# Application of Tunable Distributed Mass Dampers in Beams Subjected to Random Excitation with Peaked PSD

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**Abstract**

The problem of vibrations of the Euler-Bernoulli beam of variable cross-section with the attached distributed mass dampers, subjected to random excitations with peaked Power Spectral Densities, is presented in the paper. The problem of the beam vibrations is solved using the Galerkin method, the Lagrange's equations of second kind and the Laplace time transformation. The Power Spectral Densities of the beam deflection are determined. Numerical example presents some optimization problem of distributed mass dampers position for global objective function adopted.

***Keywords:*** distributed mass damper, beam vibration, random excitation, peaked PSD

## 1. Introduction

In civil engineering tuned mass dampers are used in slender structures of low internal damping: suspension-bridges [1], high-rise buildings [2], chimneys [3], towers of wind power plants [4], where the vibrations are caused by wind flow or seismic ground motion. They are also used in road and railway bridges [5] or footbridges [6], excited by the vehicles or pedestrian traffic. Besides these low-frequency applications intended to prevent dangerous structure vibrations, high-frequency mass dampers are used to reduce the structure borne noise (railway rails, steel bridges). Many applications concern the typical industrial problems of reducing vibrations induced by operations of compressors, gensets, electric motors.

The main aim of tuned mass dampers is the reduction of vibrations at the places of their attachment [7]. The most commonly used are passive TMDs, many theoretical studies are devoted to problems of selecting their optimal positions and parameters [8-11]. The systems of TMDs are applied in many cases, tuned to one or few frequencies, depending on the excitation force bandwidth [12, 13]. The dampers with distributed parameters can be considered as a special case of MTMDs (Multiple Tuned Mass Dampers) [14, 15].

Due to various applications, many papers were devoted to optimal choice of mass dampers positions and parameters in beams [16-18]. In the continuous system usually the best position of a damper is the place of application of the concentrated excitation force, but for the continuous load and global optimization issue the problem becomes more complex.

Because of the high sensitivity of mass dampers to inaccurate tuning (especially the dynamic vibration absorbers without damping) it is important to take into account random excitations rather than harmonic. Due to the high sensitivity of discrete mass dampers to inaccurate localization, the use of distributed mass dampers may be more effective.

In the paper the analysis of vibrations of a beam subjected to random excitations with peaked power spectral densities (PSD), with the attached distributed mass damper, is presented. The stochastic excitation of the assumed type reflects well the features of forces occurring in reality [19, 20].

The aim of the paper is to develop a computational algorithm allowing to determine the power spectral density for the displacement (and other dependent variables) at an arbitrary point of the beam. This allows to solve various optimization problems, depending on the objective function adopted.

The numerical example presents the optimization of the tunable distributed mass damper position for the global measure of vibration, as the objective function the root mean square value (RMS) of the whole beam acceleration is assumed.

## 2. Theoretical model

The Euler-Bernoulli beam with viscous damping described by the Voigt-Kelvin rheological model is assumed. Figure 1 presents a beam of length $l$, cross-section area $A(x)$, geometrical moment of inertia $I(x)$, mass density $\rho(x)$, Young modulus $E(x)$, damping coefficient $\alpha(x)$. The beam is subjected to harmonic continuous excitation $g(x,t)$ and *p* harmonic concentrated forces $P_j(t)$ applied at $x_j^0$. The system considered is equipped with *r* distributed tuned mass dampers [15].

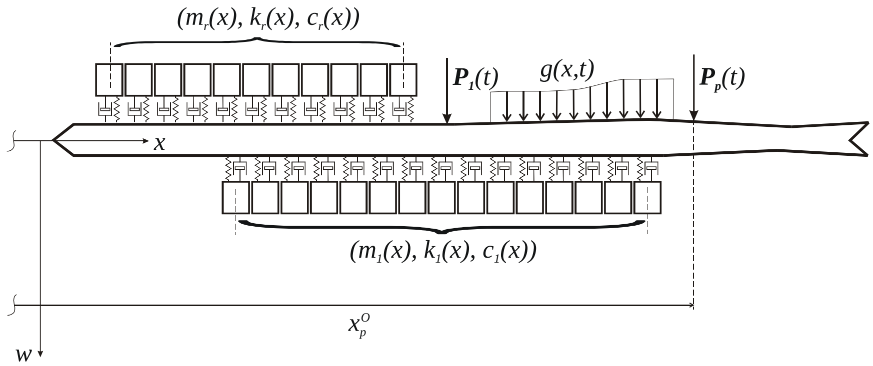


Figure 1. Beam with the system of distributed mass dampers

The beam deflection is assumed in the form of the functional series:

$$w(x,t) = \sum_{i=1}^{n} q_i(t)\varphi_i(x) \tag{1}$$

where $\varphi_i(x)$ are the modes of vibrations of the prismatic beam without attached mass dampers, for the given arbitrary boundary conditions. Time dependent functions $q_i(t)$ are generalized coordinates to be determined.

Applying the Lagrange's equations of second kind the system of differential equations is obtained:

$$\sum_{j=1}^{n} m_{ij}\ddot{q}_j + \sum_{j=1}^{n} b_{ij}\dot{q}_j + \sum_{j=1}^{n} k_{ij}q_j = H_i(t), \quad i = 1\ldots n \tag{2}$$

where the occurring numerical coefficients are given by the formulas [15]:

$$m_{ij} = \int_0^l \rho(x)A(x)\varphi_i(x)\varphi_j(x)dx \tag{3}$$

$$k_{ij} = \int_0^l E(x)I(x)\varphi_i''(x)\varphi_j''(x)dx \tag{4}$$

$$b_{ij} = \int_0^l E(x)I(x)\alpha(x)\varphi_i''(x)\varphi_j''(x)dx \tag{5}$$

The generalized force $H_i(t)$ is given by the expression:

$$H_i(t) = \sum_{k=1}^{p} P_k(t)\varphi_i(x_k^O) + \int_0^l g(x,t)\varphi_i(x)dx + \sum_{k=1}^{r}\int_0^l f_k(x,t)\varphi_i(x)dx \tag{6}$$

where $f_k(x,t)$ is the continuous load applied to the beam, originated from the *k*-th distributed damper.

After performing the Laplace time transformation with zero initial conditions the following linear algebraic system of equations is obtained:

$$\sum_{j=1}^{n} m_{ij}s^2Q_j(s) + \sum_{j=1}^{n} b_{ij}sQ_j(s) + \sum_{j=1}^{n} k_{ij}Q_j(s) = H_i(s), \quad i = 1\ldots n \tag{7}$$

The transform of the generalized force is given by the formula:

$$H_i(s) = \sum_{k=1}^{p} P_k(s)\varphi_i(x_k^O) + \int_0^l g(x,s)\varphi_i(x)dx + \sum_{k=1}^{r}\int_0^l f_k(x,s)\varphi_i(x)dx \tag{8}$$

where $P_k(s)$, $g(x,s)$ and $f_k(x,s)$ are the Laplace transforms of functions $P_k(t)$, $g(x,t)$ and $f_k(x,t)$ respectively.

The transform of the continuous force acting on the beam from the *k*-th distributed mass damper (with zero initial conditions) is given by the expression:

$$f_k(x,s) = -\frac{(c_k(x)s + k_k(x))m_k(x)s^2}{m_k(x)s^2 + c_k(x)s + k_k(x)}\sum_{j=1}^{n} Q_j(s)\varphi_j(x) \tag{9}$$

where $m_k(x)$, $k_k(x)$, $c_k(x)$ denote the linear densities of the mass, stiffness and damping coefficient of the *k*-th distributed mass damper respectively.

The system of linear algebraic equations for determining the unknown transforms $Q_i(s)$ may be written in the form:

$$\sum_{j=1}^{n}\left(m_{ij}s^2+b_{ij}s+k_{ij}+\sum_{k=1}^{r}F_{ij}^{k}(s)\right)Q_j(s)=\sum_{k=1}^{p}P_k(s)\varphi_i(x_k^0)+G_i(s),\quad i=1\ldots n \tag{10}$$

where the following notations are introduced:

$$F_{ij}^{k}(s)=\int_0^l \frac{\left(c_k(x)s+k_k(x)\right)m_k(x)s^2}{m_k(x)s^2+c_k(x)s+k_k(x)}\varphi_i(x)\varphi_j(x)$$
$$G_i(s)=\int_0^l g(x,s)\varphi_i(x) \tag{11}$$

Finally the beam deflection transform $W(x,s)$ is given by the expression:

$$W(x,s)=\sum_{i=1}^{n}Q_i(s)\varphi_i(x) \tag{12}$$

Assuming the steady state vibrations, after substituting $s=i\omega$ ($i=\sqrt{-1}$) the beam deflection in the frequency domain may be obtained.

## 3. Random excitation

Usually the problems of the mass dampers selection assumes harmonic excitation, but often the real forces are of random character. It is assumed in the paper that the beam is subjected to the random excitation being the stationary stochastic process with peaked power spectral density [19, 20].

The peaked PSD may be defined by the formula:

$$S_Y=\frac{2\beta\overline{x}}{\sqrt{\pi}}\left(\beta\Omega\right)^2\exp\left(-\left(\beta\Omega\right)^2\right),\ \Omega=\frac{\omega}{\omega_1} \tag{13}$$

where: $\overline{x}=2\int_0^{+\infty}S_Y(\Omega)d\Omega$. The maximal value of the expression (13) occurs for $\Omega=\frac{1}{\beta}$, i.e. for $\omega=\frac{\omega_1}{\beta}$.

The relation between the density functions of the random processes $Y(t)$ and $Z(t)$ takes the form:

$$S_Z(\omega)=\left|T(i\omega)\right|^2 S_Y(\omega) \tag{14}$$

where $T(i\omega)$ denotes the dynamic transfer function between input $Y(i\omega)$ and output $Z(i\omega)$ of the linear system.

The power spectral densities of random processes $\dot{Z}(t)$ and $\ddot{Z}(t)$ are given by:

$$S_{\dot{Z}}(\omega) = \omega^2 \left|T(i\omega)\right|^2 S_Y(\omega) \tag{15}$$

$$S_{\ddot{Z}}(\omega) = \omega^4 \left|T(i\omega)\right|^2 S_Y(\omega) \tag{16}$$

## 4. Numerical calculations results

The computational algorithm created allows to perform calculations for a beam with variable physical properties . The exemplary numerical calculations were performed for the system presented in Fig. 2. Steel prismatic cantilever beam of a rectangular cross-section ( 0.05 m width and 0.005 m height) has: length $l = 1.0$ m, mass density $\rho = 7800$ kg/m$^3$, Young modulus $E = 2.1 \cdot 10^{11}$ N/m$^2$ , internal damping is neglected.

The beam is subjected to homogeneous random excitation: $g(x,t) = H(t) g_0$, ( $g_0 = const$ ) within the segment: $\langle 0.3L, 0.8L \rangle$ , with the distributed mass damper attached in the range: $\langle x_1, x_2 \rangle$. Here $H(t)$ is a stationary stochastic process with the power spectral density function $S_H(\omega)$ given by expression of type (13).

It is assumed that the distributed mass damper parameters are constant along its length: $m(x) = const$, $c(x) = const$, $k(x) = const$. The total mass of the distributed damper is taken as 5% of the beam mass, i.e. 0.098 kg.

The first two natural vibrations frequencies of the considered beam are equal: $f_1 = 4.19$ Hz, $f_2 = 26.26$ Hz.

The Laplace transform of the beam deflection is given by relation:

$$W(x,s) = T(x,s) H(s) \tag{17}$$

so its power spectral density may be written as:

$$S_W(x,\omega) = \left|T(x,i\omega)\right|^2 S_H(\omega) \tag{18}$$

The following parameters describing the power spectral density $S_H(\omega)$ of the continuous load stochastic component are taken: $\bar{x} = 1.0$, $\beta = 1.0$.

## 5. Minimization of the average beam acceleration - tunable distributed vibration damper

It is assumed that the distributed mass damper applied is tuned exactly to the peak frequency of the power spectral density function. Such tunable damper formally is of a semi-active type. The value of the linear density damping coefficient $c(x)$ results from assumption that the damping ratio is taken as always equal 0.1. Damping is added to the system due to the high sensitivity of the dynamic vibration eliminators (without damping) to mistuning, which is essentially important for random loading comprising a wide range of the excitation frequency. In the numerical calculations twenty components of the series (1) were taken ( $n = 20$ ).

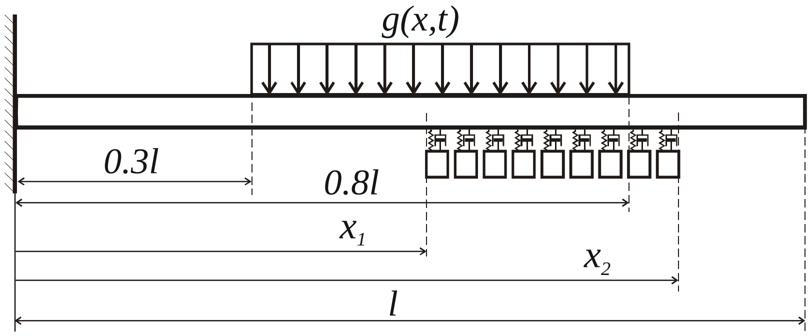


Figure 2. Beam with the attached tunable mass damper

The optimization problem is stated here as looking for the optimal position $x_1$ of the damping segment, for its given length , which minimizes the root mean square value of the average acceleration of the whole beam. The damper mass, stochastic process peak parameter, frequency ratio and damping ratio are the input parameters for the optimization procedure. The objective function is defined by the formula:

$$F(x_1) = \sqrt{\omega^4 S_H(\omega)\left(\int_0^l \left|T(x,i\omega)\right|^2 dx\right) d\omega} \tag{19}$$

The numerical calculations were carried out for cases: $x_2 - x_1 = 0.3$, $x_2 - x_1 = 0.1$, $x_2 - x_1 = 0$ (i.e. the discrete damper). Root mean square values (RMS) of the beam average acceleration versus the peak frequency of the power spectral density function are presented in diagrams of Fig. 3, for different widths of the distributed mass damper. The numbers in the figure represent the values of $x_1$ and $x_2$ (Fig. 2).

The damper segment width may be limited due to technical issues. It turns out, however, that there are frequency ranges where the discrete damper is the optimal solution. The best configuration for a given peak frequency, i.e. the width and position of the distributed mass damper, can be obtained by determining the envelope of the curves shown in Fig. 3.

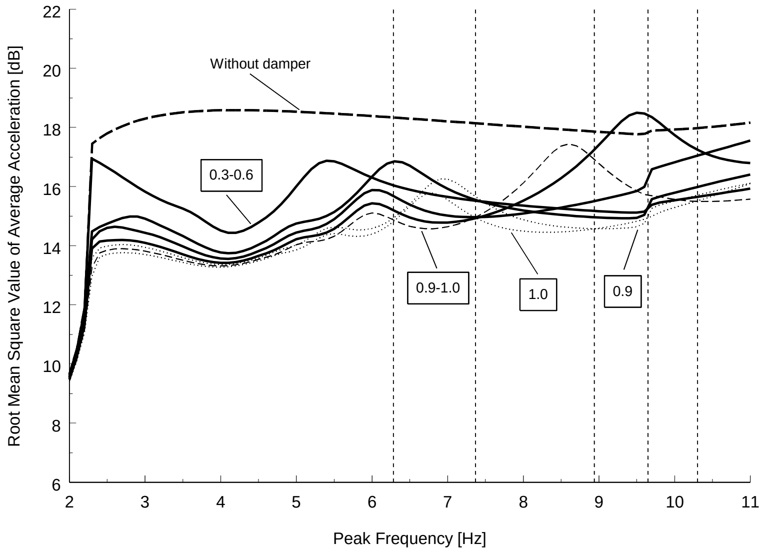


Figure 3. Root mean square values of the average beam acceleration vs. peak frequency. The damper is tuned exactly to the peak frequency of the power spectral density with parameter $\beta = 1.0$ . Damping ratio of the damper equals to 0.1

## 6. Conclusions

In the paper the computational model, which can be used for finding the optimal parameters of the distributed mass dampers in beams subjected to random excitations, is presented. It was assumed in the numerical calculations that excitation was the stationary stochastic process with the peaked power spectral density function, in many cases well describing the real forces. The calculation results show the effectiveness of vibration suppression by mounting the distributed mass dampers.

The distributed mass dampers can be used in vibration damping of other continuous structures like frames, curved beams, plates and shells when placement of one or a few discrete dampers of significant masses may be technically difficult.

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