), red curve – for full filter along the height $z$

## 3. 3D MODEL OF THE PROCESS

Fluid filtration in porous media with certain permeability is considering. Using Darcy law and conservation law these processes are described by the following equations

$$div\left(\frac{K}{\mu} grad\ p\right) = 0 \tag{14}$$

$$\vec{V} = -\frac{K}{\mu} grad\ p \tag{15}$$

with boundary conditions

$$\left.\frac{\partial p}{\partial n}\right|_S = 0, \quad p|_S = p_0 + \rho g z \tag{16}$$

where $p$ – hydraulic head in reservoir;

$$\frac{\partial C_m}{\partial t} = -\beta\theta C_r C_m \tag{17}$$

$$\frac{\partial \theta C_r}{\partial t} = div\left(\theta D grad C_r\right) - \vec{V} grad \theta C_r - \nu_1 \beta\theta C_m C_r + \sum_{i=1}^{n} q_i C_r^0 \delta\left(\vec{x} - \vec{x}_i\right) \tag{18}$$

$$\frac{\partial \theta C_p}{\partial t} = div(\theta D grad C_p) - \vec{V} grad \theta C_p + \nu_2 \beta\theta C_m C_r - \sum_{i=1}^{n} q_i C_p \delta\left(\vec{x} - \vec{x}_i\right) \tag{19}$$

where:

$\nu_1 = \nu_r R / \nu_m M,\ \nu_2 = \nu_p P / \nu_m M,$ $K$ – permeability coefficient,
$H$ – head pressure,
$\vec{V}$ – filtration rate,
$C_m$ – concentration of uranium in solid phase,
$C_r$ – initial content of mineral in layer,
– concentration of sulfuric acid in solution,
$C_r^0$ – concentration of reactant on injection well,
$C$ – concentration of useful element (uranium) in solution,
$q$ – debit of well ($q < 0$ for extraction well, $q > 0$ for inject well),
$\theta$ – porosity of layer,
$\beta$ – coefficient, characterizing reaction rate,
$D$ – hydrodynamic dispersion coefficient.

Transfer equations of reagent concentration in liquid phase (18), useful element concentration in solid phase (17), and its transition to liquid phase (19) are solved together by Crank-Nicolson scheme at initial and boundary conditions

$$
\begin{aligned}
&C_m\big|_{t=0} = C_m^0, \; C_r\big|_{t=0} = C_r^0, \; C_p\big|_{t=0} = C_p^0 \\
&\left.\frac{\partial C_m}{\partial x}\right|_G = 0, \quad \left.\frac{\partial C_m}{\partial y}\right|_G = 0, \quad \left.\frac{\partial C_m}{\partial z}\right|_G = 0, \\
&\left.\frac{\partial C_r}{\partial x}\right|_G = 0, \quad \left.\frac{\partial C_r}{\partial y}\right|_G = 0, \quad \left.\frac{\partial C_r}{\partial z}\right|_G = 0 \\
&\left.\frac{\partial C_p}{\partial x}\right|_G = 0, \quad \left.\frac{\partial C_p}{\partial y}\right|_G = 0, \quad \left.\frac{\partial C_p}{\partial z}\right|_G = 0.
\end{aligned} \tag{20}
$$

The scheme of well locations is shown in Figure 10. Owing to the symmetry calculation is realized for block of deposit, consisting of three wells: two injection and one extraction. The results of calculation for the pressure and concentration distribution are shown in Fig. 11–15.

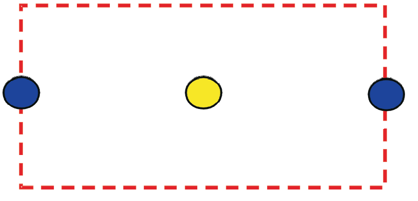

**Fig. 10.** Schematic picture of considering area
(2 injection wells on the border and one extraction well in the middle of the are)

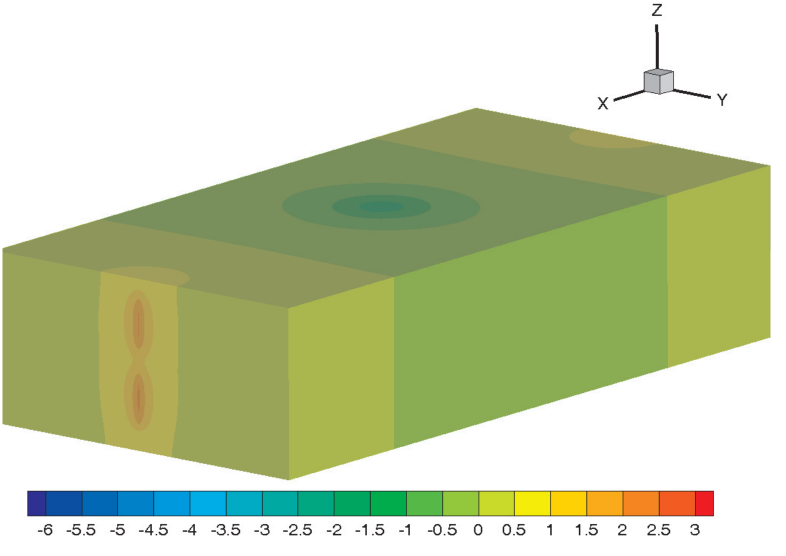


**Fig. 11.** Distribution of head pressure

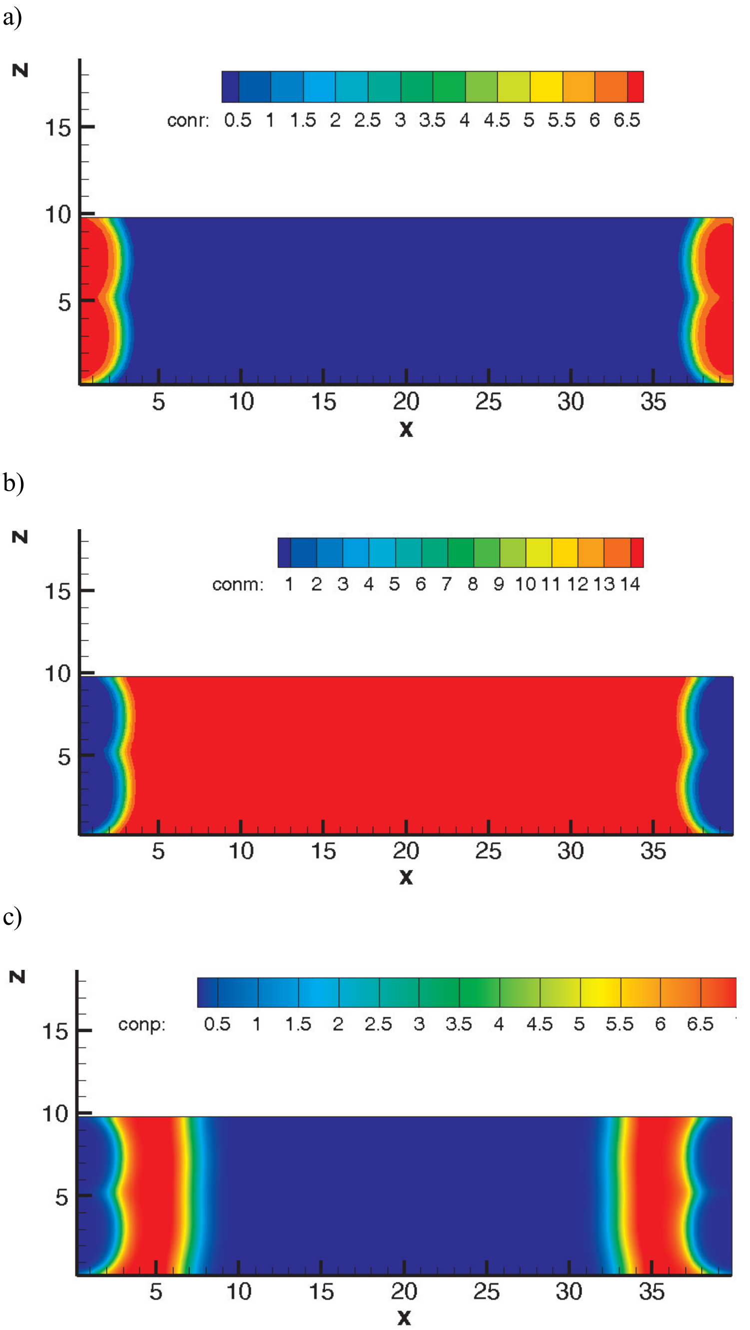


**Fig. 12.** Distribution of concentration of reagents (a), minerals in solid phase (b), minerals in liquid phase (c) in layer at T = 7.5 days (cross section along the well)

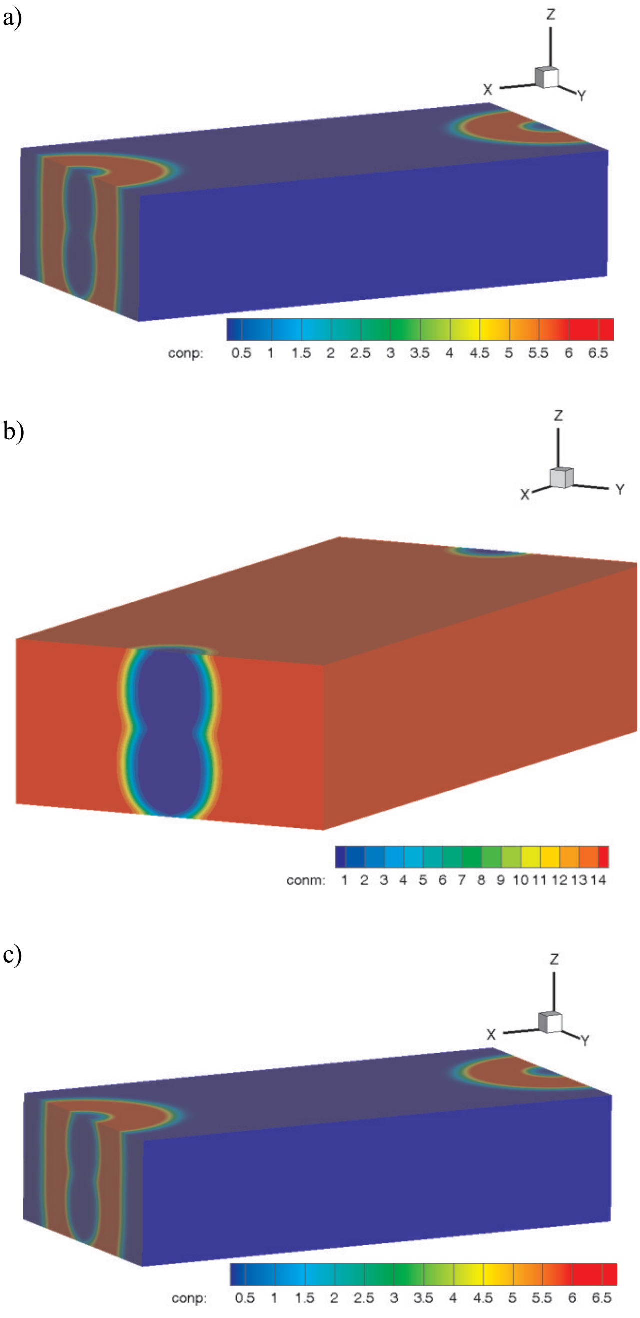


**Fig. 13.** 3D distribution of concentration of reagents (a), minerals in solid phase (b), minerals in liquid phase (c) in layer at T = 7.5 days

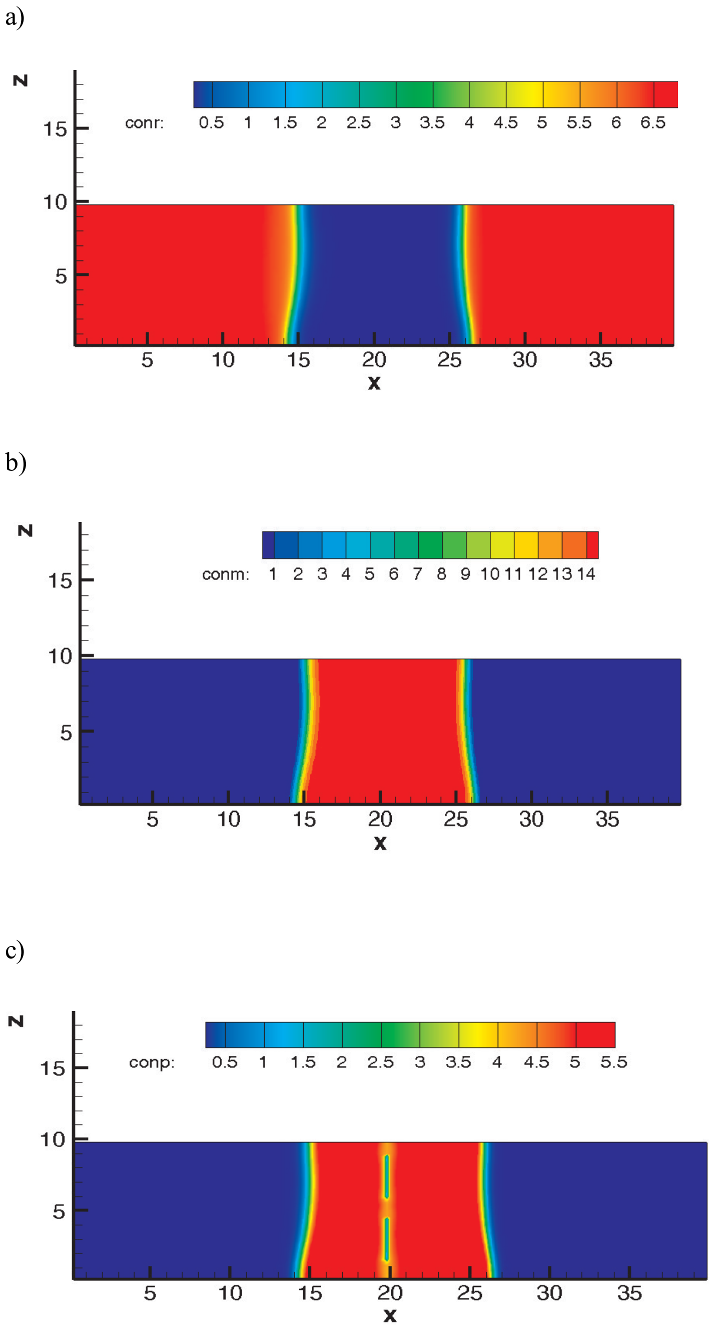


**Fig. 14.** Distribution of concentration of reagents (a), minerals in solid phase (b), minerals in liquid phase (c) in layer at T = 165 days (cross section along the well)

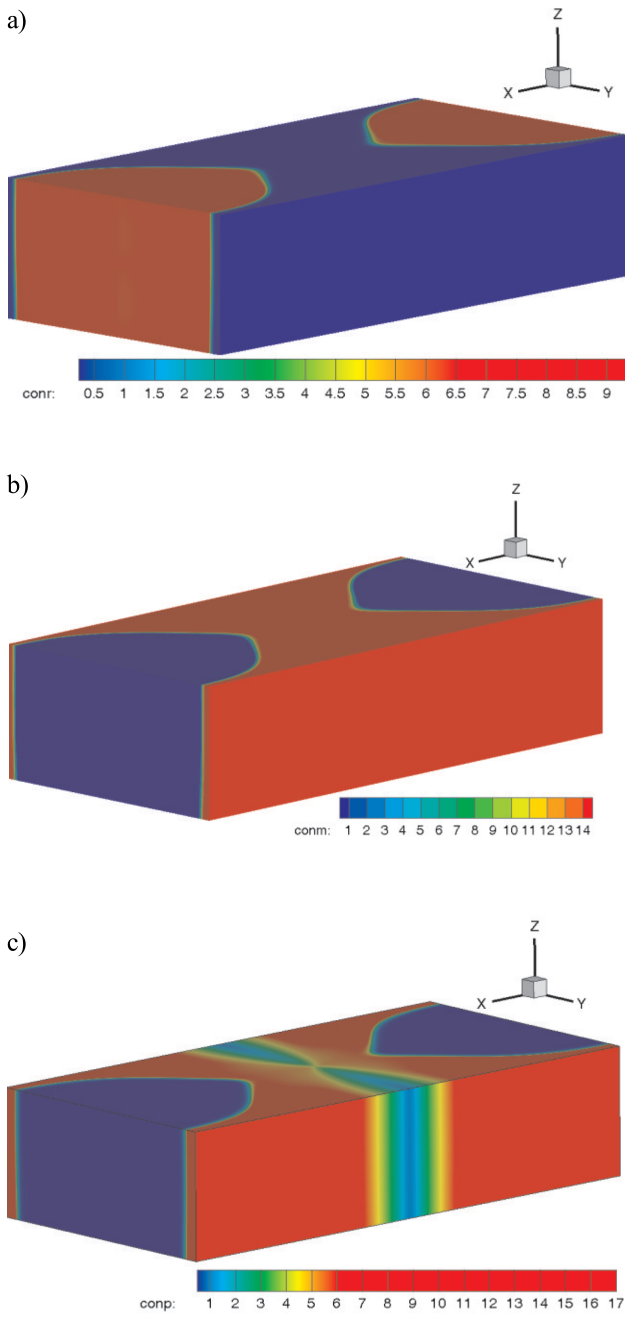


**Fig. 15.** 3D distribution of concentration of reagents (a), minerals in solid phase (b), minerals in liquid phase (c) in layer at T = 165 days

## 4. CONCLUSION

Due to the value increase of non-renewable mineral resources and the rapid growth of cost of wells repair in around the world pay special attention to the correct initial well completion. Maximum reliability and productivity have an especially importance for wells located in hard-to-reach places (sand, sand dunes). To achieve reliably and well productivity is particularly difficult where the reservoir sands are not cemented or otherwise, tend to destruction. Mechanism of carrying out of the sand is unusually complex, and it turns out influence each well completion operation (from primary drilling of layer to develop the wells for sampling or injection).

In connection with this, mathematical model is built and computer program to calculate the hydrodynamic efficient of well set with possibility of seepage control.

The results of calculations show that the distribution of flow (inflow) on well height is not uniform. In the calculations the well accepted as high-permeability channel, depending on the construction of the filter (porosity and shape of the perforations). Coefficient of fictious permeability has a strong influence on the flow of liquid to the well (see Fig. 5–8). Calculations of second section were carried out for a single well, and it is easily applied to solve three-dimensional problem with three wells in the following section (third section).

Based on the results of the solution of this problem, it can be concluded that in case of stagnation of the lower zone of the well it is appropriate to apply the multi-stage setting of filters, as the usage of such filters in stagnation zone it's appeared non-zero radial flow.

## REFERENCES


[1] Tolpayev V.A., Zaharov B.B.: *Gidrodinamicheskie osobennosti techenija zhidkosti v prizaboinoi zone skvazhiny.* Vestnik SevKavGTU., Stavrapol 2003.

[2] Danaev N.T., Korsakov N.K., Penkovskii V.I.: *Massoperenos v priskvazhnoi zone i elektromagnitnyi karotazh plasta*. Qazaq universiteti, Almaty 2005.

[3] MT3DMS: *A Modular Three-Dimensional Multispecies Transport Model for Simulation of Advection, Dispersion, and Chemical Reaction of Contaminants in Groundwater Systems*. Documentation and User's Guide, by Chunmiao Zheng, P. Patrick Wang, Department of Geological Sciences, University of Alabama 1999, 160 p.

[4] Shestakov V.M.: *Gidrogeodinamika*. Izdatelstvo MGU, Moskva 1995.

[5] Samarskii A.A., Nikolayev E.S.: *Metody reshenija setochnyh uravnenii*. Nauka, Moskva 1978.

[6] Marchuk G.I.: *Metody rasshcheplenija*. Nauka, Moskva 1988.

[7] *Dobycha urana metodom podzemnogo vyshchelachivanija*. Pod red. Mamilova V.A. Atomizdat, Moskva 1980.

[8] Beletskii V.I., Bogatkov L.K., Volkov N.I. i dr.: *Spravochnik po geotechnologii urana. Energoatomizdat*, Moskva 1997.

[9] Grabovnikov V.A.: *Geotechnologicheskie issledovanie pri razrabotke metallov*. Nedra, Moskva 1955.

[10] Alibayeva K.A., Kuldzabekov A.B., Kaltayev A.: *The modeling of uranium extraction process by the in-situ leaching method*. Krakow, Drilling Oil and Gas. Quarterly, vol. 27, No. 1–2, 2010, 49–57.

[11] Alibayeva K.A., Tungatarova M.S., Kaltayev A.: *The modeling of uranium extraction by the ISL method*. Articles of the third congress of the world mathematical society of Turkic countries. Almaty, Kazakhstan 2009.

[12] Brovin K.G., Grabovnikov V.A. and etc.: *Prognoz, poiski, razvedka i promyshlennaja otsenka mestorozhdenii urana dlja otrabotki podzemnym vyshchelachivaniem*. Gylym, Almaty 1997.