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ANALYSIS OF CORRELATION AND SPECTRAL CHARACTERISTICS OF FRACTAL COMB-STRUCTURED SIGNALS

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Abstract. *The paper presents theoretical and experimental investigations of the spectra of fractal comb-structured signals for different values of their coefficient of generation, as well as analyzes autocorrelation characteristics and phase portraits depending on the coefficient of generation. It is established that signal base increases with increasing fractal order and with decreasing coefficient of generation.*

Keywords: fractal, signal analysis, spread spectrum communication

ANALIZA CHARAKTERYSTYK KORELACYJNYCH I SPEKTRALNYCH SYGNAŁÓW FRAKTALNYCH O STRUKTURZE GRZEBIENIOWEJ

Streszczenie. *W pracy przedstawiono teoretyczne i eksperymentalne badania widm sygnałów fraktalnych o strukturze grzebieniowej dla różnych wartości ich współczynnika generowania, a także przeprowadzono analizę charakterystyk autokorelacji i portretów fazowych w zależności od współczynnika generowania. Ustalono, że baza sygnału wzrasta wraz ze wzrostem rzędu fraktali oraz ze spadkiem współczynnika generowania.*

Słowa kluczowe: fraktal, analiza sygnału, komunikacja (łączność).za pomocą widma (rozproszonego) (szerokiego)

Introduction

The comb structure of fractal signals suggested in [4] with square-shaped elementary fragments simplifies considerably their generation process and hardware implementation of generators with the use of microcontrollers as a common radioelement base.

The shape of proposed fractal comb-structured signals enables hardware implementation without the use of smoothing filter at the output of digital-to-analog converter (DAC). To generate a seventh-order fractal, it is sufficient to use an eight-bit DAC, and to store its w-th order pulse readings, it is necessary to save in memory 2^w two-byte signal code symbols. Reproduction of one elementary fragment of fractal signal is assured by one clock of machine time.

The purpose of this paper is to analyze spectral and correlation characteristics, as well as to establish the possibility of using the generated signal for data transmission under severe electromagnetic conditions.

1. Fractal comb-structured signals

A fractal comb-structured signal is composed of electric pulses of equal duration separated by equal time intervals [4]. Pulse amplitudes are formed in conformity with self-similarity laws. The structure of fractal signals of the first four orders with the coefficient of generation $m = 1/2$ is given in fig. 1.

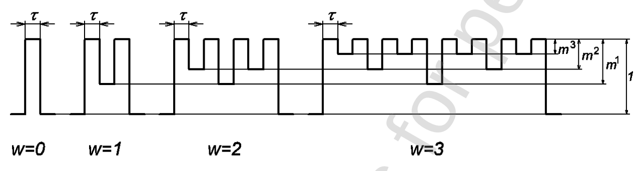


Fig. 1. Consecutive steps of fractal pulses generation with equal duration of elementary pulses with fractal orders W = 0, 1, 2, 3

A w-th order fractal is composed of $2^{w+1}-1$ elementary fragments of equal duration τ that can be divided into $w+1$ orders according to their values. Pulse amplitude of l-th level elementary fragments is m^l.

Based on the fractal structure, its mathematical model can be represented as a sum of pulse sequences with different amplitudes:

$$x(t)=\sum_{l=0}^{w} A_l \cdot \left(\sum_{k=1}^{2^{w-l}} U_l^k(t) \right) = \sum_{l=0}^{w} A_l \cdot \left\{ \sum_{k=1}^{2^{w-l}} \left[\theta\left(t-t_l^k\right) - \theta\left(t-t_l^k-\tau\right) \right] \right\} \quad (1)$$

where: w – fractal order, l – fractal level number, k – sequence number of l-th level pulse in a fractal, A_l – amplitude of l-th level elementary pulse, $U_l^k(t)$ – k-th rectangular pulse of l-th level unit amplitude, t_l^k – pulse start moment $U_l^k(t)$, $\theta(t)$ – Heaviside function, τ – elementary pulse duration (equal for all pulses).

Using direct Fourier transform for fractal comb-structured signal, its spectral density can be written as follows:

$$S(\omega)=\frac{A_0}{\omega}\cdot\sin\alpha_0-\frac{2}{\omega}\cdot\left\{\sum_{l=1}^{w} A_0\cdot\left[1-(m)^{w+1-l}\right]\cdot \sum_{k=1}^{2^{w-l-2}}\left[\sin\left(\alpha_0+\alpha_l^k\right)+\sin\left(\alpha_0-\alpha_l^k\right)\right]\right\} = \frac{A_0}{\omega}\cdot\sin\alpha_0\left\{1-\sum_{l=1}^{w}\left[1-(m)^{w-l+1}\right]\cdot\sum_{l=1}^{2^{w-l-2}}\cos\alpha_l^k\right\} \quad (2)$$

where $\alpha_0 = 0.5\cdot\omega\cdot\tau$.

2. Analysis of fractal comb-structured signal

A numerical calculation of spectral density, autocorrelation function and phase portrait will be done by the example of third-order fractal comb-structured signal with the coefficient of generation ½ and elementary fragment duration 2 μsec on condition of absence of noises in the channel the time diagram of which is given in fig. 2.

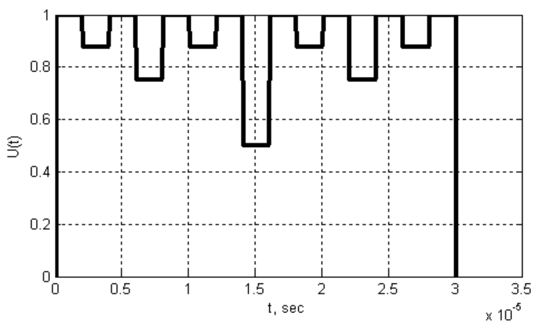


Fig. 2. Time diagram of fractal comb-structured signal of third order w = 3, m = 1/2, τ = 2 μs

The spectral density of signals considered (Fig. 3) is similar to the amplitude spectrum of square-shaped signals with pronounced local maxima at the frequencies multiple of the fundamental frequency. The harmonic components are grouped under the spectral envelope of elementary pulse.

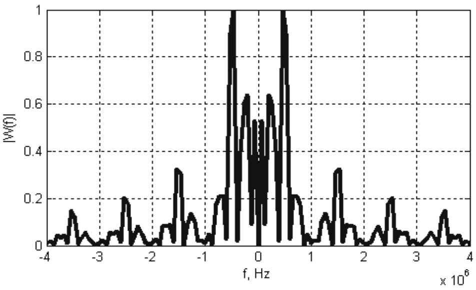
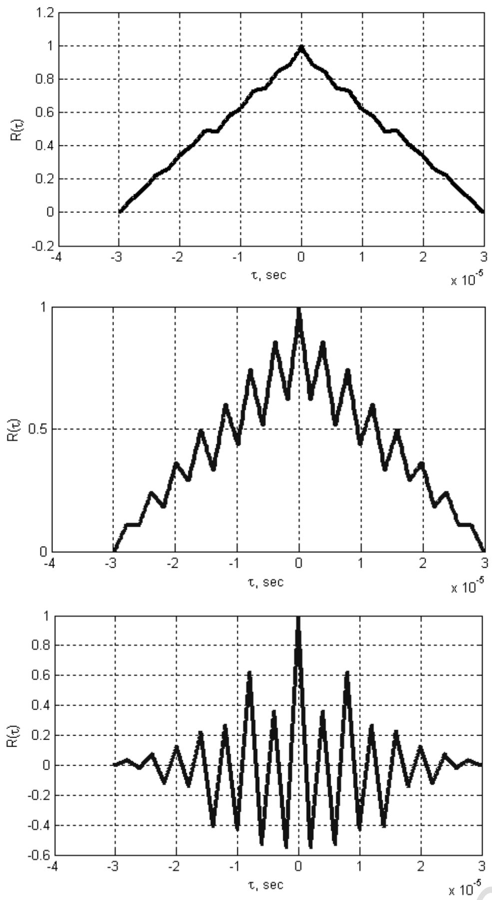


Fig. 3. Spectral density of third-order fractal comb-structured signal with the coefficient of generation m = 0.5 (direct component is removed from the plot)



a)

b)

c)

Fig. 4. Autocorrelation function of third-order fractal comb-structured signal with the coefficient of generation: a) m = 0.5; b) m = 0.8; c) m = 0.5 (direct component is removed from the signal)

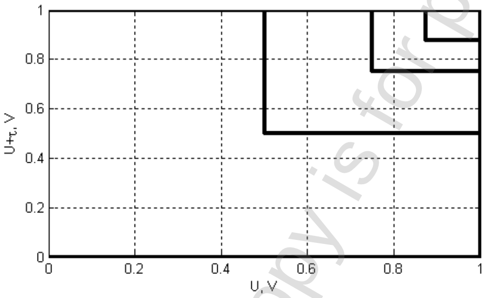


Fig. 5. Phase portrait of third-order fractal comb-structured signal with the coefficient of generation m = 0.5

Autocorrelation function of fractal comb-structured signal is similar to autocorrelation function of a rectangular pulse for the coefficient of generation $m = 0 \div 0.5$ (Fig. 4a).

With the values of the coefficient of generation within $m = 0.5 \div 1$, autocorrelation function has saw-tooth oscillations (Fig. 4b). In case of direct component removal from fractal comb-structured signal, its autocorrelation function is of the form of autocorrelation function of the oscillation function (Fig. 4c).

From the analysis of phase portrait of fractal comb-structured signal it follows (Fig. 5) that fractal signals belong to regular clock signals.

3. Experimental investigations of fractal comb-structured signal properties

The experimental investigation of the above fractal comb-structured signals was pursued with the use of our coder [5, 6] whose functional electrical diagram is shown in fig. 6.

The main element of the coder performing the functions of programmable hardcore is a microcontroller (D1) whose clock frequency is assigned by quartz resonator ZQ1.

PIC18F2550 microcontroller used in the circuit can be connected to computer through USB port, providing for data transmission from PC to the coder.

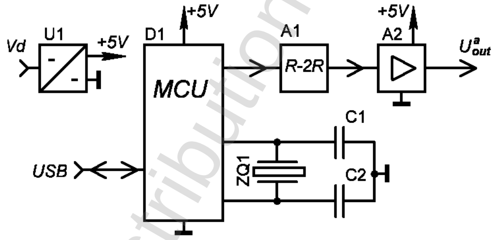


Fig. 6. Functional electrical diagram of the coder

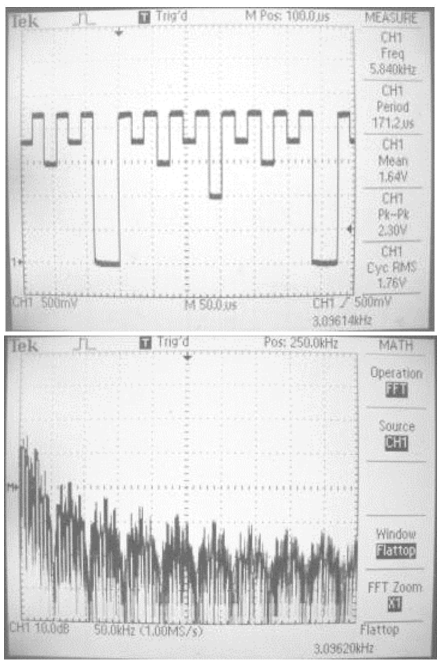
From the table fed into microcontroller memory, code sequence whose structure depends on fractal level is associated with the bytes transferred from PC.

Conversion of code sequence into fractal pulse signal of given shape is done by means of transistor keys of microcontroller port that serve as current switches of digital-to-analog converter and R-2R resistive array (A1). The necessary number of bits is defined by the order of fractal pulse (fractal comb-structured signal). For instance, for generation of fourth-order fractal comb-structured signal four coder bits will be sufficient.

Matching of the output resistance of R-2R array with the wave resistance of coaxial cable over which information fractal comb-structured signals are transmitted was achieved through use of operational amplifier (A2).

Fig. 7 shows the time diagram and spectrum of third-order fractal comb-structured signal obtained with a TEKTRONIX oscilloscope.

As can be seen, experimental results correlate well with theoretical calculations.



a)

b)

Fig. 7. Third-order fractal comb-structured signal: a) oscillogram; b) spectrum

4. Signal base and noise immunity factor

Signal base is one its main characteristics whereby one can judge on the immunity of data transfer lines to all kinds of noise.

A broadband signal is one whose modal bandwidth (the product of signal bandwidth by its duration $B = F \cdot T_S$) is more than unity [3].

In our case the duration of a fractal comb-structured signal pulse is a function of its order and is determined as follows:

$$T_S = \tau\left(2^{w+1} - 1\right) \quad (3)$$

where w – fractal order, τ – elementary pulse duration which is identical for all pulses).

In estimating the base of proposed signals, their spectral bandwidth was determined by noise equivalent method by the relation [2]

$$F_{eff} = \frac{1}{G_{\max}} \int_0^{\infty} G\ (f) df \quad (4)$$

where F_{eff} – effective signal bandwidth, $G_{\max}$ – maximum power spectral density of signal, $G\ (f)$ – signal spectral density.

Calculated dependence of the base of fractal comb-structured signals on their order for different values of formation factor is given in fig. 8.

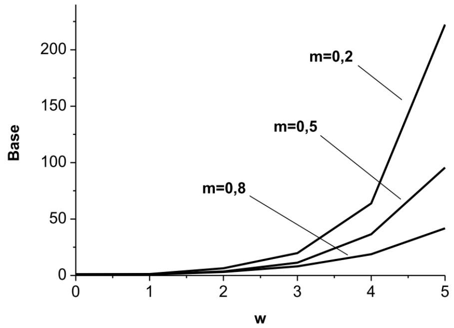


Fig. 8. Dependence of fractal signal base on its order for different values of the coefficient of generation

From the obtained results it follows that signals of this class are broadband and their base is proportional to that of broadband signals.

The values of noise immunity factor which characterizes the level of recovery of operating signals from their mixture with various kinds of noise by means of correlation device [1] were found from the expression:

$$\lambda = \int_0^{T_S} s^2(t)dt \Bigg/ \left(\int_0^{T_S} s^2(t)dt + \left| \int_0^{T_S} s(t) \cdot n(t)dt \right| \right) \quad (4)$$

where $\int_0^{T_S} s^2(t)dt$ – zero autocorrelation function of operating signal, $\int_0^{T_S} s(t) \cdot n(t)dt$ – zero cross-correlation function of operating signal and noise signal, $s(t)$ – useful signal, $n(t)$ – noise signal.

Calculated dependence of noise immunity factor of fractal comb-structured signal on its order for different values of the coefficient of generation is given in fig. 9.

In the calculations it was assumed that the powers of fractal comb-structured signal and white noise were identical.

As is evident from the plot (Fig. 8), the base of fractal comb-structured signal increases with increasing its order w, however, with increasing the coefficient of generation m, it goes down. The curves of noise immunity factor (Fig. 9) are close to each other and approach unity with increasing order of fractal comb-structured signals.

On elimination of direct component from fractal comb-structured signal, there is a reduction in the value of noise immunity factor and signal base.

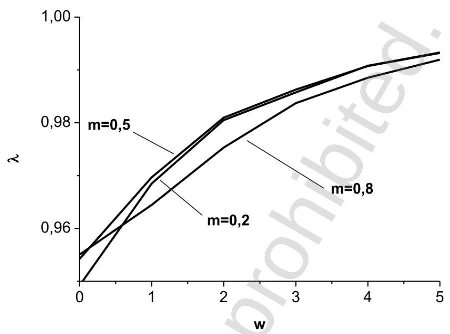


Fig. 9. Dependence of fractal signal noise factor on its order for different values of the coefficient of generation

5. Conclusions

Based on the results obtained, it can be concluded that fractal signals formed with the aid of the coder in question are broadband, and on their basis one can create communication systems that are capable of assuring high level of data protection against the impact of external electromagnetic noise.

In communication systems it is reasonable to use the proposed signals with the coefficient of generation $m = 0.2 \div 0.5$ and the order of w greater than three.

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