International Journal of Occupational Safety and Ergonomics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tose20

Theoretical Analysis of Percussive Tests of Products

Ryszard Korycki[a]
[a] Central Institute tor Labour Protection, Poland
Published online: 08 Jan 2015.

INTERNATIONAL JOURNAL OF
OCCUPATIONAL SAFETY AND ERGONOMICS 1998, VOL. 4, NO. 4, 423–448

Theoretical Analysis of Percussive Tests of Products

Ryszard Korycki

Central Institute for Labour Protection, Poland

The goal of theoretical research is to establish parameters, which have to be given in standards for percussive tests of products. Those parameters are essential for each user to be able to construct identical (equivalent) testing equipment. This would ensure identical results for identical products.

The paper presents a detailed analysis of the distribution and the value of the forces generated during percussive collisions of two bodies. Elastic, plastic, and elastoplastic collisions are considered. Parameters determining the coefficient of restitution, the courses of energy, momenta, and the values of the forces in colliding elements are determined. The dynamic force acting on a product during a percussive test was studied.

dynamics percussive tests parameters analysis energy theory

1. INTRODUCTION

Every product, especially personal protective equipment, has specific application requirements. Percussive tests make testing those features possible. This can be done at special test stands only. The kinds of tests as well as the testing methods are determined by new European standards.

Purchasing test stands for percussive tests of products imposed by the Polish standard PN-79/Z-08020 (Polski Komitet Normalizacji, Miar i Jakości, 1979) and by European standards is impossible. Test stands like that are not available. There are a few custom-built stands for dynamic tests of products in the largest and best-equipped West European research institutes. Those test stands are used for research, for

Correspondence and requests for reprints should be sent to Ryszard Korycki, Central Institute for Labour Protection, ul. Wierzbowa 48, 90-143 Łódź, Poland.

services for the industry, and for certification. All research is strictly confidential and its results are not published except for the results of certification research.

Those who create technical standards often do not know the theory of percussive tests or the dynamic interaction between elements. The standards impose parameters to be tested as well as methods to be used in testing those parameters. Frequently, however, the standards do not contain all the parameters necessary for creating identical test conditions, which makes achieving comparable test results impossible. This can be observed in the old Polish standard PN-79/Z-08020 (Polski Komitet Normalizacji, Miar i Jakości, 1979), the new European standard EN 397:1995 (Comité Européen de Normalisation, 1995), and the national standards BS 5240:Part 1:1987 (British Standard, 1987), DIN 4840:1981 (Deutsches Institut für Normung, 1981) on testing industrial safety helmets and testing harness preventing falls from a height with the use of a rigid dummy.

Using theoretical literature on percussive tests and the dynamic phenomena that occur at collision, requires mastery of technical dynamics. This drastically limits the number of users who can be critical of the implemented standards.

In international literature, there is no comprehensive discussion of the problems of theoretical and empirical percussive tests, or of the practical use of standards. There are only a few publications dealing with those problems (e.g., Hoppmann, 1988; Timoshenko & Goodier, 1951) and a relatively great number of publications devoted to the collision theory (e.g., Gryboś, 1970; Kowalski, 1976; Leyko, 1975; Timoshenko & Young, 1962). This paper is devoted to a theoretical analysis of percussive tests. The second part, which develops those problems as a practical application of the existing stands, is now in preparation.

The results of theoretical considerations are used in tests of industrial safety helmets. Two basic percussive tests of helmets imposed by both the Polish standard PN-79/Z-08020 (Polski Komitet Normalizacji, Miar i Jakości, 1979) and by foreign standards (e.g., EN 397:1995; Comité Européen de Normalisation, 1995) are

- the shock absorption test, in which the force transferred from the helmet to the model of a head is measured; and
- the resistance to penetration test, in which the fact that the punch touches the model of a head is established (this takes place when the helmet is punctured).

The operation of the stand is in both cases identical. The differences consist in the shape and the size of the falling mass m_1 and the registration of the results. Figure 1 shows a diagram of a stand for percussive tests of industrial safety helmets.

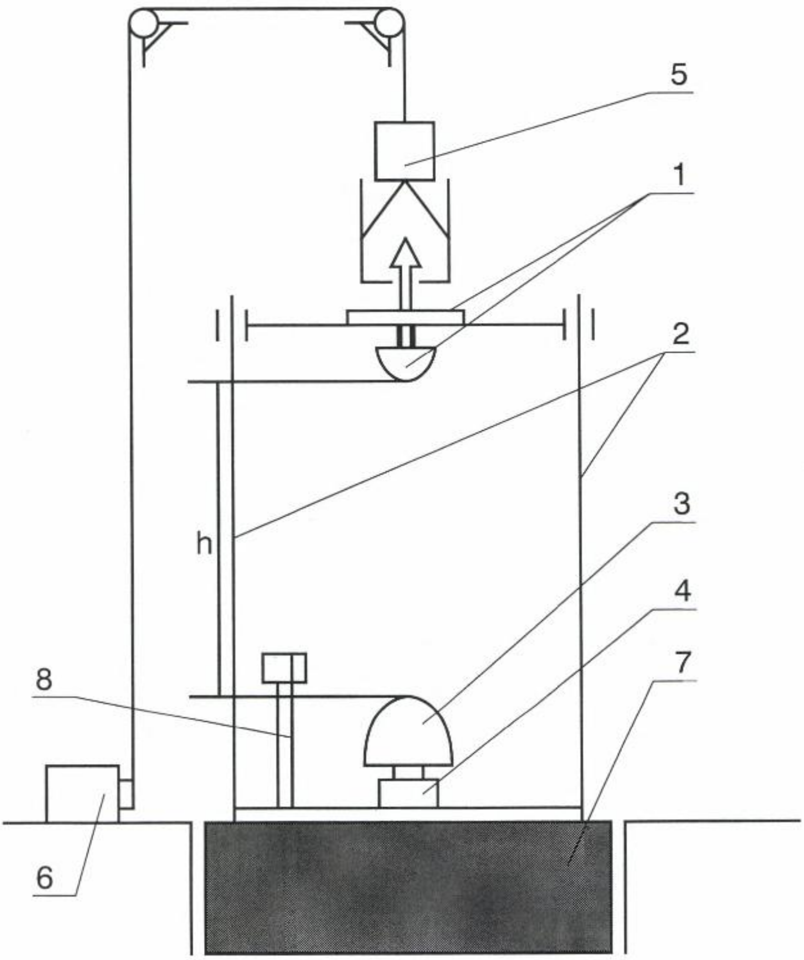

Figure 1. A stand for percussive tests of industrial safety helmets. *Notes.* 1—striking mass m_1; 2—guide poles; 3—struck mass m_2; 4—base of the helmet with a device for electronic measurement of the striking force; 5—electromagnetic release of mass m_1; 6—a jack for mass m_1; 7—monolith concrete foundation; 8—electronic measurement of the impact velocity of mass m_1; h—fall height.

The operation of the device is very simple. The striking mass m_1 lifted by a jack (No. 6 in Figure 1) falls freely on the stationary product of mass m_2 situated on the following structure: the headform—the basis—the monolith concrete foundation. All parts of this structure are fixed to one another, the summary mass is greater than 500 kg, so consequently they are practically immovable. Guide poles (No. 2 in Figure 1) and a vertical positioning of the stationary mass m_2 in relation to the striking mass m_1 ensure a simple central collision. A release device (e.g., the electromagnet, No. 5 in Figure 1) is used to release mass m_1.

It is difficult to construct

- electronic devices for measuring the velocity of mass m_1 when it strikes mass m_2, and the dynamic force acting on mass m_2 when it is struck by mass m_1; and
- a test stand with a rigid frame, maximally reduced friction of the guide poles, and a monolith concrete foundation of over 500 kg.

The helmet (the product) is fixed on the headform by using a strap system made of elastic material. The helmet during the collision can be characterized by the following assumptions:

- the punch strikes centrally in the crown part of the helmet (the product). There is also a central collision of the two masses. Thus, the dynamic force operating during the strike is located centrally in relation to the straps and is wholly transmitted on the basis, taking into consideration the small values of the strap displacements. The displacements of the strap system caused by the collision can also be neglected. The calculated and analyzed movement of the stationary mass m_2 after the strike is also a result of the collision of the two masses and is characterized by the coefficient of restitution k;
- the energy dissipation during the collision caused by the elastic straps is relatively small, too;
- there is no equality between the elastic straps and the elastic support as described, for example, by Hoppmann (1988) in relation to the mass striking an elastic plate. The straps do not generate the return impulse of the form and the value shown, for example, for this plate. In addition, it is difficult to define the interaction and the mathematical description of the dynamic response for the straps during the collision contact.

We can also assume that a model of the collision phenomenon is the falling mass m_1 striking centrally the stationary mass m_2. The physical model can be obtained by a concentration of masses m_1 and m_2 in both centres of gravity. Both masses are perfectly elastic and the model can be next described as the concentrated mass m_1 falling centrally on the

TABLE 1. Basic Parameters of Percussive Tests of Helmets

Standard	Shock Absorption Test			Resistance to Penetration		
	Falling Mass (kg)	Height of Fall (m)	Energy of Striking (J)	Falling Mass (kg)	Height of Fall (m)	Energy of Striking (J)
BS 5240:Part 1:1987	$5.0^{+0.1}_{-0}$	1 ± 0.005	49	$3.0^{+0.45}_{-0}$	1 ± 0.005	not given
DIN 4840:1981	3	1.5	not given	1.5	1.5	not given
EN 397:1995	$5.0^{+0.1}_{-0}$	1 ± 0.005	50	$3.0^{+0.05}_{-0}$	1 ± 0.005	not given
ISO 3873:1977	$5.0^{+0.1}_{-0}$	1 ± 0.005	50	$3.0^{+0.05}_{-0}$	1 ± 0.005	not given
PN-79/Z-08020	depends on type of helmet					
PN-86/Z-08110	not given	1.5	50 ± 1	not given	1.5	30 ± 0.5

Notes. BS 5240:Part 1:1987—British Standard (1987), DIN 4840:1981—Deutsches Institut für Normung (1981), EN 397:1995—Comité Européen de Normalisation (1995), ISO 3873:1977—International Organization for Standardization (1977), PN-79/Z-08020—Polski Komitet Normalizacji, Miar i Jakości (1979), PN-86/Z-08110—Polski Komitet Normalizacji, Miar i Jakości (1986).

concentrated stationary mass m_2. The falling height can be also defined as the initial distance between these two centres of gravity, that is, for velocity $V_1 = 0$.

The technical stands developed so far, impose only two or three parameters determining the conditions for conducting tests of shock absorption and resistance to penetration (Table 1).

Considerable differences in parameters can be observed in this comparison. This is caused by the fact that the authors of standards do not always quote all the parameters that have a decisive influence on the course of a dynamic collision of two bodies. The differences in the values of the dynamic forces acting on mass m_2 of the same product according to the standards listed in Table 1 reach 18%. Research resulted in establishing all the parameters of a collision of two bodies. The results of the theoretical research were used in the preparation of an updated Polish standard PN-86/Z-08110/08 (Polski Komitet Normalizacji, Miar i Jakości, 1986).

In dynamic tests—particularly percussive ones—it is difficult to determine the kinds, values, and courses of the energies, momenta, and forces in the colliding bodies. These difficulties occur in both theoretical analysis and tests. This is a result of the impulse effect of the forces in colliding bodies. They usually run from 0.01 to 0.0001 s.

The theoretical analysis consists of three basic parts:

1. In the first part, theoretical mechanics was used to analyse the influence of the coefficient of restitution on the collision of two bodies (on the kind of collision, on the loss of kinetic energy of the colliding masses, and on the transfer of kinetic energy between the colliding bodies during the collision).
2. The basic part consisted of a detailed analysis of the collision of the two masses, in order to describe the phenomena that take place and to establish all the parameters that have a decisive influence on the course and the value of these phenomena. Particular attention was paid to the courses of energy in the colliding masses. The main aim of the analysis was to determine the striking force on the struck mass during the collision. Without a knowledge of that force and the parameters that determine its volume and course, it is impossible to optimize the product (the helmet).
3. The third part consisted of an analysis of the obtained results. The courses of the functions were studied, their extreme values and all parameters describing the course of a percussive test of a product

were determined. It was decided which of the parameters have to be quoted in standards in order to enable different users to achieve identical test results for identical products.

This theoretical analysis is used in tests and research concerning the evaluation of industrial safety helmets, that is, personal protective equipment. Both changing existing standards and constructing test stands is involved. In particular,

1. It was demonstrated that percussive tests conducted according to the Polish standard PN-79/Z-08020 (Polski Komitet Normalizacji, Miar i Jakości, 1979) give higher-value results in relation to the results of tests conducted according to similar West European standards. The analysis described in this paper was used in the preparation of the Polish standard PN-86/Z-08110/08 (Polski Komitet Normalizacji, Miar i Jakości, 1986).
2. The results of the analysis were used in creating technical documentation, construction, and operation of a modernized stand for percussive tests of industrial helmets in the Central Institute for Labour Protection in Łódź, Poland.
3. The obtained results were used in creating a rigid dummy for dynamic tests of personal protective equipment used to protect against falls from a height.

2. THE INFLUENCE OF THE COEFFICIENT OF RESTITUTION ON THE DYNAMICS OF THE COLLISION OF TWO BODIES

If two material bodies collide, big instantaneous forces $F(t)$, called the instantaneous force impulse, occur in a very short time

$$\Pi = \int_{t_0}^{t_0+\tau} F(t)dt \tag{1}$$

Determination of the value and the course of this impulse by analytical methods is very difficult. All known calculation methods are approximate. That is why, in addition to theoretical analyses, precise experimental proofs are always necessary. The proofs are difficult to accomplish because the impulse occurs in a very short time of 0.01 to 0.0001 s.

In order to characterize the degree of collision elasticity, Newton introduced into mechanics the so-called coefficient of restitution. The problem was then developed as the generalized collision theory by Hertz, Sztajerman, Saint Venant, Sears, and Timoshenko (Timoshenko & Goodier, 1951; Timoshenko & Young, 1962). The coefficient of restitution has the form of a proper function and shows which part of the impulse during the first phase is recovered in the second phase of the collision (a reduction of the instantaneous force)

$$k = \frac{\Pi_2}{\Pi_1} \tag{2}$$

where k is the coefficient of restitution, Π_1 is the force impulse in the first phase of the monotonic increase, Π_2 is the force impulse in the second phase of the monotonic decrease to zero.

The coefficient of restitution k can be defined most simply by using the collision of two bodies, preferably balls. The simplest situation occurs for a simple central collision of two balls.

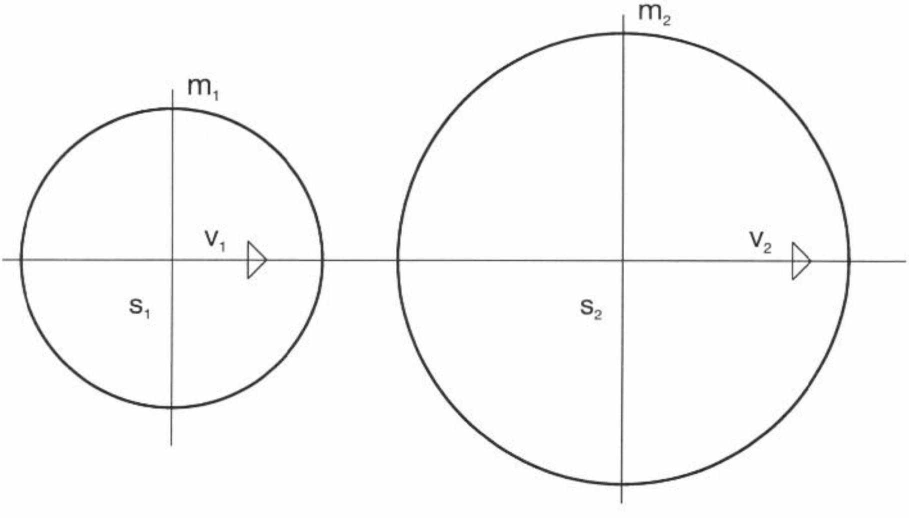

Figure 2. A simple central collision of two balls.

The first collision phase begins at the moment of the contact of the two colliding balls. It lasts until the local deformations at the contact point of the balls have caused speed equalization for the centres of the gravity. They now have the same velocity V. During this time, the interaction forces of the balls grow from zero to the maximal value.

The local deformations of the balls are reduced to zero in the second collision phase. Thus, the interaction force of the balls is reduced to zero. The common velocity V changes into two different values, V_1' and V_2'.

Using the equation of momentum, the following can be written for each ball

$$m_1 V - m_1 V_1 = -\Pi_1 \tag{3a}$$
$$m_2 V - m_2 V_2 = \Pi_1 \tag{3b}$$

where Π_1 denotes the reaction force impulse of the ball of mass m_1 on the ball of mass m_2 in the first collision phase.

On the basis of Equations 3a and 3b, the common velocity V has the following form

$$V = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2} \tag{4}$$

Denoting now as Π_2 the reaction force impulse of the ball of mass m_1 on the ball of mass m_2 in the second collision phase, the equation of momentum takes the following form

$$\begin{aligned} m_1 V_1' - m_1 V &= -\Pi_2 \\ m_2 V_2' - m_2 V &= \Pi_2 \end{aligned} \tag{5}$$

After introducing the coefficient of restitution k, the following can be expressed

$$\Pi_2 = k\Pi_1 \tag{6}$$

Using now Equations 3a, 3b, 4, and 5 in Equation 6, the values of the velocities for the balls after the collision have the following forms

$$V_1' = \frac{(m_1 - km_2) V_1 + (1 + k) m_2 V_2}{m_1 + m_2} \tag{7}$$

and

$$V_2' = \frac{(m_2 - km_1) V_2 + (1 + k) m_1 V_1}{m_1 + m_2} \tag{8}$$

From the difference between V_1' and V_2' comes, on the basis of Equations 7 and 8, after some transformations, the form of the coefficient of restitution

$$k = -\frac{V_1' - V_2'}{V_1 - V_2} \tag{9}$$

A number of very useful conclusions can be drawn from Equation 9:

1. Bodies after and before collision have the same relative velocity ratio, which depends on the two tested materials.
2. The coefficient of restitution k depends only on the materials of colliding masses, which determine their elastic properties.
3. The coefficient of restitution k does not depend on the velocity of masses m_1 and m_2 of the two colliding bodies.
4. The coefficient of restitution k does not depend on the duration of the collision.

3. THE INFLUENCE OF THE COEFFICIENT OF RESTITUTION k ON KINETIC ENERGY LOSS DURING THE COLLISION

Real bodies have the value of the coefficient of restitution k on a scale

$$0 \leqslant k \leqslant 1 \tag{10}$$

The values of the coefficient of restitution k for some materials with technical application have been listed in Table 2.

TABLE 2. Values of the Coefficient of Restitution *k*

Material of Striking Mass	Material of Struck Mass	*k*
glass	glass	.94
ivory	ivory	.85
cast iron	cast iron	.66
steel	steel	.56
wood	wood	.50
steel	lead	.20

The values referred to in Table 2 and in professional literature are defined with the following assumptions:

- both bodies are made of the same material,
- the striking body is assumed to be perfectly elastic and the struck body is made of a material listed in Table 2.

The influence of the coefficient of restitution on kinetic energy loss for the colliding masses is easy to define for a central collision. In this case, the mass strikes pointwise a stationary barrier, perpendicular to its surface. For a mass striking a stationary barrier, two values in Equation 9 are zero, that is,

$$V_2 = 0; \; V_2' = 0 \tag{11}$$

Equation 9, which defines the coefficient of restitution k, now has the following form

$$k = \frac{V_1'}{V_1}; \; V_1' = kV_1 \tag{12}$$

The difference of the kinetic energy after and before the collision is equal to

$$E_2 - E_1 = \frac{m_1(V_1')^2}{2} - \frac{m_1(V_1)^2}{2} = \frac{m_1k(V_1)^2}{2} - \frac{m_1(V_1)^2}{2} = \frac{m_1(V_1)^2}{2}(k^2 - 1) \tag{13}$$

On the basis of Equation 13, the smaller the coefficient of restitution k, the bigger the kinetic energy loss.

4. A CENTRAL COLLISION OF TWO BODIES

In the first collision phase, the colliding surfaces of the two bodies are in pointwise contact. The surfaces of the bodies at the contact point have a common normal line called the collision line. If the velocity vectors for the common points of the moving bodies are placed along the collision line, the collision is called a simple collision. For the velocity vectors inclined at an angle, the collision is called a slant collision.

During a simple collision, the instantaneous forces are situated along the collision line. If the centres of the colliding masses are placed on the collision line, the collision is called a central (middle or frontal) collision. If the centres of masses are not situated on the collision line, the collision is called an eccentric or off-centre collision.

Depending on the material elasticity of the colliding masses, there are three types of collisions: (a) elastic, (b) plastic, and (c) inelastic (elastoplastic) collisions.

During an elastic collision, there is no loss of kinetic energy. During a plastic collision, the whole kinetic energy is lost. During an elastoplastic collision, only part of the kinetic energy is lost. The lost part is transformed into heat.

In technology, practically only elastoplastic collisions are encountered. The value of kinetic energy loss can be found by means of the coefficient of restitution k. A change of the coefficient k causes a change of kinetic energy loss.

In order to execute model calculations, we can take into account a very easy, common case. A moving mass falls and strikes an elastic mass fixed on a heavy stationary base. All collisions of bodies in both vertical and horizontal planes can be explained in this way. The path of the centre of the falling freely mass is perpendicular to the surface of the base and it contains the centre of the falling mass. If this path contains the centre of the struck mass (and if the contact surface during the collision is spherical), the collision is called a central collision.

The impulse forces existing during the contact of colliding masses are internal forces. For this kind of collision, other forces can be disregarded. Now, the projection of the momentum vector on the axis connecting the centres of the masses has a constant value because the external forces are equal to zero. After the collision, the bodies begin to move with velocities V_1' and V_2' also directed along the line that connects the centres of masses. It follows that the momentum before and after the collision has the same value. Thus, we have the following equation describing the conservation of momentum law

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2' \tag{14}$$

Equation 14 has two unknowns, V_1' and V_2'. The second equation is obtained as an equation characterising the kind of existing collision, that is, elastic, plastic, or inelastic. It all depends entirely on the elasticity of the materials used in constructing the colliding bodies.

Let us analyze the particular collision cases of the masses: a central elastic and a central plastic collision. Neither is very often encountered in practice but this formulation is very convenient and can show the velocity range of the masses. It means the real existing velocities of the inelastic colliding masses should have the value from the calculated range.

5. A CENTRAL ELASTIC COLLISION OF MASSES

For a central collision, both colliding masses are treated as an isolated system. Their interactions are elastic, that is, conservative. The principles of the conservation of momentum and of the conservation of energy must be fulfilled for this system

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2' \tag{15a}$$

$$\frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2} = \frac{m_1 (V_1')^2}{2} + \frac{m_2 (V_2')^2}{2} \tag{15b}$$

The velocities of the two bodies after the collision can be obtained by solving Equations 15a and 15b

$$V_1' = -\frac{m_2 - m_1}{m_2 + m_1} V_1 + \frac{2m_2}{m_2 + m_1} V_2 \tag{16}$$

and

$$V_2' = \frac{m_2 - m_1}{m_2 + m_1} V_2 + \frac{2m_1}{m_2 + m_1} V_1 \tag{17}$$

6. A CENTRAL PLASTIC COLLISION OF MASSES

The mechanical energy of the system changes during an inelastic collision. Part of the energy turns into thermal energy, which increases the temperature of the colliding bodies. If the whole kinetic energy of the moving balls, in relation to the centre of the mass of the system, turns into heat, the central collision of masses is called a plastic collision. After the collision, both masses must have the same velocity V_s' as the centre of the mass of the system. Similarly to an elastic collision, the conservation of momentum law has the form of Equation 18

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2' \tag{18}$$

The following expression occurs for an ideal inelastic collision

$$V_1' = V_2' = V_s' \tag{19}$$

After solving this system of equations, the velocity of the centre of the mass for the system after the collision has the form

$$V_s' = \frac{m_1V_1 + m_2V_2}{m_1 + m_2} \tag{20}$$

7. A CENTRAL INELASTIC COLLISION OF MASSES

During a central inelastic collision, both colliding masses begin to move immediately after the collision, with velocities V_1' and V_2', respectively. They are directed along the line connecting the centres of masses. It follows that the momentum for the system before and after collision has the same value

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2' \tag{21}$$

The same equation of the conservation of momentum law can be obtained similarly as in section 2 from Equations 3a, 3b, 5, and 6, which describe the momentum before and after the collision and the coefficient of restitution k.

During an inelastic collision, only part of the kinetic energy is lost and turned into heat that increases the temperature of both colliding masses. In this instance, another equation is necessary in order to solve the problem and it defines the relations between velocities V_1' and V_2' immediately after the collision. It can be obtained by means of the coefficient of restitution k showing which part of the kinetic energy is lost during the collision and transformed into heat. The ratio of the relative velocities for both bodies after and before the collision is equal to the coefficient of restitution k. For a mass falling freely on a stationary mass fixed on an elastic base, the second equation has the form

$$k = -\frac{V_1' - V_2'}{V_1 - V_2} \tag{22}$$

The velocities of the bodies after the collision can be obtained by using both Equations 21 and 22, and they now have the form described by Equations 7 and 8 in section 2.

8. KINETIC ENERGY LOSS E_{ks} FOR THE STRIKING MASS DURING THE COLLISION

During the collision, each of the colliding masses moving with the specified velocity loses part of its kinetic energy. The lost kinetic energy is used for elastoplastic deformation and it is turned into heat.

Kinetic energy loss for two colliding bodies, with velocities before and after the collision $V_1 V_1'$ and $V_2 V_2'$, respectively, can be obtained from the expression

$$E_{ks} = \frac{1}{2} m_1 \left[V_1^2 - (V_1')^2 \right] + \frac{1}{2} m_2 \left[V_2^2 - (V_2')^2 \right] \tag{23}$$

In order to solve Equation 23, Equations 7 and 8 are used for the components of velocities V_1' and V_2'. Thus, the general expression determining kinetic energy loss during the collision of the masses is obtained here.

If a moving mass m_1 strikes a stationary mass m_2, both Equation 23 and Equations 7 and 8 can be simplified. The calculations according to Equation 23 are simplified and shortened. This kind of percussive action of the moving mass m_1 on the stationary mass m_2 can be used for testing safety helmets. The velocity for the struck mass m_2 at the moment of the strike by mass m_1 is equal to zero ($V_2 = 0$). Thus, Equations 7 and 8 defining the velocities of the masses after the collision have the following forms

$$V_1' = V_1 \frac{m_1 - k m_2}{m_1 + m_2} = \sqrt{2E_{k1}}\, \frac{\sqrt{m_1}\left(1 - k\dfrac{m_2}{m_1}\right)}{m_1 + m_2} \tag{24}$$

and

$$V_2' = V_1 \frac{(1 + k) m_1}{m_1 + m_2} = \sqrt{2E_{k1}}\, \frac{\sqrt{m_1}\,(1 + k)}{m_1 + m_2} \tag{25}$$

The last components on the right side of Equations 24 and 25 have been obtained by using the principle of the equality of kinetic and potential energy $E_{k1} = E_{p1}$ and the expression $V_1 = \sqrt{2gh_1}$. The equation defining kinetic energy loss for the striking mass now has the form

$$E_{ks} = \frac{1}{2} m_1 \left[V_1^2 - (V_1')^2 \right] - \frac{1}{2} m_2 (V_2')^2 \tag{26}$$

After using Equations 24 and 25 in Equation 26, the simplified form after some transformations can be described as follows

$$E_{ks} = \frac{m_1 V_1^2}{2} (1 - k^2) \frac{m_2}{m_1 + m_2} \tag{27}$$

Equation 27 can be expressed after simple transformations as a function of kinetic energy

$$E_{ks} = E_{k1}(1 - k^2) \frac{m_2}{m_1 + m_2} \tag{28}$$

Equation 27 can be also expressed differently, as a function of potential energy

$$E_{ks} = E_{p1}(1 - k^2) \frac{m_2}{m_1 + m_2} \tag{29}$$

An analysis of Equations 28 and 29 permits the conclusion that the value of the falling mass (defined by the symbol m_1) has a fundamental influence on the kinetic energy loss of the striking mass. Thus, for the constant value of the kinetic energy of the falling mass m_1

$$E_{k1} = \text{const} \tag{30}$$

changing the fall height of mass m_1 for the same value of the momentum

$$mh = m_1 h_1 \tag{31}$$

the forge impulse changes. This means the effect of its action on the struck mass m_2 changes, too.

9. KINETIC ENERGY E_{k2} OF THE STRUCK MASS m_2 IMMEDIATELY AFTER THE STRIKE

As a result of the strike of the falling mass m_1, the stationary mass m_2 begins to move with a certain velocity. The velocity of mass m_2 obtained this way depends both on the coefficient of restitution k and on kinetic energy loss E_{ks} of the striking mass m_1.

The kinetic energy of the struck mass m_2 can be expressed as follows

$$E_{k2} = \frac{1}{2} m_2 (V_2')^2 \tag{32}$$

The velocity of mass m_2 after the strike by mass m_1 can be described by Equation 8

$$E_{k2} = \frac{1}{2} m_2 \left[\frac{(1 + k) m_1 V_1}{m_1 + m_2} \right]^2 \tag{33}$$

After some transformations, Equation 33 can be expressed as follows

$$E_{k2} = E_{k1} (1 + k)^2 \frac{m_1 m_2}{(m_1 + m_2)^2} \tag{34}$$

The expression defining the kinetic energy of the struck mass m_2 can be described differently. After transformations similar to those of Equation 27, Equation 33 has the following form

$$E_{k2} = E_{p1} (1 + k)^2 \frac{m_1 m_2}{(m_1 + m_2)^2} \tag{35}$$

An analysis of Equations 34 and 35 permits the conclusion that the kinetic energy E_{k2} of the struck mass m_2 depends on the value of the falling mass m_1. The influence of mass m_1 is very big because it appears both in the numerator and in the denominator of Equations 34 and 35. If the value of the kinetic energy of the falling mass is constant (E_{k1} = const) the fall height of the striking mass has an influence on the energy of the struck mass m_2 and the effect of the interaction of the masses (see Equation 31).

10. THE ENERGY BALANCE OF THE COLLISION

Mass m_1 falling vertically or moving along another rectilinear path has, at the moment of contact with the stationary mass m_2, kinetic energy proportional to the value of the mass and the square power of the velocity at the moment of the strike. The energy of the striking mass m_1 from the moment of the contact of both masses until the moment of contact loss after the collision can be divided into three parts. These parts depend on (a) the values of masses m_1 and m_2, (b) mass ratio (m_1/m_2), and (c) the elastic properties of the used materials defined by the coefficient of restitution k.

The three parts of energy existing thanks to the division of the energy of the striking mass m_1 can be expressed in the following form

- The striking mass m_1 rebounds off the surface of mass m_2 or moves forward. In both cases m_1 reduces the velocity and conserves part of its kinetic energy. Thus, the energy can be calculated from the expression

$$E'_{k1} = \frac{1}{2} m_1 (V'_1)^2 \tag{36}$$

The expression determining the energy conserved by the striking mass m_1 can be obtained using Equation 7

$$E'_{k1} = E_{k1} \left(\frac{m_1 - km_2}{m_1 + m_2} \right)^2 \tag{37}$$

- The second part of the energy appears from the effect of a force impulse and it is turned into the work of body deformation depending on the elasticity of the bodies. This work is turned into heat. Energy loss E_{ks} can be calculated using Equation 28.
- The third part of kinetic energy is transferred to mass m_2. The kinetic energy received by mass m_2 during the collision can be calculated using Equation 34.

The sum of kinetic energies before and after the collision together with the energy lost during the collision must be equal to

$$E_{k1} = \mathrm{E}'_{k1} + E_{ks} + \mathrm{E}_{k2}$$

$$E_{k1} = E_{k1} \left(\frac{m_1 - km_2}{m_1 + m_2} \right)^2 + E_{k1} (1 - k^2) \frac{m_2}{m_1 + m_2} + E_{k1} (1 + k)^2 \frac{m_1 m_2}{(m_1 + m_2)^2} \tag{38}$$

The equation defining the kinetic energy of mass m_1 at the moment of striking mass m_2 can be obtained after some transformations as the following identity

$$E_{k1} = E_{k1} \frac{(m_1 + m_2)^2}{(m_1 + m_2)^2} = E_{k1} \tag{39}$$

The form of Equation 43 confirms the correctness of each equation used to calculate kinetic energy distribution during the collision.

11. DYNAMIC FORCE F_d, TOTAL FORCE P_c, AND THEIR INFLUENCE ON THE BASE OF MASS m_2

After collision with mass m_1, the struck mass m_2 has the velocity, according to Equation 29,

$$V_2' = \frac{(1 + k) m_1 V_1}{m_1 + m_2} \tag{40}$$

On the other hand, the kinetic energy of mass m_2 can be expressed as follows

$$E_{ks} = \frac{1}{2} m_2 (V_2')^2 \tag{41}$$

Mass m_2 is fixed on a stationary base on the elastic element. The elastic element is pressed or lengthened under the influence of the kinetic energy of mass m_2. This can be described by the compression or elongation value λ_d. Thus, the force transmitted by the elastic element onto the base grows by the value F_d

$$\lambda_d = \frac{F_d l}{E_2 A} \tag{42}$$

where l denotes the length or the thickness of the elongated elastic element, E_2 is Young's modulus, and A—the cross-section of the elongated or compressed element or the pressure surface for the pressure on the plane.

Energy E_{k2} transmitted onto the elastic element turns into the energy of elastic deformation E_{spr}

$$E_{spr} = \frac{1}{2}F_d\lambda_d = \frac{1}{2}\frac{F_d^2 l}{E_2 A} \tag{43}$$

Let us assume the conservative system for which the conservation of energy law must be fulfilled. The expression defining the value of the dynamic force F_d can be now obtained by comparing Equations 41 and 43

$$\frac{1}{2}m_2(V_2')^2 = \frac{1}{2}F_d\lambda_d = \frac{1}{2}\frac{F_d^2 l}{E_2 A} \tag{44}$$

After solving Equation 44, the expression defining F_d can be expressed in the following three forms

$$F_d = \frac{m_2 V_2'}{\sqrt{\dfrac{m_2 l}{E_2 A}}} = \frac{m_2 V_2'}{\sqrt{\dfrac{G_2 l}{g E_2 A}}} = \frac{P_2}{\sqrt{\dfrac{\lambda_s}{g}}} \tag{45}$$

where P_2 denotes the momentum of mass m_2 after the strike by mass m_1 and λ_s is the static strain of the elastic element caused by the weight $G_2 = m_2$ g.

The total pressure force on the stationary base P_c is the sum of the static load by the weight G_2 and dynamic load F_d

$$P_c = G_2 + F_d \tag{46}$$

The final form, after some transformations is

$$P_c = G_2 + \frac{(1 + k)V_1}{\sqrt{\dfrac{\lambda_s}{g}}}\frac{m_1 m_2}{m_1 + m_2} \tag{47}$$

Equation 47 can be expressed depending on the kinetic energy E_{k1} of mass m_1 at the moment of the collision with mass m_2. To this end we use the well-known expressions describing the equality of kinetic and potential energy $E_{k1} = E_{p1}$ and consequently the equation $V_1 = \sqrt{2gh_1}$

$$P_c = G_2 + \sqrt{E_{k1}}\frac{(1 + k)\sqrt{2}}{\sqrt{\dfrac{\lambda_s}{g}}}\frac{\sqrt{m_1}\, m_2}{m_1 + m_2} \tag{48}$$

12. AN ANALYSIS OF THE RESULTS OF THEORETICAL STUDIES

From the theoretical studies shown here, it follows that the collision process of two bodies can be described by five functions. They are given by Equations 25, 28, 34, 37, and 48. All five expressions contain four components: E_{k1}, m_1, m_2, and k. Those components can be divided into three groups depending on the way they were chosen. There are well-known, assumed, and calculated components dependent on the known and assumed components:

- m_2—Both the mass and the material used for the product are known. This is the initial component in the determination of all the other components.
- k—The coefficient of restitution is known, because the material, the construction of the product, and the material used for the falling mass m_1 are defined.
- E_{k1}—Kinetic energy depends on the kind of material and the value of mass m_2 of the tested product.
- m_1—This component can be defined by analysing the constant value of the kinetic energy E_{k1}.

Components m_2, E_{k1}, and k are determined univocally. They are chosen to realize a concrete purpose and cannot be optionally changed. The situation is different when determining mass m_1. Component m_1 can be changed on a large scale. There are three reasons for this:

- the equality of kinetic and potential energy $E_{k1} = E_{p1}; \frac{1}{2} m_1 V_1^2 = m_1 g h_1$,
- the velocity $V_1 = \sqrt{2gh_1}$ of mass m_1 at the moment of collision, dependent on the fall height h_1,
- the assumed constant value E_{k1} = const dependent on the kind and value of mass m_2 for the tested product.

From those reasons it follows that mass m_1 and velocity V_1 can be changed optionally. This does not change the value of kinetic energy E_{k1}. However, the expression that follows from the equality of energies $E_{k1} = E_{p1}$ must be fulfilled

$$mh = m_1 h_1 = \mathrm{m}_2 h_2 = \ldots = \text{const} \tag{49}$$

Mass m_1 occurs also as an independent component in all five expressions. It is necessary to study for the tested product, if the change of mass m_1 with the constant value of kinetic energy E_{k1} has an influence on the course and value of the energy, forces, and velocities of both masses m_1 and m_2.

In order to study the influence of the change of mass m_1 on the course of the functions expressed by Equations 25, 28, 34, 37, and 48, the derivatives of the five functions in relation to the variable parameter m_1 are determined.

- The derivative of the velocity V_2' for mass m_2 struck by mass m_1 according to Equation 25 is equal to

$$\frac{dV_2'}{dm_1} = \sqrt{2E_{k1}}\,(1 + k)\,\frac{m_2 - m_1}{2\sqrt{m_1}\,(m_1 + m_2)^2} = 0 \text{ for } m_1 = m_2$$

$$\frac{dV_2'}{dm_1^2}(m_1 = m_2) = \sqrt{2E_{k1}}\,(1 + k)\,\frac{3m_1\,(m_1 - 2m_2) - m_2^2}{4\sqrt{m_1^3}\,(m_1 + m_2)^3} < 0 \rightarrow \text{maximum} \tag{50}$$

Function V_2' has the maximum for $m_1 = m_2$.

For $\frac{dV_2'}{dm_1}\;(0 < m_1 < m_2) > 0$, V_2' is an increasing function.

For $\frac{dV_2'}{dm_1}\;(m_1 > m_2) < 0$, V_2' is a decreasing function.

- The derivative of kinetic energy loss E_{ks} for the striking mass m_1 according to Equation 28 is

$$\frac{dE_{ks}}{dm_1} = (1 - k^2)E_{k1}\,\frac{-m_2}{(m_1 + m_2)^2} < 0 \tag{51}$$

For all values $m_1 > 0$, E_{ks} is a decreasing function. When m_1 grows, E_{ks} decreases.

- The derivative of kinetic energy E_{k2} for mass m_2 after the strike by mass m_1, according to Equation 34 is

$$\frac{dE_{k2}}{dm_1} = (1 + k)^2 E_{k1}\,\frac{m_2(m_2 - m_1)}{(m_1 + m_2)^2} = 0 \text{ for } m_1 = m_2$$

$$\frac{dE_{k2}}{dm_1}(m_1 = m_2) = (1 + k)^2\,E_{k1}\,\frac{-2m_2\,(2m_2 - m_1)}{(m_1 + m_2)^4} < 0 \rightarrow \text{maximum} \tag{52}$$

Function E_{k2} has the maximum for $m_1 = m_2$.

For $\dfrac{dE_{k2}}{dm_1}$ $(0 < m_1 < m_2) > 0$, E_{k2} is an increasing function.

For $\dfrac{dE_{k2}}{dm_1}$ $(m_1 > m_2) < 0$, E_{k2} is a decreasing function.

- The derivative of kinetic energy E'_{k1} for mass m_1 after the strike by mass m_2, according to Equation 37 is

$$\frac{dE'_{k1}}{dm_1} = 2(1 + k)E_{k1}\frac{m_2(m_1 - km_2)}{(m_1 + m_2)^3} = 0 \text{ for } m_1 = km_2$$

$$\frac{d^2E'_{k1}}{dm_1^2}(m_1 = km_2) = 2(1 + k)E_{k1}\frac{m_2[m_2(3k + 1) - 2m_1]}{(m_1 + m_2)^4} > 0 \rightarrow \text{maximum} \tag{53}$$

Function E'_{k1} has the minimum for $m_1 = km_2$.

For $\dfrac{dE'_{k1}}{dm_1}$ $(0 < m_1 < km_2) > 0$, E'_{k1} is a decreasing function.

For $\dfrac{dE'_{k1}}{dm_1}$ $(m_1 > km_2) > 0$, E'_{k1} is an increasing function.

- The derivative of the total pressure force P_c of mass m_2 on the stationary base, according to Equation 48 is

$$\frac{dP_c}{dm_1^2} = \sqrt{E_{k1}}\frac{\sqrt{2}(1 + k)}{\sqrt{\dfrac{\lambda_s}{g}}}\,\frac{m_2(m_2 - m_1)}{2\sqrt{m_1}(m_1 + m_2)^2} = 0 \text{ for } m_1 = m_2$$

$$\frac{d^2P_c}{dm_1^2}(m_1 = m_2) = \sqrt{E_{k1}}\frac{\sqrt{2}(1 + k)}{\sqrt{\dfrac{\lambda_s}{g}}}\,\frac{m_2\left[3m_1(m_1 - 2m_2) - m_2^2\right]}{4\sqrt{m_1^3}(m_1 + m_2)^3} < 0 \rightarrow \text{maximum} \tag{54}$$

Function P_c has the maximum for $m_1 = m_2$.

For $\dfrac{dP_c}{dm_1}$ $(0 < m_1 < m_2) > 0$, P_c is a decreasing function.

For $\dfrac{dP_c}{dm_1}$ $(m_1 > m_2) < 0$, P_c is an increasing function.

The realized analysis of the results of theoretical studies has demonstrated that for all five functions expressed in Equations 25, 28, 34, 37, and 48, the following are true

1. The variations and values of the determined five functions depend on four components: E_{k1}, m_1, m_2, and k.
2. The value of the fifth function (Equation 48) depends on λ_s—the coefficient of compression or elongation of an elastic element that transfers an impulse of mass m_2 on the base—as well as on E_{k1}, m_1, m_2, and k.

13. CONCLUSIONS

1. The whole process of the percussive test of a product can be described by five functions: V_2' (the velocity of the struck mass m_2 of the product), E_{ks} (kinetic energy loss of the striking mass m_1 during the collision with mass m_2), E_{k2} (the kinetic energy of mass m_2 after the collision with mass m_1), E_{k1}' (the kinetic energy of mass m_1 after the collision with mass m_2), and P_c (the total pressure force of the struck mass m_2 on the stationary base).
2. The parameters determining the values and the courses of the five functions are E_{k1}(the kinetic energy of mass m_1 at the moment of the strike by mass m_2), m_1 (the mass of the falling body), m_2 (the mass of the tested product), and k (the coefficient of restitution for the colliding masses m_1 and m_2). The same parameters and λ_s (the coefficient of deformation for an elastic element that transfers an impulse of mass m_2 on the base) have an influence on the value of the total pressure force P_c.
3. Three parameters, E_{k1}, m_1, and k, must be listed in the standards regulating percussive tests of products, which are also used in the construction of testing equipment. This is necessary in order to obtain identical results using equipment constructed by different producers for testing the same products. There is a group of parameters characterising both the falling body (E_{k1} and m_1) and the kind of collision (k). If one of these three parameters changes, it will cause a change of the force for the falling mass m_1. Thus, it changes consequently the value and the distribution of all dynamic forces acting during the collision. Two other parameters describing the reaction of the tested mass after the strike (m_2 and P_c) are not taken

into account in the imposed standard. They can give us some information concerning the tested product and the test stand and will be very useful to analyse the results of the collision.

4. Test results of the same product according to standards DIN 4840:1981 (Deutsches Institut für Normung, 1981) and PN-86/Z-08110/08 (Polski Komitet Normalizacji, Miar i Jakości, 1986) cannot be compared with the results obtained by using the three other standards listed in Table 1. The collision force of the falling mass m_1 striking the stationary product m_2 is greater for the conditions described in both the DIN and the PN standards. The differences reach 18%. Thus, the dynamic strength and dynamic requirements for the tested product are greater for those two standards, and the difference reaches 18%, too.
5. In order to compare the same products of the different countries it is necessary to determine the standard, the test stand, and the parameters of the dynamic test of the certified products. For the existing differences of both the used standards and the parameters of the test stand, comparison calculations according the theory described in this paper can be made.

The analysis of the collision problem should be developed as an analysis of the courses of all five parameters and a practical application of the discussed parameters characterising the strike (both E_{k1}, m_1, and k, and m_2 and P_c) for the real existing test conditions. It should determine the courses and the practical influence of all discussed parameters on the real strike between the punch and the tested product (the helmet) during shock absorption or resistance to the penetration test.

REFERENCES

British Standard. (1987). *Industrial safety helmets. Part 1: Specification for construction and performance* (Standard No. BS 5240:1987). London: British Standard Institution.

Comité Européen de Normalisation. (1995). *Industrial safety helmets* (Standard No. EN 397:1995). Brussels, Belgium: Author.

Deutsches Institut für Normung. (1981). *Industrial safety helmets* (Standard No. DIN 4840:1981). Berlin, Germany: Author. (In German).

Gryboś, H. (1970). *Collision theory in discrete mechanical systems*. Warsaw, Poland: Państwowe Wydawnictwa Naukowe. (In Polish).

Hoppmann, W.H., II. (1988). Effects of impact on structures. In C.M. Harris (Ed.) *Shock and vibrations* (pp. 9.1–9.11). New York: Mc Graw-Hill.

International Organization for Standardization. (1977). *Industrial safety helmets* (Standard No. ISO 3873:1977). Geneva, Switzerland: Author.

Kowalski, J. (1976). *Mechanical problems with applications to the calculations of machine elements*. Warsaw, Poland: Państwowe Wydawnictwo Naukowe. (In Polish).

Leyko, J. (1975). *Theoretical mechanics*. Warsaw, Poland: Państwowe Wydawnictwo Naukowe. (In Polish).

Polski Komitet Normalizacji, Miar i Jakości. (1979). *Test methods of the resistance to shock and resistance to penetration and determination of the shock absorption properties of the helmets* (Standard No. PN-79/Z-08020). Warsaw, Poland: Wydawnictwa Normalizacyjne. (In Polish).

Polski Komitet Normalizacji, Miar i Jakości. (1986). *Industrial safety helmets. Test methods* (Standard No. PN-86/Z-08110/08). Warsaw, Poland: Wydawnictwa Normalizacyjne. (In Polish).

Timoshenko, S., & Goodier, J.N. (1951). *Theory of elasticity*. New York: Mc Graw-Hill.

Timoshenko, S., & Young, D. (1962). *Engineering mechanics*. New York: Mc Graw-Hill.